ACOUSTICS OF BELLS

Edited by

Thomas D. Rossing

Benchmark Papers in Acoustics Series

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ACOUSTICS OF BELLS

Edited by

THOMAS D. ROSSING
Northern Illinois University

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SERIES EDITOR'S FOREWORD

The "Benchmark Papers in Acoustics" constitute a series of volumes that make available to the reader in carefully organized form important papers in all branches of acoustics. The literature of acoustics is vast in extent and much of it, particularly the earlier part, is inaccessible to the average acoustical scientist and engineer. These volumes aim to provide a practical introduction to this literature since each volume offers an expert's selection of the seminal papers in a given branch of the subject—that is, those papers that have significantly influenced the development of that branch in a certain direction and introduced concepts and methods that possess basic utility in modern acoustics as a whole. Each volume provides a convenient and economical summary of results as well as a foundation for further study for both the person familiar with the field and the person who wishes to become acquainted with it.

Each volume has been organized and edited by an authority in the area to which it pertains. Each volume provides an editorial introduction summarizing the technical significance of the field being covered. Each article is accompanied by editorial commentary, with necessary explanatory notes, and an adequate index is provided for ready reference. Articles in languages other than English are either translated or abstracted in English. It is the hope of the publisher and editor that these volumes will constitute a working library of the most important technical literature in acoustics and value to students and research workers.

The present volume, *Acoustics of Bells,* has been edited by Thomas D. Rossing of Northern Illinois University. Professor Rossing is a well-known teacher and research investigator in acoustics and is a Fellow of the Acoustical Society of America. In its 26 carefully selected seminal articles this volume covers thoroughly the development of the researches on the acoustical properties of bells from the end of the last century to the present. Particular emphasis is placed on the modes of vibration of bells and the search for the strike note. The history of the subject is not overlooked, but nearly half of the articles refer to research done during the past two decades. Each of the eight groups of articles into which the volume is divided is provided not only with an introduction but also with editorial comments on

the significance of the papers in the group and their relation to other papers in the volume. Both in its coverage and method of presentation this volume should command the enthusiastic attention of all workers in acoustics and all devotees of music.

R. BRUCE LINDSAY

PREFACE

Although the subject of this volume would appear to be narrower in scope than most of the previously published Benchmark volumes, it is a subject that has attracted writers from many different countries of the world. I have found well in excess of 100 books and papers dealing with the acoustics of bells.

Unfortunately, many of the most significant papers have been published in journals that are not generally found in American libraries. A number of them are in Dutch or German. Thus it seemed that a collection of these papers or their English translations would be of considerable interest to carillonneurs, acousticians, bell founders, and others.

I wish to thank Wilfried Reinicke, Andre Lehr, Hervey Bagot, Miss J. Sanderson, Tim Martin, and Robert Perrin for calling my attention to several papers, and to Hera Leighton and R. Selmen for their translations. My own studies of bell acoustics have benefited greatly from associations with André Lehr, Robert Perrin, John Sathoff, and my graduate students in acoustics.

THOMAS D. ROSSING

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ACOUSTICS OF BELLS

INTRODUCTION

The bell dates from prehistoric times. One of the world's oldest bells, found near Babylon, is thought to be about 3000 years old. A set of 65 bronze bells, discovered in the Chinese province of Hubei in 1978, dates from the fifth century B.C. Bells of some type were important to most ancient civilizations.

Ancient bells tended to be elongated cylinders with either straight or convex sides. The sound bow, which added mechanical strength and a louder sound, appeared about the sixth century. The flared mouth and thick sound bow of the modern Western bell apparently developed during the fourteenth century.

Bells developed as musical instruments in the fifteenth and sixteenth centuries, when bell founders learned how to tune the partials harmonically. Founders in the Low Countries took the lead in tuning bells; their fine bells and bell towers remain to this day a source of national pride.

In the seventeenth century, Jacob van Eijck, Dutch carillonneur, discovered that the partials of a bell could be excited individually, and the brothers Francois and Pieter Hemony used tuned rods to measure the partials of a bell during tuning. Many of the finest bells in the Low Countries today were tuned by the Hemonys.

Although in theory it is possible to cast a bell exactly to ring with the desired harmonic partials, it is more practical to cast a bell slightly thicker than necessary and turn it down on a bell lathe. This practice has been followed since the seventeenth century. Each partial has its own vibration pattern, and thinning the bell at any particular place will change the frequencies of the partials by varying amounts. The bell tuner, therefore, must know exactly how frequencies will change with thinning at each location. Herein lies much of the art of producing fine bells.

Because of the strategic importance of their constituent metals, bells fare rather badly in time of war. World War II was no exception. The German army removed many historic bells from bell towers in the Low Countries, France, Poland, and other countries as well as in Germany itself. Those not destroyed were rehung after the war, but

in the meantime many were studied in the laboratory. Results of many of these studies are reported by E. W. van Heuven (Paper 9).

The publication of van Heuven's thesis renewed interest in the acoustics of bells, especially in Germany and The Netherlands. Acoustical measurements became more precise using electronic equipment developed in the 1950s. Foundries in England and the Low Countries were overwhelmed with orders to replace bells destroyed during World War II, and bell founding prospered.

BELL RINGING

The earliest bells were rung singly, to call people to worship, to herald a great event, to warn of fire or flood or enemy attack. Later, sets of tuned bells were hung in towers to announce the time of day or to play tunes. During the fourteenth and fifteenth centuries two special techniques for ringing bells developed in Flanders and England.

The Flemish developed a remote keyboard mechanism that allowed musicians to perform on sets of bells, or carillons, as they came to be called. The clavier of a carillon consisted of wooden rods, played with the hands or feet, to which ropes or rods were attached to operate the clappers. This technique has not changed greatly to the present day.

About the same time, English bell ringers perfected systems for "change-ringing" bells. The basic strategy is to ring a set or peal of bells in all possible sequences, to move in an orderly fashion from sequence to sequence, and to avoid repeating any sequence. (There are 24 possible ways to ring 4 bells, but with 8 bells the number grows to 40,320, and with 12 bells 479 million sequences are possible!) To control the ringing rate, and thus make it possible to ring bells of different sizes in a sequence, bells must be mounted to swing in full circle. Most English church bells are hung that way.

HANDBELLS

Although the earliest bells were probably hand-held, tuned handbells as we know them developed in England during the eighteenth century, primarily to give tower bell ringers a convenient instrument on which to practice change ringing.

Eventually, handbells came to be used for ringing tunes, and bands of ringers appeared all over England. Handbell ringing has become very popular in this century, and over two thousand handbell choirs are reported in the United States alone.

THE FUTURE

Advances in digital computers have made it possible, for the first time in over one hundred years of research on the acoustics of bells, to calculate modes of vibration of real bells. The results of one such calculation are reported in Paper 8. Further refinements of finite element analysis and other numerical methods can be expected in the future.

Equally important to bell researchers has been the development of sophisticated real-time spectrum analyzers. These instruments allow acousticians to study the spectral characteristics of transient, as well as steady-state, sounds. Many of these analyzers can feed data directly into digital computers and graphics terminals, which can be programmed to diagram modes of vibration, for example.

It is somewhat ironic that these significant advances in theoretical and experimental techniques come at a time when relatively little research is being done on the acoustics of bells. It is unlikely that any large research effort will develop in the near future, as the market for new bells will probably remain small. Yet the interest in research on bell acoustics will continue as long as bells hold their unique fascination, which has lasted for centuries.

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Part I

EARLY INVESTIGATIONS

Editor's Comments on Papers 1, 2, and 3

1 **STRUTT (LORD RAYLEIGH)**
On Bells

2A **SIMPSON**
On Bell Tones

2B **SIMPSON**
On Bell Tones. No. II

3 **JONES**
The Vibrations of Bells

The acoustical properties of bells received Lord Rayleigh's attention in the latter part of the nineteenth century. In Paper 1 he reported on his experiments with eight bells, including five in his own parish church at Terling. He was able to identify the pitches of four to six partials in each bell by comparing them to the notes of a harmonium, and he correctly concluded that the nominal pitch of each bell lay an octave below the fifth partial. The bells were not good examples of the tuning art, however, and he noted many dissonances and imperfect relationships between partials. He correctly predicted the possibility of bringing important partials into a harmonic relationship by careful design, apparently unaware that the Dutch masters had worked on this over 200 years earlier.

The author of Paper 2, an English clergyman, was aware of the fine bells in the Low Countries, and his investigations were intended to discover why most English bells failed to measure up to them in quality of musical sound. His criticism of English founders, as well as his suggestions for improving bell tuning, apparently were heeded, for English bells became the equal of any in musical quality.

The pioneer of bell acoustics in the United States was Professor A. T. Jones of Smith College, who published many papers on the subject. In Paper 3, his second on the subject, he reported relative frequencies of ten partials measured in the bells of the Carlile Chime at Smith College and the Harkness Chime at Yale University. He concluded that the first five partials of the large Taylor bells in the

Harkness Chime are tuned harmonically, but those of the smaller Meneely bells in the Carlile Chime are not. (The imperfect partials of the Carlile bells are quite similar to those Rayleigh found in the Terling peal.)

By carefully examining each bell with a stethoscope connected to a Helmholtz resonator, Jones mapped the nodal lines for the first seven partials. He also discussed the origin of the strike note. Of the several hypotheses he suggested, the one he considered second most likely is the one ultimately accepted by most authorities.

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1

Reprinted from *Philos. Mag.* **29**:1–17 (1890)

ON BELLS

J. W. Strutt (Lord Rayleigh)

THE theory of the vibrations of bells is of considerable difficulty. Even when the thickness of the shell may be treated as very small, as in the case of air-pump receivers, finger-bowls, claret glasses, &c., the question has given rise to a difference of opinion. The more difficult problem presented by church bells, where the thickness of the metal in the region of the *sound-bow* (where the clapper strikes) is by no means small, has not yet been attacked. A complete theoretical investigation is indeed scarcely to be hoped for; but one of the principal objects of the present paper is to report the results of an experimental examination of several church bells, in the course of which some curious facts have disclosed themselves.

In practice bells are designed to be symmetrical about an axis, and we shall accordingly suppose that the figures are of revolution, or at least differ but little from such. Under these circumstances the possible vibrations divide themselves into classes, according to the number of times the motion repeats itself round the circumference. In the gravest mode, where the originally circular boundary becomes elliptical, the motion is once repeated, that is it occurs *twice*. The number of nodal meridians, determined by the points where the circle intersects the ellipse, is *four*, the meridians corresponding (for example) to longitudes 0° and 180° being reckoned separately. In like manner we may have 6, 8, 10 . .

nodal meridians, corresponding to 3, 4, 5 . . . cycles of motion. A class of vibrations is also possible which are symmetrical about the axis, the motion at any point being either in or perpendicular to the meridional plane. But these are of no acoustical importance.

The meaning here attached to the word *nodal* must be carefully observed. The meridians are not nodal in the sense that there is no motion, but only that there is no motion *normal to the surface.* This can be best illustrated by the simplest case, that of an infinitely long thin circular cylinder vibrating in two dimensions*. The graver vibrations are here purely flexural, the circumference remaining everywhere unstretched during the motion. If we fix our attention upon one mode of vibration of n cycles, the motion at the surface is usually both radial and tangential. There are, however, $2n$ points distributed at equal intervals where the motion is purely tangential, and other $2n$ points, bisecting the intervals of the former, where the motion is purely radial. There are thus no places of complete rest; but the first set of points, or the lines through them parallel to the axis, are called nodal, in the sense that there is at these places no normal motion.

The two systems of points have important relations to the place where the vibrations are excited. "When a bell-shaped body is sounded by a blow, the point of application of the blow is a place of maximum normal motion of the resulting vibrations, and the same is true when the vibrations are excited by a violin-bow, as generally in lecture-room experiments. Bells of glass, such as finger-glasses, are, however, more easily thrown into regular vibration by friction with the wetted finger carried round the circumference. The pitch of the resulting sound is the same as that elicited by a tap with the soft part of the finger; but inasmuch as the tangential motion of a vibrating bell has been very generally ignored, the production of sound in this manner has been felt as a difficulty. It is now scarcely necessary to point out that the effect of the friction is in the first instance to excite tangential motion, and that the point of application of the friction is the place where the tangential motion is greatest, and therefore where the normal motion vanishes"†.

When the symmetry is complete, the system of nodal meridians has no fixed position, and may adapt itself so as to suit the place at which a normal blow is delivered. If the

* 'Theory of Sound,' § 232.

† 'Theory of Sound,' § 234. That the rubbing finger and the violin-bow must be applied at different points in order to obtain the same vibration was known to Chladni.

point of application of the blow be conceived to travel round a circle symmetrical with respect to the axis, say, for brevity, a circle of *latitude*, the displacement will make no difference to the vibration considered as a whole, but the effect upon an observer who retains a fixed position will vary. If the bell be situated in an open space, or if the ear of the observer be so close that reflexions are relatively unimportant, the sound disappears as nodes pass by him, swelling to a maximum when the part nearest to the ear is one of the places of maximum normal motion, which for brevity we will call *loops*. In listening to a particular note it would thus be possible to determine the number of nodal meridians by watching the variations of intensity which occur as the place of the blow travels round a circle of latitude.

In practice the symmetry is seldom so complete that this account of the matter is sufficient. Theoretically the slightest departure from symmetry will in general render determinate the positions of the nodal systems. For each number of cycles n, there is one determinate mode of vibration with $2n$ nodes and $2n$ intermediate loops, and a second determinate mode in which the nodes and loops of the first mode exchange functions. Moreover the frequencies of the vibrations in the two modes are slightly different.

In accordance with the general theory, the vibrations of the two modes as dependent upon the situation and magnitude of the initiating blow are to be considered separately. The vibrations of the first mode will be excited, unless the blow occur at a node of this system; and in various degrees, reaching a maximum when the blow is delivered at a loop. The intensity, as appreciated by an observer, depends also upon the position of his ear, and will be greatest when a loop is immediately opposite. As regards the vibrations of the second mode, they reach a maximum when those of the first mode disappear, and conversely.

Thus in the case of n cycles, there are $2n$ places where the first vibration is not excited and $2n$ places, midway between the former, where the second vibration is not excited. At all $4n$ places the resulting sound is free from beats. In all other cases both kinds of vibration are excited, and the sound will be affected by beats. But the prominence of the beats depends upon more than one circumstance. The intensities of the two vibrations will be equal when the place of the blow is midway between those which give no beats. But it does not follow that the audible beats are then most distinct. The condition to be satisfied is that the intensities shall be equal *as they reach the ear*, and this will depend upon the situation

of the observer as well as upon the vigour of the vibrations themselves. Indeed, by suitably choosing the place of observation it would be theoretically possible to obtain beats with perfect silences, wherever (in relation to the nodal systems) the blow may be delivered.

There will now be no difficulty in understanding the procedure adopted in order to fix the number of cycles corresponding to a given tone. If, in consequence of a near approach to symmetry, beats are not audible, they are introduced by suitably loading the vibrating body. By tapping cautiously round a circle of latitude the places are then investigated where the beats disappear. But here a decision must not be made too hastily. The inaudibility of the beats may be favoured by an unsuitable position of the ear, or of the mouth of the resonator in connexion with the ear. By travelling round, a situation is soon found where the observation can be made with the best advantage. In the neighbourhood of the place where the blow is being tried there is a loop of the vibration which is most excited and a (coincident) node of the vibration which is least excited. When the ear is opposite to a node of the first vibration, and therefore to a loop of the second, the original inequality is redressed, and distinct beats may be heard even although the deviation of the blow from a nodal point may be very small. The accurate determination in this way of two consecutive places where no beats are generated is all that is absolutely necessary. The ratio of the entire circumference of the circle of latitude to the arc between the points represents $4n$, that is four times the number of cycles. Thus, if the arc between consecutive points proved to be 45°, we should infer that we were dealing with a vibration of two cycles—the one in which the deformation is elliptical. As a greater security against error, it is advisable in practice to determine a larger number of points where no beats occur. Unless the deviation from symmetry be considerable, these points should be uniformly distributed along the circle of latitude*.

In the above process for determining nodes we are supposed to hear distinctly the tone corresponding to the vibration under investigation. For this purpose the beats are of assistance in directing the attention; but with the more difficult subjects, such as church bells, it is advisable to have recourse to resonators. A set of Helmholtz's pattern, manufactured by Koenig, are very convenient. The one next higher

* The bells, or gongs, as they are sometimes called, of striking clocks often give disagreeable beats. A remedy may be found in a suitable rotation of the bell about its axis.

in pitch to the tone under examination is chosen and tuned by advancing the finger across the aperture. Without the security afforded by resonators, the determination of the octave is in my experience very uncertain. Thus pure tones are often estimated by musicians an octave too low.

Some years ago I made observations upon the tones of various glass bells, of which the walls were tolerably thin. A few examples may be given :—

I.	c',	$e''\flat$,	$c'''\sharp$.
II.	a,	$c''\sharp$,	b''.
III.	$f'\sharp$,	b''.	

The value of n for the gravest tone is 2, for the second 3, and for the third 4. On account of the irregular shape and thickness only a very rough comparison with theory is possible ; but it may be worth mention that for a thin uniform hemispherical bell the frequencies of the three slowest vibrations should be in the ratios

$$1 : 2{\cdot}8102 : 5{\cdot}4316 ;$$

so that the tones might be

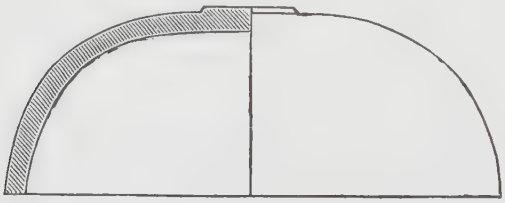
$$c, \quad f'\sharp, \quad f'', \text{ approximately.}$$

More recently, through the kindness of Messrs. Mears and Stainbank, I have had an opportunity of examining a so-called hemispherical metal bell, weighing about 3 cwt. A section is shown in fig. 1. Four tones could be plainly heard,

$$e\flat, \quad f'\sharp, \quad e'', \quad b'',$$

the pitch being taken from a harmonium. The gravest tone

Fig. 1.



has a long duration. When the bell is struck by a hard body, the higher tones are at first predominant, but after a time they die away, and leave $e\flat$ in possession of the field. If the striking body be soft, the original preponderance of the higher elements is less marked.

By the method above described there was no difficulty in showing that the four tones correspond respectively to $n=2$, 3, 4, 5. Thus for the gravest tone the vibration is elliptical with 4 nodal meridians, for the next tone there are 6 nodal meridians, and so on. Tapping along a meridian showed that the sounds became less clear as the edge was departed from, and this in a continuous manner with no suggestion of a nodal circle of latitude.

A question, to which we shall recur in connexion with church bells, here suggests itself. Which of the various coexisting tones characterizes the pitch of the bell as a whole? It would appear to be the third in order, for the founders give the pitch as E nat.

My first attempts upon church bells were made in September 1879, upon the second bell (reckoned from the highest) of the Terling peal; and I was much puzzled to reconcile the pitch of the various tones, determined by resonators, with the effective pitch of the bell, when heard from a distance in conjunction with the other bells of the peal. There was a general agreement that the five notes of the peal were

$$\sharp, \quad g\sharp, \quad a\sharp, \quad b, \quad c\sharp,$$

according to harmonium pitch, so that the note of the second bell was b. A tone of pitch $a\sharp$ could be heard, but at that time nothing coincident with b or its octaves. Subsequently, in January 1880, the b was found among the tones of the bell, but at much higher pitch than had been expected. The five gravest tones were determined to be

$$d', \quad a'\sharp, \quad d'', \quad g''\sharp+, \quad b'';$$

so that the nominal note of the bell agreed with the fifth component tone, and with no graver one. The octaves are here indicated by dashes in the usual way, the c' immediately below the d' being the middle c of the musical scale.

Attempts were then made to identify the modes of vibration corresponding to the various tones, but with only partial success. By tapping round the sound-bow it appeared that the minima of beats for d' occurred at intervals equal to $\frac{1}{8}$ of the circumference, indicating that the deformation in this mode was elliptical ($n=2$), as had been expected. In like manner $g''\sharp$ gave $n=3$; but on account of the difficulty of experimenting in the belfry, the results were not wholly satisfactory, and I was unable to determine the modes for the other tones. One observation, however, of importance could be made. All five tones were affected with beats, from which

it was concluded that none of them could be due to symmetrical vibrations, as, till then, had been thought not unlikely.

Nothing further worthy of record was effected until last year, when I obtained from Messrs. Mears and Stainbank the loan of a 6-cwt. bell. Hung in the laboratory at a convenient height, and with freedom of access to all parts of the circumference, this bell afforded a more convenient subject for experiment, and I was able to make the observations by which before I had been baffled. Former experience having shown me the difficulty of estimating the pitch of an isolated bell, I was anxious to have the judgment of the founders expressed in a definite form, and they were good enough to supply me with a fork tuned to the pitch of the bell. By my harmonium the fork is d''.

By tapping the bell in various places with a hammer or mallet, and listening with resonators, it was not difficult to detect 6 tones. They were identified with the following notes of the harmonium*:—

e',	c'',	$f''+$,	$b''\flat$,	d''',	f'''.
(4)	(4)	(6)	(6)	(8)	

As in the former case, the nominal pitch is governed by the fifth component tone, whose pitch is, however, an octave higher than that of the representative fork. It is to be understood, of course, that each of the 6 tones in the above series is really double, and that in some cases the components of a pair differ sufficiently to give rise to somewhat rapid beats. The sign + affixed to f'' indicates that the tone of the bell was decidedly sharp in comparison with the note of the instrument.

I now proceeded to determine, as far as possible, the characters of the various modes of vibration by observations upon the dependence of the sounds upon the place of tapping in the manner already described. By tapping round a circle of latitude it was easy to prove that for (each of the approximately coincident tones of) e' there were 4 nodal meridians. Again, on tapping along a meridian to find whether there were any nodal circles of latitude, it became evident that there were none such. At the same time differences of intensity were observed. This tone is more fully developed

* In comparisons of this kind the observer must bear in mind the highly compound character of the notes of a reed instrument. It is usually a wise precaution to ascertain that a similar effect is not produced by the octave (or twelfth) above.

when the blow is delivered about midway between the crown and rim of the bell than at other places.

The next tone is c''. Observation showed that for this vibration also there are four, and but four, nodal meridians. But now there is a well-defined nodal circle of latitude, situated about a quarter of the way up from the rim towards the crown. As heard with the resonator, this tone disappears when the blow is accurately delivered at some point of this circle, but revives with a very small displacement on either side. The nodal circle and the four meridians divide the surface into segments, over each of which the normal motion is of one sign.

To the tone f'' correspond 6 nodal meridians. There is no well-defined nodal circle. The sound is indeed very faint, when the tap is much removed from the sound-bow; it was thought to fall to a minimum when the tap was about halfway up.

The three graver tones are heard loudly from the sound-bow. But the next in order, $b''\flat$, is there scarcely audible, unless the blow be delivered to the rim itself in a tangential direction. The maximum effect occurs at about halfway up. Tapping round the circle, we find that there are 6 nodal meridians.

The fifth tone, d''', is heard loudly from the sound-bow, but soon falls off when the locality of the blow is varied, and in the upper three fourths of the bell it is very faint. No distinct circular node could be detected. Tapping round the circumference showed that there were here 8 nodal meridians.

The highest tone recorded, f''', was not easy of observation, and I did not succeed in satisfying myself as to the character of the vibration. The tone was perhaps best heard when the blow was delivered at a point a little below the crown.

All the above tones, except f'', were tolerably close in pitch to the corresponding notes of the harmonium.

Although the above results seemed perfectly unambiguous, I was glad to have an opportunity of confirming them by examination of another bell. This was afforded by a loan of a bell cast by Taylor, of Loughborough, and destined for the church of Ampton, Suffolk, where it now hangs. Its weight is somewhat less than 4 cwt., and the nominal pitch is d. The observations were entirely confirmatory of the results obtained from Messrs. Mears's bell. The tones were

$c'\flat-2$,	$d''-6$,	$f''+4$,	$b''\flat-b''$,	d''',	g''';
(4)	(4)	(6)	(6)	(8)	

the correspondence between the order of the tone and the number of nodal meridians being as before. In the case of d'' there was the same well-defined nodal circle. The highest

tone, g''', was but imperfectly heard, and no investigation could be made of the corresponding mode of vibration.

In the specification of pitch the numerals following the note indicate by how much the frequency for the bell differed from that of the harmonium. Thus the gravest tone $e'\flat$ gave 2 beats per second, and was flat. When the number exceeds 3, it is the result of somewhat rough estimation and cannot be trusted to be quite accurate. Moreover, as has been explained, there are in strictness two frequencies under each head, and these often differ sensibly. In the case of the 4th tone, $b''\flat$—b'' means that, as nearly as could be judged, the pitch of the bell was midway between the two specified notes of the harmonium.

The sounds of bells may be elicited otherwise than by blows. Advantage may often be taken of the response to the notes of the harmonium, to the voice, or to organ-pipes sounded in the neighbourhood. In these cases the subsequent resonance of the bell has the character of a pure tone. Perhaps the most striking experiment is with a tuning-fork. A massive $e'\flat$ (e' on the $c'=256$ scale) fork, tuned with wax, and placed upon the waist of the Ampton bell, called forth a magnificent resonance, which lasted for some time after removal and damping of the fork. The sound is so utterly unlike that usually associated with bells that an air of mystery envelops the phenomenon. The fork may be excited either by a preliminary blow upon a pad (in practice it was the bent knee of the observer), or by bowing when in contact with the bell. In either case the adjustment of pitch should be very precise, and it is usually necessary to distinguish the two nearly coincident tones of the bell. One of these is to be chosen, and the fork is to be held near a loop of the corresponding mode of vibration. In practice the simplest way to effect the tuning is to watch the course of things after the vibrating fork has been brought into contact with the bell. When the tuning is good the sound swells continuously. Any beats that are heard must be gradually slowed down by adjustment of wax, until they disappear.

Observations upon the two bells in the laboratory having settled the modes of vibration corresponding to the five gravest tones, other bells of the church pattern can be sufficiently investigated by simple determinations of pitch. I give in tabular form results of this kind for a Belgian bell, kindly placed at my disposal by Mr. Haweis, and for the five bells of the Terling peal. For completeness' sake the Table includes also the corresponding results for the two bells already described.

Mears, 1888.	Ampton, 1888.	Belgian Bell.	Terling (5), Osborn, 1783.	Terling (4), Mears, 1810.	Terling (3), Graye, 1623.	Terling (2), Gardiner, 1723.	Terling (1), Warner, 1863.
Actual Pitch by Harmonium.							
e'	e'b−2	d'−4	g−3	a+3	a♯+3	d'−6	d'+2
c'	d'''−6	c''♯—d''	g'−4	g'♯−4	a'+6	a'♯−5	b'+2
f'''+	f''+4	f'+1	a'+6	b'+6	c''♯+4	d''+8	e''
b''b	b''b—b''	a''−6	d'−3	d''♯—e''	e''+6	g'♯+(10)	g'♯+4
d'''	d'''	.....	f''♯−2	g''♯−6	a''♯	b''+2	c'''♯+3
f'''	g'''						
Pitch referred to fifth tone as *c*.							
d	c♯−2		c♯−3	c♯+3	c+3	eb−6	c♯+2
bb	c−6		c♯−4	c−4	bb+6	b−5	bb+2
eb+	cb+4		eb+6	eb+6	eb+4	eb+8	eb
ab	ab—a		ab−3	g—g♯	f♯+6	a+8	g+4
c	c		c−2	c−6	c	c+2	c+3

It will be seen that in every case where the test can be applied, it is the fifth tone in order which agrees with the nominal pitch of the bell. The reader will not be more surprised at this conclusion than I was, but there seems to be no escape from it. Even apart from estimates of pitch, an examination of the tones of the bells of the Terling peal proves that it is only from the third and fifth tones that a tolerable diatonic scale can be constructed. Observations in the neighbourhood of bells do not suggest any special predominance of the fifth tone, but the effect is a good deal modified by distance.

It has been suggested, I think by Helmholtz, that the aim of the original designers of bells may have been to bring into harmonic relations tones which might otherwise cause a disagreeable effect. If this be so, the result cannot be considered very successful. A glance at the Table shows that in almost every case there occur intervals which would usually be counted intolerable, such as the false octave. Terling (5) is the only bell which avoids this false interval between the two first tones; but the improvement here shown in this respect still leaves much to be desired, when we consider the relation of these two tones to the fifth tone, and the nominal pitch of the bell. Upon the assumption that the nominal pitch is governed by that of the fifth tone, I have exhibited in the second part of the above Table the relationship in each case of the various tones to this one.

One of my objects in this investigation having been to find out, if possible, wherein lay the difference between good and bad bells, I was anxious to interpret in accordance with my results the observations of Mr. Haweis, who has given so much attention to the subject. The comparison is, however, not free from difficulty. Mr. Haweis says * :—"The true Belgian bell when struck a little above the rim gives the dominant note of the bell; when struck two-thirds up it gives the third; and near the top the fifth; and the 'true' bell is that in which the third and fifth (to leave out a multitude of other partials) are heard in right relative subordination to the dominant note."

If I am right in respect of the dominant note, the *third* spoken of by Mr. Haweis must be the minor third (or, rather, major sixth) presented by the tone third in order, which it so happens is nearly the same interval in all cases. The only *fifth* which occurs is that of the tone fourth in order. Thus, according to Mr. Haweis's views, the best bell in the series would be Terling (1), for which the minor chord of the last

* 'Times,' October 29, 1878.

three tones is very nearly true. It must be remarked, however, that the tone fourth in order is scarcely heard in the normal use of the bell, so that its pitch can hardly be of importance directly, although it may afford a useful criterion of the character of the bell as a whole. It is evident that the first and second tones of Terling (1) are quite out of relation with the higher ones. If the first could be depressed a semitone and the second raised a whole tone, harmonic relations would prevail throughout.

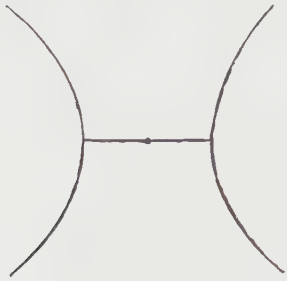
Judging from the variety presented in the Table, it would seem not a hopeless task so to construct a bell that all the important tones should be brought into harmonic relation; but it would require so much tentative work that it could only be undertaken advantageously by one in connexion with a foundry. As to what advantage would be gained in the event of success, I find it difficult to form an opinion. All I can say is that the dissonant effect of the inharmonious intervals actually met with is less than one would have expected from a musical point of view; although the fact is to a great extent explained by Helmholtz's theory of dissonance.

One other point I will touch upon, though with great diffidence. If there is anything well established in theoretical acoustics it is that the frequencies of vibration of similar bodies formed of similar material are inversely as the linear dimensions—a law which extends to *all* the possible modes of vibration. Hence, if the dimensions are halved, all the tones should rise in pitch by an exact octave. I have been given to understand, however, that bells are not designed upon this principle of similarity, and that the attempt to do so would result in failure. It is just possible that differences in cooling may influence the hardness, and so interfere with the similarity of corresponding parts, in spite of uniformity in the chemical composition of the metal; but this explanation does not appear adequate. Can it be that when the scale of a bell is altered it is desirable at the same time to modify the relative intensities, or even the relative frequencies, of the various partials?

Observations conducted about ten years ago upon the manner of bending of bell-shaped bodies—waste-paper baskets and various structures of flexible material—led me to think that these shapes were especially stiff as regards the principal mode of bending (with four nodal meridians) to forces applied normally and near the rim, and that possibly one of the objects of the particular form adopted for bells might be to diminish the preponderance of the gravest tone. To illustrate this I made calculations, according to the theory of the paper

already alluded to, of the deformation by pure bending of thin shells in the form of hyperboloids of revolution, and in certain composite forms built up of cylinders and cones so as to represent approximately the actual shape of bells. In the case of the hyperboloid of one sheet (fig. 2), completed by a

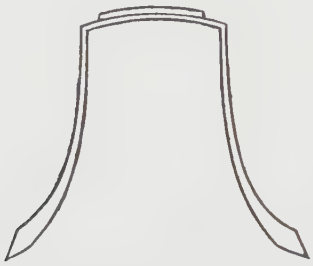
Fig. 2.



crown in the form of a circular disk through the centre, and extending across the aperture, it appeared that there was no nodal circle for $n=2$. The investigation is appended to this paper.

The composite forms, figs. 4 and 5, represent the actual bell (fig. 3 *) as nearly as may be. At the top is a circular disk,

Fig. 3.



and to this is attached a cylindrical segment. The expanding part of the bell is represented by one (fig. 4), or with better approximation by two (fig. 5), segments of cones. The calculations are too tedious to be reproduced here, but the results are shown upon the figures. In both cases there is a circular node N for $n=2$, not far removed from the rim, and in fig. 5 very nearly at the place which represents the sound-bow of an actual bell. In the latter case there is a node N′ for $n=3$ near the middle of the intermediate conical segment.

* Copied from Zamminer, *Die Musik und die musikalischen Instrumente.* Giessen, 1855.

The nodal circle for $n=2$ has been verified experimentally upon a bell constructed of thin sheet zinc in the form of fig. 5. The gravest note, G♯, and the corresponding mode of vibration, could be investigated exactly in the manner already described. In each mode of this kind there were four nodal

Fig. 4.

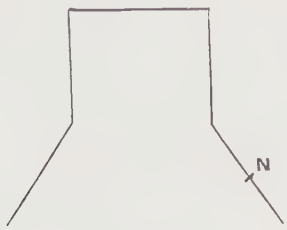
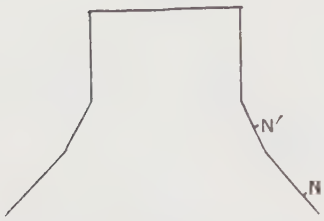


Fig. 5.



meridians, and a very well defined nodal circle. The situation of this circle was not quite so low as according to calculation; it was almost exactly in the middle of the lower conical segment. By merely handling the model it was easy to recognize that it was stiff to forces applied at N, but flexible higher up, in the neighbourhood of N′.

It is clear that the actual behaviour of a church bell differs widely from that of a bell infinitely thin; and that this should be the case need not surprise us when we consider the actual ratio of the thickness at the sound-bow to the interval between consecutive nodal meridians. I think, however, that the form of the bell does really tend to render the gravest tone less prominent.

Appendix.

On the Bending of a Hyperboloid of Revolution.

The deformation of the general surface of revolution was briefly treated in a former paper *. The point whose original

* "On the Infinitesimal Bending of Surfaces of Revolution," Proc. Math. Soc. xiii. p. 4 (1881).

cylindrical coordinates are z, r, ϕ, is supposed to undergo such a displacement that its coordinates become

$$z+\delta z, \quad r+\delta r, \quad \phi+\delta\phi.$$

The altered value $(ds+d\delta s)$ of the element of length traced upon the surface is given by

$$(ds+d\delta s)^2=(dz+d\delta z)^2+(r+\delta r)^2(\delta\phi+d\delta\phi)^2+(dr+d\delta r)^2.$$

Hence, if the displacement be such that the element is unextended,

$$dz\,d\delta z+r^2 d\phi\,d\delta\phi+r\delta r(d\phi)^2+dr\,d\delta r=0.$$

Now

$$d\delta z=\frac{d\delta z}{dz}dz+\frac{d\delta z}{d\phi}\,d\phi,$$

$$d\delta r=\frac{d\delta r}{dz}dz+\frac{d\delta r}{d\phi}\,d\phi,$$

$$d\delta\phi=\frac{d\delta\phi}{dz}dz+\frac{d\delta\phi}{d\phi}\,d\phi\,;$$

and by the equation to the surface

$$dr=\frac{dr}{dz}dz+\frac{dr}{d\phi}\,d\phi,$$

in which, by hypothesis, $dr/d\phi=0$. Thus

$$(dz)^2\left\{\frac{d\delta z}{dz}+\frac{dr}{dz}\frac{d\delta r}{dz}\right\}+(d\phi)^2\left\{r^2\frac{d\delta\phi}{d\phi}+r\delta r\right\}$$
$$+dz\,d\phi\left\{\frac{d\delta z}{d\phi}+r^2\frac{d\delta\phi}{dz}+\frac{dr}{dz}\frac{d\delta r}{d\phi}\right\}=0.$$

If the displacement be of such a character that no line traced upon the surface is altered in length, the coefficients of $(dz)^2$, $(d\phi)^2$, $dz\,d\phi$, in the above equation, must vanish separately, so that

$$\frac{d\delta z}{dz}+\frac{dr}{dz}\frac{d\delta r}{dz}=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$r\frac{d\delta\phi}{d\phi}+\delta r=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\frac{d\delta z}{d\phi}+r^2\frac{d\delta\phi}{dz}+\frac{dr}{dz}\frac{d\delta r}{d\phi}=0. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From these, by elimination of δr,

$$\frac{d\delta z}{dz}-\frac{dr}{dz}\frac{d}{dz}\left(r\frac{d\delta\phi}{d\phi}\right)=0, \quad . \quad . \quad . \quad . \quad (4)$$

$$\frac{d\delta z}{d\phi}+r^2\frac{d\delta\phi}{dz}-r\frac{dr}{dz}\frac{d^2\delta\phi}{d\phi^2}=0\,; \quad . \quad . \quad . \quad . \quad (5)$$

from which again, by elimination of δz,

$$\frac{d}{dz}\left(r^2\frac{d\delta\phi}{dz}\right)-r\frac{d^2r}{dz^2}\frac{d^2\delta\phi}{d\phi^2}=0. \quad . \quad . \quad . \quad . \quad (6)$$

For the purposes of the present problem we may assume that $d\phi$ varies as $\cos s\phi$, or as $\sin s\phi$; thus,

$$r^2\frac{d}{dz}\left(r^2\frac{d\delta\phi}{dz}\right)+s^2r^3\frac{d^2r}{dz^2}\delta\phi=0 \quad . \quad . \quad . \quad . \quad (7)$$

is the equation by which the form of $\delta\phi$ as a function of z is to be determined.

When application is made to the hyperboloid of one sheet

$$\frac{r^2}{a^2}-\frac{z^2}{b^2}=1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

we find, since

$$r\frac{dr}{dz}=\frac{a^2z}{b^2}, \quad r^3\frac{d^2r}{dz^2}=\frac{a^4}{b^2},$$

$$r^2\frac{d}{dz}\left(r^2\frac{d\delta\phi}{dz}\right)+\frac{s^2a^4}{b^2}\delta\phi=0. \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The solution of this equation is expressed by an auxiliary variable χ, such that

$$z=b\tan\chi, \quad r=a\sec\chi \quad . \quad . \quad . \quad . \quad (10)$$

in the form

$$\delta\phi=\mathrm{A}\cos s\chi+\mathrm{B}\sin s\chi. \quad . \quad . \quad . \quad . \quad (11)$$

In order to verify this it is only necessary to observe that by (10)

$$r^2\frac{d}{dz}=\frac{a^2}{b}\frac{d}{d\chi}.$$

We will now apply this solution to an inextensible surface

formed by half the hyperboloid and a crown stretching across in the plane of symmetry $z=0$ (fig. 2). The deformation of this crown can take place only in the direction perpendicular to its plane, so that $\delta r=0$, $\delta\phi=0$. These conditions must apply also to the hyperboloid at the place of attachment to the crown. Hence $\delta\phi$ must vanish with z, or, which is the same, with χ. Accordingly $A=0$ in (11); and dropping the constant multiplier we may take as the solution

$$\delta\phi = \sin s\chi \cos s\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and in correspondence therewith by (2) and (3)

$$\delta r = sr \sin s\chi \sin s\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\delta z = -\frac{a^2}{b}\{\cos s\chi + s \tan\chi \sin s\chi\} \sin s\phi. \quad . \quad . \quad (14)$$

It is evident from these equations that, whatever may be the value of s, there is no circle of latitude over which both $\delta\phi$ (or δr) and δz vanish *. Hence there can be no circular nodal line in the absolute sense. But just as there are meridians ($\sin s\phi=0$) on which the *normal* motion vanishes, so there may be nodal circles in this more limited sense. The condition to be satisfied is obviously

$$\delta r/\delta z = dr/dz\,;$$

or in the present case

$$\sin 2\chi + 2s \tan s\chi\,(\sin^2\chi + b^2/a^2) = 0. \quad . \quad . \quad (15)$$

In this equation the range of χ is from 0 to $\frac{1}{2}\pi$; and thus there can be solutions only when $\tan s\chi$ is negative.

In the case $s=2$ the equation reduces to

$$1 + 2\sin^2\chi + 4b^2/a^2 = 0,$$

which can never be satisfied.

When $s=3$, the roots, if any, must lie between $\chi=30°$ and $\chi=60°$. A more detailed consideration shows that there is but one root, and that it occurs when χ is a little short of 60°.

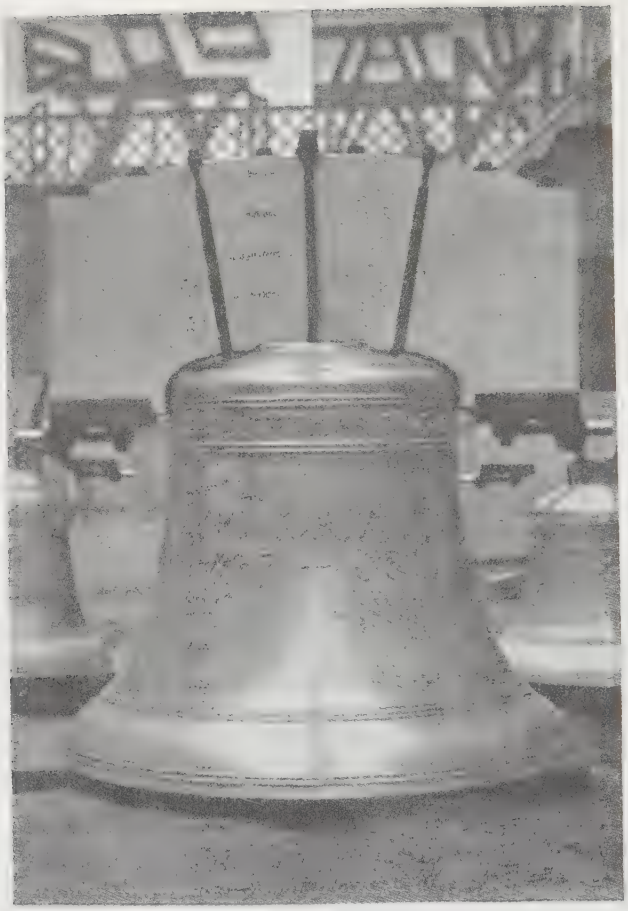
* A corresponding proposition may be proved more generally, that is without limitation to the hyperboloid.

2A

Reprinted from *Nash's Pall Mall Mag.* **7**:183–194 (1895)

ON BELL TONES

A. B. Simpson



A Bell by Messrs. Taylor, of Loughborough.

ON BELL TONES.

THE present writer has nothing to say on the ancient history of bells, nor will he attempt to make any addition to the many pretty things which have been said as to their sentimental power. His object is more prosaic. It is simply to place on record certain facts which have come under his notice, during a course of observation extending over many years, and which he has reason to think would prove interesting to many of the readers of this Magazine.

We have the bells with us everywhere, and few people with musical ears have not, at one time or another, amused themselves, and (we will venture to suggest) *puzzled* themselves, in attempting to determine accurately the notes of their own church bells.

Many of us, also, have been struck by the apparent want of harmony in the famous carillons of Bruges and other Belgian towns; and some few have been at great expense to set up carillons of their own, and have been reluctantly driven to the vexatious conclusion that they are "painfully out of tune."

St. Mary's Church, Stoke-by-Wayland.

To all these we think we have that to say which will interest them. And we are not without hope that, through their influence, our bell founders and tuners may be roused to study their work more closely, to try to understand better what was the purpose of the original designers of the present form of bell, and endeavour to fulfil that purpose more nearly than they have done in the past.

We begin by boldly asserting, as the result of a pretty wide experience, that there is probably not a single bell in England that is really "in tune *with itself*," and almost certainly not a single "peal" of bells that are properly in tune with each other.

We do not say that there are not many peals which are in excellent tune as to the most important note in each, and their general musical effect very pleasing. But we do assert that the best of these might be much better; and, in the majority of cases, the irregularities we complain of are such as seriously to mar their musical effect, and such as ought to be, and might be, avoided by more intelligent founding, or (in most cases) rectified by more intelligent tuning after founding.

Now, this whole matter turns on the meaning of the expression "in tune with itself." Most people have an idea that every bell has one prominent, unmistakable note which characterises it, and as to whose pitch no two people with musical ears could differ. Thus, in the article on Bells in the latest edition of the *Encyclopædia Britannica*, we read the following: "A good bell when struck yields one note, so that any person with an ear for music can say what it is."

If for "a good bell" we may read "a bell in good tune," this statement is true; but, as it stands, it condemns as "not good" some of the finest and best bells in the world.

One example, and that a notable one, will suffice at present to support us in this.

In the *Times* of July 20th, 1887, there is an account of the "inauguration" of the great bell "Gloriosa," made out of French cannon and hung in the Cathedral of Cologne. The account concludes thus: "The opinions of experts are divided as to whether the note which the bell sounds is C sharp or D."

We feel sure that many of our readers have felt a similar difficulty in determining the note of a familiar bell.

Now, to account for this, and to clear the way for further observations, we must understand what is the true "theory" of a bell, if we may be allowed the expression.

It would surely be unreasonable to suppose that the very peculiar form of bell which (with slight modifications) has been preserved for so many hundreds of years, both here and on the Continent, was adopted without the deliberate purpose of ensuring that the various tones and sub-tones of each bell should be in some fixed musical relation to each other.

What is that relation?

We make bold to suggest that it is this: Every true bell should give out, when fairly struck, a fundamental note or "tonic," its third, fifth, and octave above, and its octave below, thus sounding the full chord—*do*, *mi*, *sol*, *do*, with the bass *do* below.

This is the "theory" which was, we are satisfied, before the minds of the original designers of the present form of bell. Almost forgotten (if ever realised) by many of their successors, it is still recognised by some, and irresistibly forced upon the acceptance of those who, like the present writer, have made a study of the tones of bells as they are.

The following extract from the article on Founding of Bells in the *Encyclopædia Britannica*, 5th Edition, 1815, is of importance as showing that such a theory was

Stoke-by-Wayland Church, Suffolk.

recognised in this country not so very long ago, though, it is true, there is no reference to the lowest note we have spoken of.

"The height of the bell in proportion to its diameter (is) as 12 to 15, or in the proportion of the fundamental sound to its third major: whence it follows (?) that the sound of a bell is principally composed of the sound of its extremity, or brim, as a *fundamental*—of the sound of the crown, which is an *octave* to it—and of that of the height, which is a *third*."

But now, to bring this paper within reasonable limits, we must dismiss all consideration of thirds and fifths, and confine our attention to the three more important notes—*i.e.*, the tonic, its octave above, and its octave below. For convenience' sake, and for reasons which will appear further on, let us call the first of these the "fundamental," the second (or octave above) the "nominal," and the third (or octave below) by the name by which it is known in English foundries, the "hum-note."

If, then, a bell corresponded to its "theory," these three would sound the same note, in three consecutive octaves, and the bell would, so far, be "in tune with itself."

"But, alas! where shall we find such a bell? Whatever the cause may be—whether founders, in ignorance or indifference as to the importance of having these notes in accord, have, (1) for convenience of *ringing*, altered the original proportions of bells, or, (2) to obtain greater power, put more metal into them—certain it is that it is quite the exception to find a bell which has *any two* of these notes in unison, and we have not yet met with *one* in which *all three* were in accord.

By far the commonest state of things is this: The "fundamental" is almost always the flattest of the three—irrespective, of course, of octave. The "hum-note" is almost always the sharpest, and the "nominal" generally between the two. Thus, if the nominal of a bell is C, the fundamental will probably be somewhere between C and B in the octave below, while the hum-note will probably be between C and C sharp in the octave below that. (It is not unusual for the hum-note to be much sharper than this.)

In support of this statement let us take a few examples. Take first the peal at Terling, in Essex, which consists of five bells in the key of F sharp, by five different makers, and of various dates, covering a period of 240 years. This is an excellent example, as, from the variety of makers and dates, any general characteristics that we may observe cannot be considered as peculiarities of "time" or "foundry." And it has further this great value, that the tones of these bells have been carefully analysed by Lord Rayleigh, and tabulated in his most valuable paper "On the Tones of Bells," printed in the *Philosophical Magazine* for January 1890. An examination of these tables gives the following results.

1. In the first three bells, including the oldest and the newest, the tones follow just the rule which we have called the "common" one—*i.e.*, they are nearly in octaves, but the fundamental is the flattest, the nominal sharper, and the hum-note the sharpest. In the fourth bell, the fundamental and the nominal are true octaves and the hum-note is sharper by a long semitone. In the tenor, the fundamental and the hum-note are true octaves, and the nominal a semitone *flatter* (very unusual).

We claim this peal as a powerful witness to the truth of our position—(1) that the fundamental, nominal, and hum-note were *meant* to be in octaves, and (2) that, as a matter of fact, it is the exception to find a bell in which any two of them are in accord.

2. Take next the little peal of six bells in the church of Fittleworth, Sussex.

Church of SS. Peter and Paul, Lavenham

Of these, three are new, the fourth is about fifty years old, the fifth and sixth very ancient. In each of the six, without exception, the nominal is a quarter of a tone, more or less, sharper than the fundamental. And, in all but one, the hum-note is a trifle sharper than the nominal; the exception being the tenor, a very ancient bell, in which the hum-note and the nominal are true octaves. Can we help feeling that the general small defection from perfect octaves is an error from a *design* which is *fulfilled* in the exceptional case?

3. In the tower of Eastry, in Kent, are five bells, the tenor about a ton in weight. In *all* these the common rule, as stated above, holds good, with the exception that, in the fourth bell, the nominal is a shade *flatter* than the fundamental—an unusual case, but still tending to confirm the theory that these notes were meant to be true octaves.

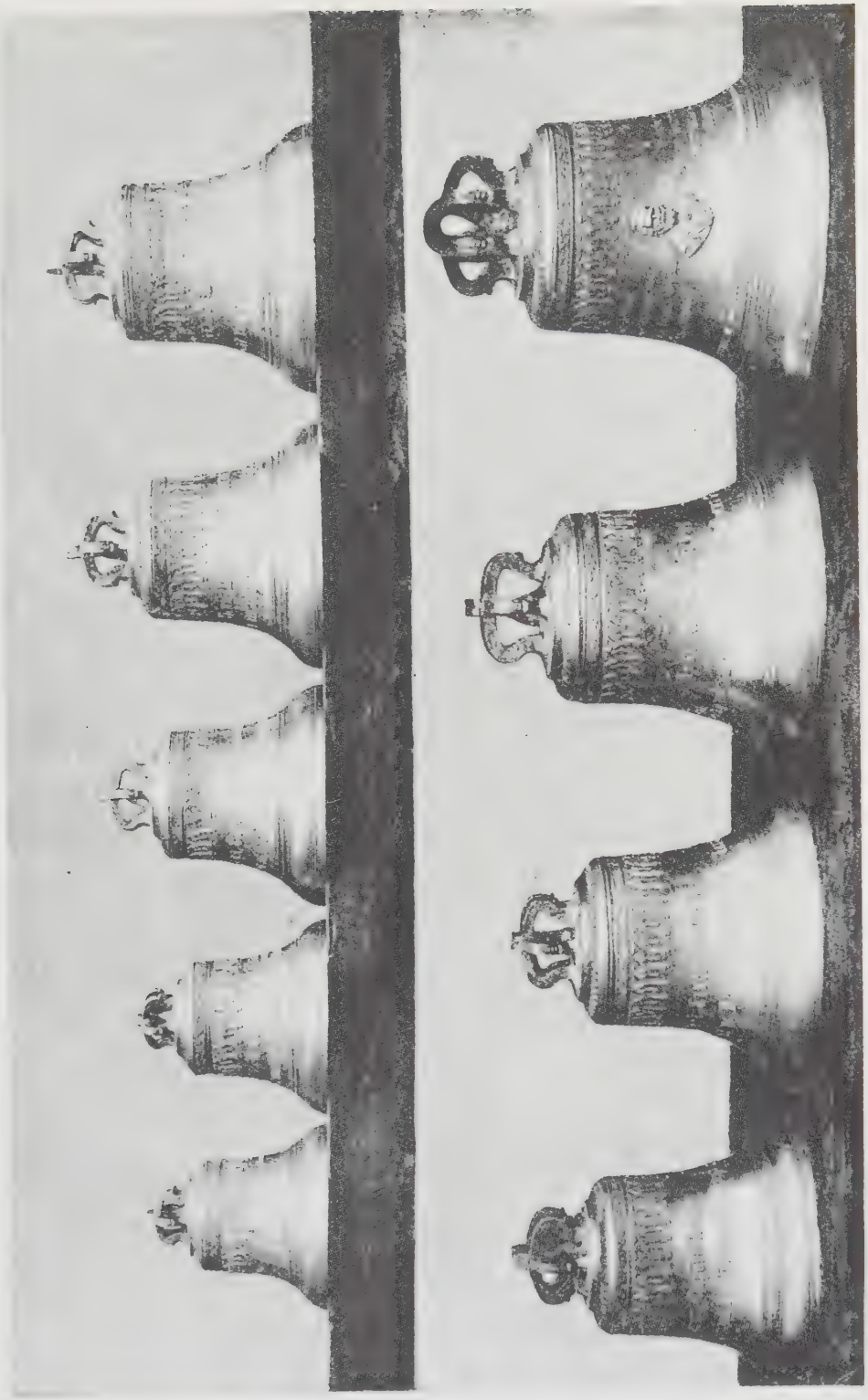
4. There is a very fine peal of six bells in the tower of Stoke-by-Nayland, in Suffolk. Every one of these, with possibly one exception, follows the same rule: the nominals, fundamentals, and hum-notes being nearly in octaves, the hum-notes being the sharpest, the fundamentals the flattest, and the nominals between the two. The possible exception is the treble, which is slightly cracked, and the hum-note not observable.

5. One of the finest peals of eight bells in the kingdom is that of Lavenham, in Suffolk. Here also *every one*, with one notable exception, follows the same general rule of slightly imperfect octaves between the three principal notes; the errors in each bell being of the same kind—*i.e.*, all the hum-notes a little sharper, and the fundamentals a little flatter, than their respective nominals. The one exception is the tenor, a marvellously fine-toned bell, which has, indeed, the reputation of being about the finest in England. In this, the fundamental and the nominal are apparently in perfect octave—a very noticeable fact, taken in connection with the reputation of this bell.

These are all instances of English bells. But, to guard against the supposition that these coincidences, or irregularities, are peculiar to our bells, we will mention, lastly, a peal of eight bells cast at Louvain about eight years ago, and placed in the tower of Lower Beeding, near Horsham, in Sussex. We had an opportunity of examining these bells on their arrival in this country. There was much to learn from them, which we may hereafter refer to. But with respect to the point now before us—viz., the relative positions of the three principal tones—there was nothing to distinguish them from an ordinary English peal. There was the usual approximation to octaves, but we only noticed one instance in which the relation was true. In the seventh bell, the fundamental was a true octave below the nominal. But it was evident that this bell had been greatly altered, and we have reason to feel sure that, originally, the fundamental had been flatter than the nominal, as in most English bells, and as was, and is, the case with the tenor bell by its side.

If now we have carried our readers with us so far, they will no longer be surprised at any difficulty they may have experienced in determining the note of any particular bell. For here we see that, in the *majority* of bells, we have three notes, very near to each other in pitch, though in different octaves, all struggling for the mastery, and each able—let us here say—under given circumstances, to assert its supremacy. Is it any wonder that even a skilled musician, if not learned in bell tones, should be in doubt as to the true note of a bell submitted to him? Is it so very surprising that "experts differed as to whether the note of the 'Gloriosa' bell was C sharp or D"?

We have not a doubt that the explanation of this is, that the sharper tone heard



Church of S Pieove, Ghent.

was the nominal, and the flatter the fundamental: this great bell following, in this respect, the common rule which we have so fully illustrated above.

"But"—we imagine our musical readers exclaiming—"if this be so, how is it that the sounds of bells are even tolerable? Any other instrument which gave forth simultaneously, *e.g.*, C, with a rather flat C sharp above and a rather flat D below, would be unbearable,—and the succession of a series of bells of this imperfect character would surely produce nothing but a hideous noise."

So one might think; but it is not so. For reasons which satisfy the learned in acoustics, the difference in quality of these sets of tones is such that they do not *interfere* with one another, so as to give the sense of discord, which we should expect. The tones of nominals, fundamentals, and hum-notes, seem to move, as it were, in three separate spheres. And though any discord, between at least the nominal and fundamental, in a bell cannot but seriously detract from the purity and fulness of its tone, it does not produce "beats," nor affect the ear painfully. Consequently, there are thousands of bells, having their principal tones quite out of tune with each other, which, *considered individually*, are good and pleasing. They might be made much better, perhaps; but as long as they stand *alone*, no alteration is necessary in order to satisfy the ordinary musical listener. You may listen to whichever tone you like—sometimes to one, sometimes to another; all are pleasing, and no *one* interferes with another.

But when it comes to peals and carillons, the case is different. In order to get any really musical effect, it is absolutely necessary that some *one*, at least, of the *sets* of tones in the series of bells should be properly in tune with itself—*i.e.*, all the nominals in tune with each other, or, similarly, all the fundamentals, or all the hum-notes. This, it is evident, is the very least that can be accepted. And, as a matter of fact, the choice is more restricted than this; for, important as are the hum-notes—far more important, as we are prepared to maintain, than either foreign or English founders seem to think—we are quite ready to allow that, unless *all three*

The Belfry of Ghent.

sets are to be made to move in true octaves (which is devoutly to be wished, but is more than can at present be demanded), the "hum-note" set must be neglected, in comparison with either of the other two.

At this point, therefore —again to "lighten the ship"—we drop all reference to the hum-note till a more favourable time, and confine our attention to nominals and fundamentals.

We have then before us, let us suppose, a series of bells, direct from the foundry, all of the *common* character—*i.e.*, with the fundamental in each bell more or less flatter than its nominal—and, as is sure to be the case, with neither its nominals nor its fundamentals quite in tune with each other. Now, what shall we do? Plainly, the only really satisfactory operation, if it could be done, would be to bring the nominal in each bell into unison with its own fundamental, and then to tune each bell, so rectified, to its neighbour. This would indeed be "something like" tuning, and we are prepared to maintain that, in all ordinary cases, it can be done.

But, as a matter of fact (speaking generally) no one attempts such a thing. What is done is this: the tuner (consciously or unconsciously) selects one of the two sets of notes (either the nominals or the fundamentals), puts them in proper tune with each other, and leaves the other set untuned, either to the other set, or to each other.

To be more explicit—and we now ask the attention of our readers, *especially of those interested in carillons*, to what we now state—"The Englishman tunes the nominals, and neglects the fundamentals; while the foreigner tunes the fundamentals, and (comparatively) neglects the nominals."

Now, this difference of practice is a very remarkable fact, which we have never seen referred to in any of the many treatises on bells, nor have we ever met with any one who seemed to be aware of it. Yet it is as certain as it is interesting and important.

1. That the Englishman tunes a peal by the (so-called) "nominals" is shown beyond question by Lord Rayleigh in the paper referred to. The Terling peal is pronounced by English bell-experts to be in the key of F sharp. Well, the *upper* series of notes in those bells *is* in that key; and indeed we call this note the

"nominal," because the Englishman *names* each bell according to the pitch of this note in it. The fundamentals of this peal form no musical series at all, and evidently have not engaged the attention either of the founder or the tuner.

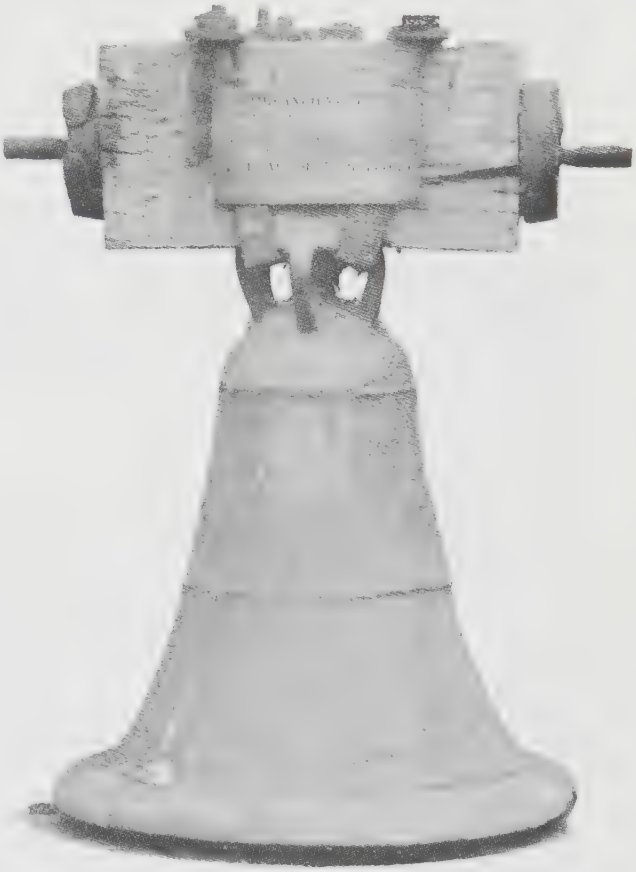
But *any* peal of English bells will prove the same point. If the bells are what an English tuner calls "in tune," you will find that it is the nominals that are in tune, and not the fundamentals. We have often tried to call the attention of professional tuners to the fundamental of the bell they were tuning, but they invariably treated it with indifference. And there is, further, this curious fact: that while a tuner always gave the nominal as the note of any Bell, he invariably gave the pitch an octave lower than it really was. We have, for many years past, lost no opportunity of calling the attention of founders and tuners to these things, but we know not with what effect.

2. That the foreigner, and in particular the Belgian, takes the fundamental as the principal note, and tunes the bells by it, is equally certain and unquestionable. The very fact that he calls it, as he does, the "fundamental" is almost conclusive. And no one can visit a Belgian foundry and engage in discussion about the bells without being convinced on this point. But we have clearer proof than this. The Belgian peal at Beeding was pronounced by the founders to be correctly in tune, and the seventh bell, in particular, they declared to be *exactly* in tune with the eighth. So it was, as to the fundamental; but, as regards the nominal, it was very flat, and the whole peal, generally, painfully out of tune to the English ear.

Moreover, the internal form of foreign bells is such as to indicate how to sharpen them. But the effect of so doing is to sharpen the fundamental, without altering the nominal at all, or very partially.

We could adduce much more evidence on this point, and, on the whole, we feel safe in asserting what we have stated, as to the foreign method, to be true.

The foreigner is, indeed, quite aware of the existence of the nominal, and, to hear him talk, you might think that he brought both sets into unison.



This bell, which formerly hung in the church of Mitford, in Northumberland, is of unknown, but certainly of great antiquity. It is very interesting as marking a distinct stage in the evolution of the present form of bells. The chief points of interest are: (1) The height is EQUAL *to the width—instead of four-fifths or less; and (2) The bell, instead of terminating in an* OUTER *cusp, terminates in an* INNER *one; so that, as seen in the illustration, it stands on its* INNER *rim.*

But, except perhaps in very large bells, he really does not do so, as any one with an ear may judge, from the inharmonious character of Belgian carillons, as well as from such tangible examples as the Beeding bells.

One point more remains to be cleared up, before we can arrive at the practical conclusion we are seeking to reach. The foreigner tunes by the fundamental, the Englishman by the nominal: which is right?

A direct unqualified answer to this question is, as might be expected, impossible. Both are right in their way. But there is this difference: that while it *never* can be allowable to neglect the nominals, the fundamentals *may* be neglected in English *ringing* peals, and in the *upper* bells of carillons, not without some loss of purity and fulness of tone, but without *painful* injury to the harmonious effect. The reason for this distinction will appear from the following consideration.

When bells are struck at considerable intervals of time, most persons would be apt to take the fundamentals as the notes of the bells, on account of their full and *persistent* character. But the case is different when one bell follows another in rapid succession. At the instant of striking, the keen sound of the (higher) "nominal" is most perceptible; and, if followed immediately by another, there is no time for the "fundamental" to force itself into prominence, and so the ear keeps following the nominals all through. This would be the case with an English peal, in which the bells follow one another very rapidly. And so also with the *higher* bells of a carillon, which take the "air." Consequently the nominals *must* be put into tune with each other in these cases. And it is the want of attention to this necessity which is the cause of the lack of harmony observable in foreign carillons.

But with the *lower* Bells of a carillon, the case is different. These strike, generally, at longer intervals, and the fundamental has time to assert itself, and to remain, so to say, master of the field. It therefore becomes necessary, in these bells, to tune the fundamentals also; and, of course, to bring them into unison with their respective nominals.

To sum up, then (omitting all reference to the hum-note)—

1. It is *essential* that *all the nominals*, throughout any peal or carillon, should be in tune with each other.

2. It is *very desirable*, in a *carillon*, that at least all the *heavier* bells (say, those above 7 cwt.) should have their fundamentals also brought into unison with their respective nominals.

3. It is *best*, in *all* cases, to bring the fundamental of each bell into true octave with its nominal, and then to tune the whole series of bells, *so rectified*, to each other.

Can this be done? We suggest, with some confidence, that, in all ordinary cases, it can. But this is another question, which we are not concerned to deal with in this paper. Bell founders and tuners naturally do not care to take any more trouble than is necessary in order to satisfy the public. It has been our object *to move the public to move the tuners*. If we succeed in that, we feel sure that the tuners will find out how to satisfy the demands made upon them, so far as the nature of the case admits.

Nevertheless, if we thought that the publication of such knowledge as we possess —as to the possibilities and methods of Bell-tuning—would tend at all to the perfecting of the art in this country, we should be glad to return to the subject.

2B

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ON BELL TONES
NO. II

A. B. Simpson

No. II.

THE object of my former paper (*vide P. M. M.*, October. 1895) was to place on record certain facts concerning bells as they are, and to call the attention of the musical public to them. And I did so, not merely in order to offer an explanation of the want of harmony observable in peals,—and still more in carillons,—but also in order to bring public opinion to bear on founders and tuners, so as to induce them to bring their bells into a more perfect agreement with the (presumed) intentions of the original designers of the present form of bell than has been thought necessary in the past.

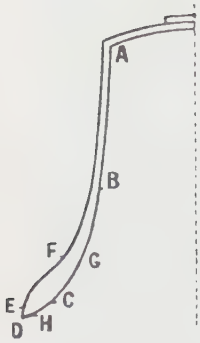
But I did something more than this. I ventured on a pretty confident opinion that existing bells and peals might (generally) be so rectified as to be much more completely in tune, both with themselves and with each other, than we have yet found them.

And here I desire to say, at the outset, that I am not a bell-founder. Had I been such, no doubt I should have kept any special knowledge to myself, and have made use of it for my own advantage. But, having no personal interests to serve, and believing it to be for the best interests of the art that knowledge—which I hold to be essential to any real progress—should be at the command equally of all the profession, I have preferred to make a clean breast of all that I think I have learned, and let bell-founders take it or leave it as they may think best.

If, as I venture to hope, my account of things should prove correct and of value, I shall feel amply repaid if this is acknowledged (as no doubt it would be) by those to whom it may prove an important assistance in what is now a very obscure department of the bell-founders' art.

It will be remembered that the main point insisted on in the former article was that the tones which I have there designated as the nominal and the fundamental should in each bell be brought into unison with each other.

According to theory the nominal should be an exact octave above the fundamental, But, as a matter of fact, we rarely find it so, the nominal being very generally about a quarter of a tone (more or less) sharper than the fundamental; and the question is, "How to bring them together?"



In order to explain my answer to this satisfactorily, I must direct attention to the figure.

This figure represents a half-section of a normal English bell.* The line A B C D represents the *inner* surface of the bell, and forms a continuous curve, the curvature of which becomes more acute as we approach the lip at D.

The line of the *outer* surface is more complicated, but is so ordered that the thickness of the bell shall be uniform from A to B, then gradually increase up to a point C, and then rapidly diminish until the bell terminates in a sharp "cusp" at D.

The points E and F indicate the position of certain "bead lines," which may be observed running all round a bell, which mark the boundaries of what is called the "sound-bow," or principal zone of the bell; and the points H and G are the corresponding points respectively on the inner surface.

We are now in a position to explain how to tune a bell—*i.e.*, how to alter its tones so as to bring them into harmony with each other, or with the corresponding tones in other bells.

It would evidently be inconsistent with the character of this Magazine to enter into a detailed description of the various processes, and of the effect of each; but here is the whole matter "in a nutshell":—

The point H is the "turning-point" for tuning the nominal.

The point C is the "turning-point" for tuning the fundamental.

To be a little more particular:—

1. To *sharpen* the nominal, metal must be taken away (of course, all round the bell) from the little triangular portion H D E; by this means the nominal may be safely sharpened about $\frac{1}{8}$th of a tone. But the process rather spoils the *look* of a bell, and, for that and other reasons, is not popular with tuners. Nevertheless it is effective, and, within reasonable limits, quite allowable.

2. To *flatten* the nominal, metal must be taken off, all round the bell, from H to C, or to G, or even to B if necessary, thus *thinning* the main part of the bell; by this means the note may well be lowered as much as half a tone if required.

Now, these two processes are well known to all English experts, and I have never met with one who used any other; which shows, as I said before, that (consciously or unconsciously) English experts tune their bells to each other by the *nominals*, and by *no other* note.

And now for the fundamentals:—

1. To *sharpen* the fundamental, take off metal along the line C D; this may be continued until the line C E is reached.

2. To *flatten* the fundamental, take off metal from C to G, or even to B if necessary.

And these are the processes used by foreign experts in tuning their carillons or peals; which, again, shows (as I said before) that the main object which *they* have in view, is to tune their bells to each other by the fundamentals, and *not* by the nominals.

If, now, the above rules are carefully studied, in connection with the figure, the following Possibilities, or Impossibilities—which I am unable to establish at length—will, I think, be sufficiently apparent:—

* Foreign bells (which probably represent an earlier type) differ from English in having the line C D straight instead of curved.

1. That the fundamental cannot be flattened without also flattening, to some extent, the nominal.

2. That the nominal *can* be flattened considerably without altering the fundamental —viz., by taking off metal, on both sides of C.

3. That the nominal can be flattened a *little*, while by the same process the fundamental will be a little sharpened, thus bringing the two towards each other—viz., by reducing the *rounded* surface C E to a flat surface.

4. That the fundamental can be sharpened considerably, while leaving the nominal nearly, if not quite, unaltered.

5. That although the process of sharpening the nominal will also raise the fundamental, it will not do so in the same degree; and would tend, therefore, to bring the two together, in the unusual (and very objectionable) case of the fundamental being originally the sharper note.

With these possibilities within our reach, it is surely evident that bells *can*, as regards these two principal tones, be put into tune with themselves and with each other. And if so, they can be so cast in the first instance.

Of course, in all this I am speaking *generally*. There are many bells so radically wrong that there is nothing to be done with them except "put up with" them, or send them to the melting-pot. And there are numberless cases of treble bells which, in order to make their leading tones *powerful* enough, have been made of a disproportionate thickness, with the inevitable result that their tones have been thrown very far out of due relation to each other. I fear that there is no help for this, and no effective method of dealing with such cases. There must be a *compromise* somewhere, and we have a tolerably clear idea as to what had best be sacrificed in order to effect it. All that I care to say here is, that the compromise must *not* be at the expense of the *nominal*. Whatever else has to give way, the nominals of these treble bells *must* be in tune with those of the rest of the peal.

And now, with due allowance for exceptional cases, I hope I have made out at least a very strong case to justify my appeal to the musical public—and especially to such as are incumbents or churchwardens—to demand from bell-founders a more perfect harmony in the bells supplied to them or tuned for them.

Why are we to submit to listen all our lives to a church peal which gives out two (not to say three) series of sounds, which ought to be in octaves, but which, all through the peal, are *about* a quarter of a tone apart? And I emphasise that word "about," because, if the error were *constant*, each of the series would be at least true in itself, though false to the others.

Are we to be told: "Oh! never mind the fundamentals,—the nominals are all right, and that is all that is necessary"?

Well, but here is all Europe (practically) telling us to "never mind the *nominals* if the fundamentals are in tune."

They may be wrong in bidding us be indifferent to the nominals, but surely it savours of ignorance, or of "insular arrogance," to dismiss in this summary fashion the consensus of European opinion on the value of the fundamentals.

If the result of doing so were *satisfactory*, we might be content. But it is not so. No one who pays any intelligent attention to English peals can be satisfied with them as they are.

If the dissonance were *unavoidable*, we might submit to it. But it is not so. I am *certain*—and I think I have justified my opinion—that in most cases, probably in all important cases, it might be corrected.

Let incumbents and churchwardens, in particular, insist on a greater accuracy of tuning, and in time they will get it. Tuners will find out how to alter bells, if necessary, so as to bring about this unison between nominals and fundamentals; and then founders will learn experimentally how to alter their designs, so that bells should "come out" more nearly right at first.

Of course, this means a good deal of intelligent painstaking, and some little expense. But is not the object worth it? We may be sure of this, that the founder who first has the courage to grapple with, and the perseverance to solve, this problem, will get a name and reap a harvest which will amply repay him, and he will, further, earn the gratitude of all true lovers of music.

And here I must add a word in justice to myself, as well as to foreign professors of bell-tuning. I should be sorry to be thought ignorant of the intelligence and pains which many of them have bestowed on the regulation of the subordinate tones in their bells, and especially in the larger ones. I believe that they know a great deal that I am ignorant of. But I am certain that they make a great mistake in underrating the importance of the tone which I have called the nominal, especially in the case (so common with us) where it is sharper than the fundamental.

I believe that if they would only recognise the conspicuous importance of this tone—in such cases especially—even at the expense of the comparative neglect of some other tone, they would leave little to be desired in the carillons which are their peculiar pride.

It was no part of my original intention to enter upon the consideration of "thirds" and "fifths." But I may just say, in passing, that the "third" is far too important a tone to be ignored in the tuning, at least, of a large bell. When once its strident sound has caught the ear, it almost obliterates the other tones, and is sometimes, indeed, mistaken for the principal tone.

All that I can say about it here is—

1. That some bells are cast with *maior* "thirds" (more or less imperfect), and some with *minor*.

2. That a "third" cannot be sharpened; but may be flattened, by thinning the bell all round, from G towards B.

3. That in a peal of (say) eight bells, the "thirds" should be all major or all minor. This is considered correct; but we should rather like to hear a peal in which the "thirds" of the treble 4th, 5th and tenor bells are "major," and those of the rest "minor."

The "fifth," in like manner, cannot be sharpened; but may be flattened by thinning the bell all round, from B towards G.

And now a word about the tone which we have called the "hum" note – the deepest and most persistent of all the tones in a bell.

I have stated that this tone should be a true octave below the fundamental, and a double octave, therefore, below the nominal, thus forming the Bass note of the whole system. That it was really intended so to be, is surely manifest from the fact that it is found *nearly* to fulfil these conditions, in almost any bell; and that, in most cases, it is not more than half a tone sharper than the fundamental and a quarter of a tone sharper than the nominal.

I believe also that I shall be supported in my contention by Belgian experts, and moreover we have the authority of Helmholtz for the following statement:

"According to the observations of the organist Gleitz, the bell cast for the Cathedral at Erfurt in 1477 has the following proper tones—*E*, *e*, *g*♯, *b*, *e'*, *g'*♯, *b'*, *c'*♯." And "Hemony of Zütphen, a master in the seventeenth century, required a good bell to have three octaves, two 'fifths,' one major and one minor 'third.'"

This I think should settle the question, not only of the hum-note, but of the whole theory of bell tones as set forth in these papers.

Before speaking of the method of altering the hum-note, so as to bring it into line with the other tones, I should like to say a word as to its practical importance.

The fact that it is the bass of the whole system of tones, in any perfectly attuned bell, ought to be sufficient. And though it is not so loud as the fundamental, nor so clear as the nominal, it is impossible that any want of agreement with these should not injure the general effect, especially if in a *series* of bells these great "booming" tones do not rise and fall fairly evenly with the others. In the case of large slow-striking bells of clocks, or carillons, this want of agreement with the other tones must be specially objectionable, as the hum-notes are, as we have said, the most *persistent* of all.

But their general importance may be made clearer by the following illustration :—

Sitting in my dining-room, with outer and inner doors shut, I was struck by the singularly sweet sound of our six little Fittleworth bells as heard down the chimney. On opening the doors, so as to hear them *directly*, I observed with surprise that the scale was different; and I finally discovered that what I had heard down the chimney were the hum-notes, which *alone* found their way to me by this devious course.

After this experience, we cannot dismiss the hum-notes as unworthy of careful attention.

How to govern them and bring them into unison with their proper fundamentals and nominals? is a question which I am not prepared to answer with quite the same certainty as in the case of the other tones. To be able to do so, we must first determine on what proportions of the bell its "pitch" mainly depends.

Now, in the case of the other tones this is sufficiently known; and it is from this knowledge that we are able to deduce the rules for altering them, as given above.

But there is a mystery about the production of the hum-note which has not yet been quite cleared up. I give here my own belief, founded on observations which I cannot here describe at length. I believe, then, that, whereas each of the other tones may be referred principally to some particular *portion*, or zone, of the bell, the hum-note is, in a peculiar sense, the tone of the *whole mass* of the bell. And I have come to the following conclusions as to its government :—

1. That this tone can never be sharpened.

2. That it may be flattened to any reasonable extent by thinning the walls of the bell near to the crown—*i.e.*, near to A in the figure.

3. That, possibly, the proportion of the "length" of the bell to its "width" may have some bearing on the relation of the hum-note to the other tones; any shortening of the length in proportion to the other dimensions having the effect of sharpening the hum-note.

These are points, however, which can be easily settled by any bell-founder; and, should they turn out as I suggest, the whole problem will be solved; and, thenceforward, no founder ought to permit at least any large clock-bell to leave his hands which has not its hum-note, fundamental, nominal, "third" (and perhaps "fifth"), in harmony with each other.

I know of no such bell at present. Who will be "first in the field" to produce it?

But though I do not know of any completely attuned bell, I should wish to do justice to modern founders; and I am bound to say that, as a rule, the bells produced nowadays by our best founders are more nearly in harmony than are the majority of our *old* church bells; especially is this the case with *large* bells.

A remarkable instance of this may be heard any day by dwellers in London. I refer to the Great Bell of St. Paul's Cathedral. This bell, weighing nearly seventeen tons, hangs in one of the western towers, and is rung every day at 1 p.m. Its position in the tower, sunk as it is so much below the orifices, is very unfavourable to its "carrying" power, and to the *even* development of its tone. Nevertheless, it can be heard very well from any point of the space round Queen Anne's Statue.

I have tested this bell, as far as my instruments would permit, with the result that I have found all the tones, so far, in perfect accord: *i.e.*, the fundamental and the nominal are in true octave, each being a true E♭. The tone next above the fundamental is also in perfect tune with it, but with this peculiarity—that the note, instead of being a "third" above the fundamental, is a "fourth," *i.e.*, A♭ instead of G or G♭. This is a pity; and the more so that, from the position of the bell in its tower, this tone is heard in undue proportion to the others. Nevertheless, the general effect is musical, and very pleasing.

The hum-note of this great bell was too deep for a scientific test by any instrument within my reach, but it appears to be in complete unison with its fundamental. Altogether, this bell is a very noble specimen of modern English bell-founding.

There still remains one very important matter to be considered, which has already, no doubt, suggested itself to those who have had the patience to read these articles. "How," it will be asked, "are we to discriminate, with accuracy, all these various tones, so as to be able to compare them, and note the various degrees of error?"

This is a most pertinent inquiry, for unless this can be done I am confident that any knowledge of the methods of *altering* these several tones will be practically useless. And more, had I not possessed some method of eliciting *each* tone, separately, I should never have attained what knowledge I have on this subject.

The method which I have used for about twenty-three years, I have reason to think, is not unknown to foreign experts; but, as far as I am concerned, is my own invention, quite effective, and very simple to those who know how to use it.

But it is not merely a piece of information to be imparted by words, but rather an art to be learnt; and I am unwilling to risk the probability of its being pronounced "a failure" by those who, acting on a mere verbal description, should make nothing of it.

If any of my readers are really desirous of making themselves masters of this "art," a letter addressed to the writer at Fittleworth Rectory, Sussex, shall receive every attention; and I can promise to founders a most valuable help in their difficult work, and to any lover of music a fruitful source of interest and pleasure

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THE VIBRATION OF BELLS

By Arthur Taber Jones

Abstract

Relative frequencies, nodal meridians, and nodal circles of the bells of the Harkness Memorial Chime.—This is a study of the ten bells of the Harkness Memorial Chime at Yale University. The large size of these bells makes it possible to examine partial tones of higher order than could be studied on smaller bells. For the first ten partials the average values found for the relative frequencies, the numbers of nodal meridians, and the numbers of nodal circles are shown in the following table.

Partial	1	2	3	4	5	6	7	8	9	10
Relative Frequency	0.50	1.00	1.20	1.49	2.00	2.56	2.96	3.28	3.80	4.06
Nodal Meridians	4	4	6	6	8	8	10	8	10	12
Nodal Circles	0	1	1	1	1	2	2			

The nodal circles for the 2d, 3d, 4th, and 5th partials are at levels about 1/3, 1/2, 1/6, and 1/2 the height of the bell, those for the 6th partial are roughly at 1/5 and 3/5, for the 7th at 2/5 and 4/5 of the height.

Amplitudes of the partials.—Curves showing the vibration in the air near the bells were obtained by a microphone, amplifier, and oscillograph. There is first an irregular motion which may arise from noises in the mechanism by which the bells are struck. The clapper probably strikes after some 0.01 sec. to 0.03 sec., for after this time the amplitude is larger and the curve more regular. Since the partial tones of a bell are not harmonic a Fourier analysis is not directly applicable, but a *modified Fourier analysis* may be employed. An approximation to the amplitude of each component is obtained by proceeding as if a Fourier analysis were being made, the range of integration for each component being however independently chosen as a whole number of periods of that component. A method is given for obtaining corrections to the approximate amplitudes thus found.

The strike note of bells.—The curious phenomenon of the *strike note* is considered. The hypothesis that the pitch of the strike note is determined by that of the fifth partial, but that it is very generally judged an octave lower, is supported by several facts which have been already known and by the new fact that the fifth partial appears with surprising promptness and vigor. One possible cause for such a misjudging of the octave is the fact that the fifth, seventh, and tenth partials are all brought out well when a bell is struck on the soundbow, and that the relative frequencies of these partials are not far from 2:3:4, thus perhaps giving rise in the ear of the observer to the pitch of the strike note.

Introduction

THE vibrations of bells are not well understood. Even on so elementary a matter as the number and position of nodal lines the very meager published values are not in satisfactory agreement. Moreover the different partial tones of a single bell usually bear inharmonious relations to each other, and the relative frequencies of the partial tones in one bell often differ from the relative frequencies in other bells—even when the bells were cast

by the same founder. As to the frequencies which it is desirable for the partials to have, there seems now to be more or less agreement[1] that in the best bells the first seven partials have frequencies in the ratio 1:2:2.4:3:4:5:6, although the third partial is sometimes represented by 2.5 instead of 2.4, being thus a major third above the second partial instead of a minor third above it.[2]

In addition to the partials of a bell certain other tones can be heard. The most important of these is called the "strike note." The strike note is the most prominent note heard when bells are sounded in fairly rapid succession. It is usually this note that it meant when the pitch of a bell is given. Griesbacher[3] states that the strike note is audible at distances at which none of the partial tones can be heard. In bells that are well tuned the strike note coincides with the second partial, but in most cases the two tones differ in pitch, often to the extent of a major second or more. If a tuning fork has nearly the pitch of a partial tone, beats are easily detected; but no beats can be obtained from a fork and the strike note. The strike note is not a combination tone, and Blessing[4] states that when sounding with other notes it is not capable of producing combination tones. The partial tones are produced, as we should expect, by division of the bell into various numbers of transversely vibrating segments separated by nodal lines. The strike note appears to be produced in some other way. The partial tones can be elicited by resonance, and when a bell has been struck they can be reinforced in the usual manner by Helmholtz resonators. The strike note is not reinforced by a resonator, and Biehle[5] has even called it an "imaginary tone." Neither is it easy to obtain the strike note by resonance. A tuning fork may be adjusted until its pitch is that of one of the partial tones of a bell. If the fork is then struck and its stem pressed against the bell the partial tone comes out very clearly. When the strike note has a pitch different from that of the second partial it is not easy to obtain any such response from the strike note. Griesbacher[6] has found however that by tuning a fork to the pitch of the strike note, setting the fork into vibration, and then pressing either

[1] J. Biehle, Archiv für Musikwiss. **1**, 306 (1919); W. W. Starmer, Beiaardkunst, Handelingen van het Eerste Congress, p. 69, Mechelen (1922); William Gorham Rice, Carillons and Singing Towers of the Old World and the New, p. 226, Dodd, Mead, and Company (1925); P. Griesbacher, Glockenmusik, pp. 14, 23, Alfred Coppenrath (1927).

[2] It is possible to tune the partial tones of a bell to some extent by cutting out material from the inside of the bell at different levels. This fact was known in the seventeenth century to the two Hemony brothers, who cast bells that are still famous for the quality of their sound. But the art was lost and not rediscovered until some thirty years ago. The rediscovery was made independently by Canon Simpson (Pall Mall Magazine, Oct., 1895, and Sept., 1896) and more recently by Johannes Biehle. Simpson's method has been developed by John Taylor and Company of Loughborough, England, and Gillett and Johnston of Croydon, England. So far as I know, all of the dozen or more carillons that have been installed in this country and Canada during the past four or five years have been cast by one or the other of these two firms.

[3] Griesbacher, ref. 1, p. 40.

[4] P. J. Blessing, Phys. Zeits. **12**, 597 (1911).

[5] Biehle, ref. 1, p. 296.

[6] Griesbacher, ref. 1, pp. 53–55.

the stem or the lowest part of a prong against the lip, where the inner and outer surfaces of the bell meet, it is possible to obtain the strike note.[7]

Griesbacher appears to think[6] that in the production of the strike note the metal on the inner and outer sides of the soundbow, on which the clapper strikes, vibrates in opposite directions. That would seem to mean that the waves which produce the strike note are compressional rather than flexural. But this explanation seems to be ruled out[8] by work which I did[9] some years ago on the Dorothea Carlile Chime at Smith College. Moreover if the strike note were due to such compressional waves it would seem that it should be possible to pick it up by a Helmholtz resonator held just beyond the lip in the region where flexural vibrations of the bell would give a minimum of intensity. I do not find this to be the case.

In a number of bells which Rayleigh[10] examined he found that the strike note was an octave below the fifth partial.[11] Biehle[12] and Griesbacher,[13] both of whom have examined many bells, state that the strike note is almost always an exact octave below the fifth partial, but that in rare cases there are slight deviations from this relationship. In my study of the Dorothea Carlile Chime I found the strike note an octave below the fifth partial, and I suggested as a possible explanation of the strike note that the fifth partial may perhaps come out with especial promptness and prominence, the octave in which it lies being however very generally misjudged.[14]

[7] Griesbacher states that he has been able in this way to obtain the strike notes of hundreds of bells. His book did not reach me until most of the experimental work reported in this paper had been completed. I was skeptical about the possibility of getting the strike note to respond to a tuning fork, but I have succeeded on two bells in verifying his statement that it is possible. I have also succeeded in getting the strike note to respond by holding the fork at an oblique angle to the surface of the bell, and then touching the stem lightly to the soundbow. If my explanation of the strike note as a misjudged octave is correct, I do not see why this effect should be obtained.

[8] Suppose that we have a bell and also a ring made of bell metal. Then from measurements on particular bells it is not difficult to show (See ref. 9 and Lamb, Dynamical Theory of Sound, ed. of 1910, p. 136) that if the ring is to vibrate in the compressional mode which has the lowest frequency, and if the frequency of the ring is the same as the strike note of the bell, then the diameter of the ring must be nearly three times the diameter of the mouth of the bell. Now it is true that the normal modes of vibration for a ring differ from those for a bell, but it hardly seems likely that for compressional modes of the same frequency the dimensions would differ so widely.

[9] A. T. Jones, Phys. Rev. **16**, 247 (1920).

[10] Rayleigh, Phil. Mag. **29**, 1 (1890), and Theory of Sound, ed. 2, vol. 1, p. 393.

[11] The statements in Rayleigh's paper in the Philosophical Magazine have been quoted (Winkelmann, Hdbuch d. Physik, ed. of 1909, vol. 2, p. 405) and seemed to indicate that the strike note was usually two octaves lower than the fifth partial, but in his treatment of the same question in the Theory of Sound he is careful to state definitely that "the nominal pitch, as given by the makers, is an octave below" the fifth partial.

[12] Biehle, ref. 1, pp. 299–300.

[13] Griesbacher, ref. 1, pp. 54, 57.

[14] Cases of error in the judgment of an octave are not uncommon. Helmholtz states (Sensations of Tone, tr. Ellis, ed. 4, p. 62) that when Tartini discovered combination tones he gave the pitch an octave too high, and that Henrici gave for some of the partial tones of tuning forks pitches an ocatve too low. Rayleigh states (ref. 10, p. 5) that "pure tones are often

The work described in this paper was undertaken in the hope that it would clear up some of the discrepancies as to the nodal lines of bells, and would also contribute to an understanding of the strike note. This latter purpose was to be attained by securing information as to the relative intensities of the different partials, and especially by finding out whether the fifth partial really is very prompt and vigorous. Most of the work was carried out on the bells of the Harkness Memorial Chime[15] at Yale University.

The Frequencies of the Partials

The frequencies of the partials were determined by counting the beats which they made with notes from a vacuum tube oscillatory set. This set was calibrated by comparison with thirteen tuning forks made by König, and during the work on the bells the oscillator was compared frequently with a tuning fork of known frequency. The frequencies obtained are not as reliable as had been hoped. They can probably be trusted to one percent.

The results are summarised in Table I, which also includes a summary of the corresponding results from the earlier work on the Dorothea Carlile

Table I. *Relative frequencies of the partials.* The frequencies are calculated with reference to the fifth partial, and the frequency of the fifth partial is taken as 2, so as to make the frequency of the strike note equal to 1.

Partial	Ideal Ratios	Dorothea Carlile Chime			Harkness Chime		
		No. of Bells Examined	Av.	Av. Dev.	No. of Bells Examined	Av.	Av. Dev.
1	0.50	12	0.58	0.010	10	0.50	0.004
2	1.00	12	0.92	0.037	10	1.00	0.008
3	1.20	12	1.19	0.016	10	1.20	0.013
4	1.50	12	1.71	0.023	10	1.49	0.014
5	2.00	12	2.00		10	2.00	
6	2.50	6	2.74	0.069	10	2.56	0.045
7	3.00	6	3.00	0.028	10	2.96	0.022
8	3.33				7	3.28	0.036
9	3.75	2	3.77	0.177	7	3.80	0.113
10	4.00	1	4.07		7	4.06	0.052

Chime. It will be seen that the relative frequencies of the first five partials on all the bells of the Harkness Chime are very close to those indicated above as given by the best bells. The tenth partial is about an octave above the fifth, and the seventh, eighth, ninth, and tenth do not differ widely from the series *sol*, *la*, *ti*, *do*. If these values were precise, and we use a scale that is not tempered, the relative frequencies of the first ten partials of an ideal bell would be those given in the second column of the table.

estimated by musicians an octave too low." Obata (Frank. Inst. Journ. **203**, 659 (1927)) has recently found that the sounds from the striking of bricks and from xylophones "give a feeling of sounds about two octaves lower than those determined by the oscillographic records."

[15] The Harkness Memorial Chime consists of ten bells cast in 1921 by John Taylor and Company. The largest bell is $f\#_2$, with a strike note of about 182 cycles/sec. It weighs about 6300 kg, and has a maximum diameter of 219 cm. The smallest bell is g_3, of about 385 cycles/sec. This bell weighs about 680 kg, and has a maximum diameter of 104 cm.

Nodal Lines

General considerations. Two systems of nodal lines are generally recognized in bells. One is a system of meridians which run up and down the bell at different azimuths, and the other is a system of circles which lie at different levels. Rayleigh[10] and Vas Nunes[16] examined a few of the nodal lines on a few bells. Biehle[12] has examined the partial tones of several hundred bells, and has published some brief statements about nodal lines. van der Elst[17] is developing a bar resonator which is intended in part for studying the nodal lines of bells, but I think he has as yet published nothing as to results obtained with it.

On the Harkness Chime I have examined the nodal lines in practically the same way that I described in connection with the work on the Dorothea Carlile Chime.[9] That is, I connected a Helmholtz resonator through a piece of rubber tubing to the binaurals of a stethoscope, tuned the resonator by covering the mouth to a greater or less extent with a finger, moved the resonator about near the surface of the bell, and listened for the positions of minimum intensity.

Nodal meridians. The numbers of nodal meridians which I found for the first ten partials on the bells of the Harkness Chime are shown in Table II.

Table II. *Nodal lines.* The positions of the nodal circles are indicated by fractions. The unit chosen is the distance measured along the outer surface of the bell from the lip to the shoulder. Numbers in parentheses give average deviations. The middle of the soundbow, on which the clapper strikes, is at 0.12 (0.007). Except where otherwise noted the results are from observations on all of the bells.

Partial	No. of Nodal Meridians	No. of Nodal Circles	Location of Nodal Circles	Remarks
1	4	0		
2	4	1	0.32 (0.037)	
3	6	1	0.53 (0.010)	Position of circle determined on all bells except two of the largest.[18]
4	6	1	0.16 (0.026) 0.15 (0.010)	These two values were obtained by measuring along the outer and inner surfaces of the bells.
5	8	1	0.47 (0.045)	
6	8	2	0.21 (0.022) 0.55 (0.021)	
7	10	2	0.43 (0.031) 0.76 (0.067)	Locations of these two circles are avs. for all except the two smallest bells.
8	8			
9	10			
10	12			

[16] Abraham Vas Nunes, Experimenteel Onderzoek van Klokken van F. Hemony, pp. 95–97. Dissertation, Amsterdam (1909).

[17] W. van der Elst, Physica, **6**, 42 (1926).

[18] Most of the determinations of the location of the nodal circle for the third partial were made after the rest of the work was completed and ladders had been removed from the tower. See footnote 21.

These numbers agree with those which I found on the Dorothea Carlile Chime.[19] My results agree with those of Rayleigh up to the fifth partial. The largest bell that he used was considerably smaller than the smallest on the Harkness Chime, so that for higher partials he was not able to examine nodal meridians. Biehle gives numbers of nodal meridians for the first eight partials. For the first seven my results agree with his.[20] My results do not agree entirely with those of Vas Nunes. But not only did Vas Nunes work without resonators—which makes any determination of this sort very difficult—but the series of frequencies which he found is somewhat different from the series that I have found, and it may be that there are also differences in the nodal lines.

Nodal circles. The nodal circles are more difficult to locate than the meridians. My results for the first seven partials are given in Table II. On comparing with Table I in my former paper it will be seen that there is good agreement, and that on the Harkness Chime I have been able to locate circles for the fourth and seventh partials in regions of the bells where I was not able to detect those partials on the smaller bells of the Dorothea Carlile Chime. In the case of the fourth partial there is a nodal surface that cuts through the material of the bell in the neighborhood of the sound-bow. Mechanically it seems remarkable to have a surface of minimum transverse motion so close to a parallel free edge.

Biehle[12] gives the numbers of nodal circles for the first eight partials as 0, 1, 0, 1, 0, 1, 0, 1. It will be seen that except for the first, second, and fourth partials my numbers do not agree with his. For any one partial Vas Nunes[16] was not able to detect more than one nodal circle, although he supposed that the higher partials would have more. Since the partials which he found differ from those which I found, it is somewhat difficult to compare results. But in so far as I am able to compare tones that seem to correspond, the results for the nodal circles appear to agree. Rayleigh[10] gives results as to nodal circles for the first four partials. His results are definite for only the first two, but in no case is there disagreement between his results and mine.[21]

[19] Comparison with my former paper does not show agreement in the numbers of nodal meridians for the highest partials. In preliminary work on the Harkness Chime I missed the eighth partial, and it turns out that in the study of the Dorothea Carlile Chime I also missed it. On checking over the higher partials on the two largest bells of the Dorothea Carlile Chime—the only ones on which I was able to detect partials higher than the seventh—I have found this eighth partial, and I now find that the numbers of nodal meridians on the Dorothea Carlile Chime—so far as I am able to examine them—agree with those which I have found on the Harkness Chime.

[20] For the eighth partial Biehle gives ten nodal meridians—the same number that I published for the Dorothea Carlile Chime when I thought that the ninth partial was the eighth.

[21] In the case of the third partial Rayleigh gives a result for one bell. He says, "There is no well-defined nodal circle. The sound is indeed very faint, when the tap is much removed from the sound-bow; it was thought to fall to a minimum when the tap was about halfway up." In my paper on the Dorothea Carlile Chime I indicated that the third partial has one nodal circle. On the Harkness Chime I had great difficulty in deciding whether this partial really has a nodal circle or not. There was no doubt that the note heard in a resonator tuned to this partial disappeared and then reappeared when the resonator was moved up and down along the

I have not found the nodal circles for partials higher than the seventh. But I may note that the first, second, third, fifth, seventh, and tenth partials are well brought out by tapping on the soundbow, whereas the fourth, sixth, eighth, and ninth are better obtained by tapping higher up. The eighth and ninth are separated by tapping at slightly different levels on the waist of the bell. These facts may perhaps mean that when a bell is struck by the clapper the fourth, sixth, eighth, and ninth partials do not come out as strongly as the others, so that in tuning a bell especial care should be given to the first, second, third, fifth, and seventh partials.[22]

The Amplitudes of the Partials

Experimental arrangement. Curves representing the sounds from the different bells were obtained by using a high quality microphone, an amplifier, and an oscillograph with recording camera. The accuracy of transmission was tested by holding successively near the microphone three tuning forks which differed considerably in pitch, each provided with the proper resonator. The distortion proved not to be important. The speed at which the photographic film moved was obtained by records from current in a shunt around an electrically driven tuning fork. But as the oscillograph which was available had only a single vibrator, curves from the fork and a bell could not be taken at the same time, and this casts some doubt on the accuracy of the timing.

Analysis of the curves. Since the periods of the partial tones of a bell do not form a harmonic series, a Fourier analysis of the curves is not directly applicable. The method finally adopted was the following. Some fifty to eighty equally spaced ordinates were measured and then multiplied by $\sin nt$ and $\cos nt$ respectively, where n stands for 2π times the frequency of some chosen partial tone. By adding these products approximations were obtained to the values of the integrals $\int y \sin nt\, dt$ and $\int y \cos nt\, dt$, the integration extending over a whole number of periods of the chosen partial. The amplitude was calculated as if a Fourier analysis were being made. The process was repeated for other sets of ordinates, so that from two to five values for the amplitude were obtained. The results were then averaged. The values obtained are in error, partly because of experimental inaccuracies

waist of a bell. But I came to think that this effect was due to the second partial overpowering the third in this region, and consequently stated, in a preliminary report on a part of this work (Phys. Rev. **29**, 616 (1927)), that the third partial has no nodal circle. When I attempted to check this over again on the Dorothea Carlile Chime I did not feel so sure, and after trying several methods that did not lead to the desired certainty I went back to Rayleigh's method of holding a resonator at one point while I tapped up and down the bell. This method seemed to show rather definitely a nodal circle for the third partial, and after checking over again several of the bells of the Harkness Chime by this method I now feel fairly sure that the third partial does have a nodal circle after all.

[22] On the bells of the Harkness Chime the first few partials have been tuned, on the bells of the Dorothea Carlile Chime they have not. And it is interesting to see from Table I that on the Dorothea Carlile Chime the average ratios of the frequencies for the third, fifth, and seventh partials are close to the ideal values, whereas the fourth and sixth partials deviate widely from those values.

in determining the speed of the film and the frequencies of the partials, partly because of the slight distortion in the transmission from microphone to oscillograph, partly because of the approximation involved in finding the integrals, partly because the range of integration is not a whole number of periods for partials other than the one in question, and probably partly because of an irregular motion to be mentioned shortly.

The error due to inharmonic relations between the partials may be corrected as follows.[23] Suppose at first that there are only two partials, and that the curve is represented by

$$y = a_1 \cos n_1 t + b_1 \sin n_1 t + a_2 \cos n_2 t + b_2 \sin n_2 t. \tag{1}$$

Let s be an integer, and let p be a quantity defined by the relation $sT_2 = pT_1$, where $T_1 = 2\pi/n_1$ and $T_2 = 2\pi/n_2$. Then it is not difficult to show that

$$a_2 = \frac{2}{sT_2}\int_0^{sT_2} y \cos n_2 t dt - \frac{p}{\pi(p^2 - s^2)}\left\{a_1 \sin 2\pi p + b_1(1 - \cos 2\pi p)\right\} \tag{2}$$

and

$$b_2 = \frac{2}{sT_2}\int_0^{sT_2} y \sin n_2 t dt + \frac{s}{\pi(p^2 - s^2)}\left\{a_1(1 - \cos 2\pi p) - b_1 \sin 2\pi p\right\}. \tag{3}$$

From (2) and (3) it will be seen that if p is an integer the correction terms to the right of the integrals vanish, that if p is close to an integer these terms are small, and that if p and s are very different, i.e. if n_1 and n_2 are very different, these terms are small. When (1) contains more than two components each new component adds to (2) and (3) terms of the same form as the correction terms that have been written.

A first approximation is obtained by neglecting the correction terms in (2) and (3), a second approximation can be obtained by putting into the correction terms in the extended expressions which take the place of (2) and (3) the values found in the first approximation, and further approximations might be obtained by repeating the process. Results of applying these corrections are shown in Figs. 1 and 2. In Fig. 1 no important change is produced. In Fig. 2 the importance of the third partial is brought out more strongly, but the uncorrected values already indicated that it had the largest amplitude. The calculation of these corrections takes considerable time, and it has not been carried through except in these two cases.

Results. On curves which showed the beginning of the sound there was at first an irregular motion that lasted for from 0.01 sec. to 0.03 sec., and was followed by a considerable increase in amplitude and the development of a more regular type of vibration. In the initial irregular motion there was no evident periodicity, but there were suggestions of frequencies in the neighborhood of 1000 to 2000 or more cycles/sec. Such frequencies are higher than the tenth partials of the bells that were concerned, but not nearly high enough to arise from waves running back and forth through the ball of the

[23] I find that this method of correction is not new.

clapper. It is entirely possible that they may be due to noises in the mechanism by which the bells are struck. To what extent these irregular motions continued to be superposed on the more regular part of the curves I do not know.

Only a small number of satisfactory films were obtained, and the amplitudes have been calculated for only six. Three of these were exposed about the time a bell was struck, and in all three the analysis—beginning after the initial irregular motion—shows that the fifth partial has a considerably larger amplitude than the others. Fig. 1 shows the analysis of one of these films. On a few other films that were exposed when a bell was struck there was evident without analysis a prominent vibration which had a frequency about that of the fifth partial.

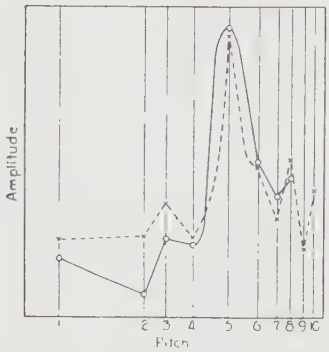

Fig. 1. Amplitudes of partials of a bell very soon after it was struck.

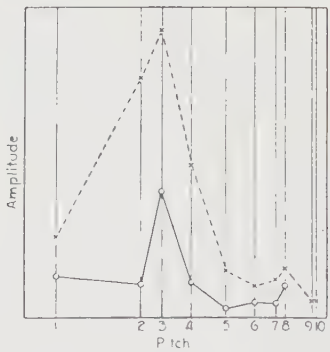

Fig. 2. Amplitudes of partials of another bell about 3 sec after it was struck.

In both these figures the abscissas are proportional to the logarithms of the frequencies, and the ordinates are proportional to the amplitudes. The numbers marked along the axes of abscissas indicate the partials. The scales for the amplitudes are arbitrary, and are different for the two figures. The points indicated by crosses and joined by dotted lines show the uncorrected amplitudes, and the points indicated by circles and joined by full lines show the corrected amplitudes.

A number of films were exposed about three seconds after the bells were struck. The general appearance of these films differs according to the size of the bell. For the three or four largest bells they show that several partials were still important. For the smaller bells they show that most of the partials had died down a good deal, leaving only one that had much amplitude. An examination of these films showed that the frequency of the tone that was now most prominent is about that of the third partial. Analyses were made of three of the films that were exposed about three seconds after the bells were struck. Two of these were cases where the curve showed only one prominent frequency, and in both of these cases the third partial does turn out to have a larger amplitude than any of the others. Fig. 2 shows one of these cases. The third curve was from one of the larger bells, and the analysis indicates that the third partial is prominent but does not have the largest amplitude.

This prominence of the third partial surprised me. I had supposed that the higher partials died out more quickly than the lower, and that the tone which persisted longest was always the first partial. I still suppose that in most bells the first partial lasts the longest, but in the cases of a number of bells to which I have recently listened I find that the third partial becomes very prominent soon after the bell is struck.

Conclusion

The work described in this paper extends our knowledge of the nodal lines of bells, and shows that, although the relative frequencies of the partial tones in the Taylor bells which form the Harkness Chime differ from those of the Meneely bells which form the Dorothea Carlile Chime, nevertheless the nodal lines of both chimes are very similar.

Although the work is not sufficiently accurate to give quantitative values for the relative amplitudes of the different partials, it does seem to show rather definitely that in the bells which have been studied the fifth partial really comes out with surprising strength in the very early stages of the vibration. Just why the fifth partial should be so prominent at first is not entirely clear. It seems to be true however that most of the vibration that gives rise to the fifth partial occurs in the neighborhood of the soundbow. One evidence of this is the fact that the surface of minimum intensity which extends outward from the nodal circle for this partial does not pass directly away from the bell, but runs upward not far from the surface.

Whatever the cause for the early prominence of the fifth partial, the fact that it is at first so prominent supports the suggestion that the strike note is determined by the fifth partial, the octave in which it lies being however misjudged. Another fact which supports this hypothesis is the short time that the strike note lasts. If the proposed hypothesis is correct it would be expected that the strike note should disappear as soon as other partials become as prominent as the fifth. Still another bit of evidence for the hypothesis is a statement by Blessing[4] that if the soundbow of a bell is turned thinner and thinner the strike note decreases in intensity, and that by the time the soundbow is cut off to such an extent that the inner and outer surfaces are parallel the strike note has disappeared. It would be interesting to know what happens to the fifth partial while this process is carried out.

If the above hypothesis is correct, why is the octave so generally misjudged? This question is really one for the psychologist, but in my former paper I hazarded two suggestions, and I may now point out further that the frequencies of the fifth, seventh, and tenth partials—all of which come out strongly from the soundbow—are nearly in the ratio 2:3:4. Now Fletcher[24] has found that when three pure tones with frequencies in the ratio 2:3:4 reach the ear together a note of frequency 1 may also be heard. This would be the frequency required for the strike note, but whether the frequencies of the fifth, seventh, and tenth partials are close enough to the ratio 2:3:4 to produce this effect I am somewhat doubtful.

[24] Harvey Fletcher, Phys. Rev. **23**, 427 (1924).

Against the proposed hypothesis we have Griesbacher's observation[6] that it is possible to get the strike note to respond to a tuning fork. Whether there is some way of reconciling this fact with the hypothesis I do not yet feel sure. The fact that the strike note will not beat with other tones of nearly the same pitch seems incompatible with any actual vibration that has the frequency of the strike note. The best hypothesis that I see at present is that of the misjudged octave.

Before concluding I wish to express my hearty thanks to all who have cooperated in making this study possible: To the Corporation of Yale University, for permission to make use of the Harkness Chime; to the Departments of Physics and Electrical Engineering at Yale University, especially to Professor W. F. G. Swann and Professor H. M. Turner, and to Mr. Enerson and Mr. Pechar, mechanicians at the Sloane Laboratory; to the General Electric Company, especially to Dr. C. W. Hewlett; to the Doolittle Radio Corporation; to the Bell Telephone Laboratories; and to Miss Evelyn H. Puffer, who has carried through a considerable amount of calculation for me.

Smith College,
January 13, 1928.

Part II

MODES OF VIBRATION AND TONAL CHARACTERISTICS

Editor's Comments on Papers 4 Through 8

4 **TYZZER**
Characteristics of Bell Vibrations

5 **CURTISS and GIANNINI**
Some Notes on the Character of Bell Tones

6 **SLAYMAKER and MEEKER**
Measurements of the Tonal Characteristics of Carillon Bells

7 **GRÜTZMACHER, KALLENBACH, and NELLESSEN**
Acoustical Investigations on Church Bells

8 **PERRIN, CHARNLEY, and dePONT**
Normal Modes of the Modern English Church Bell

Much work, beginning with Rayleigh's (Paper 1), has been devoted to determining the modes of vibration and the various factors that determine the tonal characteristics of bells. Early investigators were handicapped by the lack of electronic instruments to isolate the modes of vibration. Modes were usually driven by direct contact with a tuning fork and detected by listening through a Helmholtz resonator moved about near the surface of the bell.

A major breakthrough came when Franklin Tyzzer and his colleagues at the Riverbank Laboratories developed a means for exciting single modes by using a vacuum tube oscillator to drive an earphone in contact with the bell. In Paper 4 the vibration frequencies and nodal configurations of three bells are described. Sixty-eight different modes are identified in one of the bells, and they are classified into seven families, one particular family having a nodal circle near the sound bow. Tyzzer's families form the basis of much subsequent work on mode classification.

Three years later, in Paper 5, Curtiss and Giannini reported measurements of sound level and decay times made with an early RCA noise meter, and they also determined the spectrum of partials by beating against the output of the F_3 bell with a variable oscillator. A stopwatch was used to determine when each partial started beating

with the oscillator and when it stopped. The authors concluded that better tone quality might result from using thinner bells in the high register to more nearly match the scale of those in the low register, even though this might necessitate electrical amplification of the high register. (The results of an experiment along these lines are reported in Paper 22.)

A comparative study of bells in the United States cast by four bell founders in the United States and Europe is reported in Paper 6 by Slaymaker and Meeker. Frequencies and decay rates were measured for the principle partials in a number of bells using "modern" equipment such as a tape recorder, a sound analyzer, and a chart recorder. The bells were compared with some of the Dutch bells described in van Heuven's thesis (see Paper 9).

An extensive investigation by Grützmacher, Kallenbach, and Nellessen at the Physikalisch-Technische Bundesanstalt in Braunschweig is reported in Paper 7. The modal frequencies and modal configurations are in general agreement with those of Tyzzer (Paper 4). Extensive experiments on the influence of the mass, material, and form of the clapper on the bell sound are reported. Finally, the damping of the partials is studied. Three contributions to the damping are considered: (1) internal damping; (2) air friction; and (3) sound radiation. There is a range of maximum radiation damping which depends on the relative wavelengths of the bending waves in the bell as compared to the sound wavelength in air.

The mode classification schemes used in papers by Tyzzer (Paper 4) and by Grützmacher, Kallenbach, and Nellessen (Paper 7) are in good agreement, except for the modes that have a nodal circle or a line of minimum motion (which is not quite a node) at the sound bow. Tyzzer designates them as (m,1♯); Grützmacher, Kallenbach, and Nellessen as (m,0), where m is the number of nodal meridians. Lehr (Paper 11), Schouten and 't Hart (Paper 16), and Perrin, Charnley, and DePont (Paper 8) all agree that the hum (which has no nodal circle) belongs in the same family as the third, the octave, the upper fifth, and so forth, each of which has a nodal circle near the waist of the bell. The various families and designations are indicated in Table 1.

Schouten and 't Hart use the same designations as Lehr. In the designations of Tyzzer and Grützmacher, Kallenbach, and Nellessen, the first number is the number of nodal meridians, and the second the number of nodal circles. In the designations of Lehr (and also Schouten and 't Hart) the Roman numeral designates a family group, and the second number is the number of complete over-the-crown nodes. In the designation of Perrin, Charnley, and DePont, the first number is the family group and the second number of nodal meridians.

Table 1. Classifications of bell modes in various acousticians' papers.

Name of mode	Tyzzer (Paper 4)	Grützmacher et al. (Paper 7)	Lehr (Paper 11)	Perrin et al. (Paper 8)
Hum	4,0	4,0	I-2	RIR,4
Prime (fundamental)	4,1	4,1	II-2	1,4
Third (tierce)	6,1	6,1	I-3	RIR,6
Fifth (quint)	6,1#	6,0	II-3	1,6
Octave (nominal)	8,1	8,1	I-4	RIR,8
Upper third	8,1#	8,0	II-4	1,8
	4,2		III-2	2,4
	6,2		III-3	2,6
Upper fifth	10,1	10,1	I-5	RIR,10

In Paper 8 the modes of a modern English church bell are compared to those calculated on a computer using a finite element technique. The computer-calculated modes are described as ring-driven bending modes, shell-driven bending modes, and twisting modes (of no acoustical significance). Three of the most musically significant modes are ring-driven.

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CHARACTERISTICS OF BELL VIBRATIONS.

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A bell is a metallic structure designed for the production of sound of a pleasing quality. The quality depends upon the relative amplitudes and frequencies of the series of tones of the bell, which in turn depend on its shape and thickness. Most bells are essentially a cylinder with one end closed by a flat plate and with a flared out portion at the other end. The closed end is called the crown and the open end, where the thickness is considerably increased, is called the sound bow.

Considering the bell as a modified cylinder or as a curved shell, it is to be expected that its flexural vibrations would be characterized by nodal lines, which are equidistant meridians, and by parallel circles of latitude. The crown is of course a circular node for all vibrations. For this class of vibrations the number of meridian nodes cannot be less than four, and must be even, but there can be any number of circular nodes. In this paper a vibration will be designated by two numbers; the first, its number of meridian nodes; and the second, its number of circular nodes. Thus a 6–3 has six meridian nodes and three circular nodes, excluding the node at the crown.

In addition to the above form of vibration, two other types are possible; the first, an expansion and contraction without deformation of the circular shape, having any number of circular nodes; and the second, a vibration which resembles the compressional or longitudinal vibrations of a rod with two meridian lines of no radial motion. The frequencies of all of these vibrations can be calculated for certain vibrating systems, such as a cylinder or a flat plate, but for an irregular shape like a bell this calculation is impossible.

In the sound of a bell distinct beats can often be heard in one or more of the partials. According to Lord Rayleigh this is due to lack of symmetry in the bell. The same effect can be

produced by applying a load to the bell. If the system of meridian nodes is located so that this load is at an antinode, the frequency of the tone will be slightly lower than if the load is at a node. When the bell is struck at a node for either one of these tones, the other is excited alone, but when the blow is struck at any other point, both tones are excited simultaneously and beats are heard.

This phenomenon of beats was used by Lord Rayleigh in his investigation of bell tones. In his method the bell is tapped at various points around the circumference and the tone in question is selected by the use of Helmholtz resonators. Due to a local load fastened to the bell or to its natural lack of symmetry, beats are produced. If the point of excitation is moved around the circumference these beats will disappear for a number of points equal to double the number of meridian nodes. The circular nodes are found by tapping the bell along a meridian, and noting the points at which the intensity for the tone in question is a minimum.

Another method of investigating bells is that used by A. B. Simpson, in which the stem of a tuning fork is held in contact with the bell at different points. If the frequency of the fork coincides with one of the partials of the bell, a tone will be produced, the loudness depending on the location of the point of contact. This gives a method for finding circular nodes, but for the location of meridian nodes the bell must be restrained at one or more points while the fork is moved around the circumference to determine the number of points of least sound excitation.

Professor A. T. Jones in his work on bell vibrations, has used a resonator connected by a rubber tube to a stethoscope with which he listened to the sound coming from different parts of the bell, which is always struck at the same spot.

All of these methods depend on observations made on a sound which is constantly diminishing, and are therefore somewhat uncertain. Simpson's method is limited by the necessity of using a tuning fork of the same frequency as the tone. The methods in which the bell is excited by tapping are open to objection because of the difficulty of picking out one tone from the great number produced, even with the use of a resonator. This is especially true when working with high

frequencies. Besides this difficulty, Jones' method of listening to the sound in the air is made very uncertain by the interference with the sound from other parts of the bell. This is especially confusing when working in a room where standing waves are present.

In order to increase the range of tones to be investigated, and to get more certain and easily obtained results, the excitation should be continuous, it should excite one frequency at a time, and it should not change the frequencies of the free bell tones by loading or otherwise. In regard to the detection of the motion of the bell, the best results should be obtained by using a device which responds to the motion of the bell itself, and to motion in one direction only, as the nodes for radial motion are antinodes for tangential motion.

The tone was excited in these experiments by means of a vacuum tube oscillator, the current from which passed through an ear-phone whose pole pieces rested on a small piece of rubber covered iron in contact with the bell. The frequency of the oscillator was varied until it coincided with one of the partials of the bell; this condition being indicated by a sound maximum. To make sure that the frequency thus found was the same as that excited by tapping, a tuning fork was loaded until it gave a certain number of beats per second with the oscillator tone, and then the number of beats was counted when the tone was produced by tapping. The number of beats was found to be the same except in the case of the smallest bell tested, and even here the difference was removed by supporting the excitor so that its weight was not an appreciable load on the bell.

The first instrument used in this test to observe the motion of the bell was the pickup of a phonograph connected to the ears by rubber tubes. In this device the motion of a pin is transmitted by a lever to a diaphragm which varies the air pressure in a small cavity behind it, producing sound. This was quite sensitive, but since one side of the diaphragm was open to the air, confusing phase differences between the air vibrations and those of the bell were produced, depending upon the way the diaphragm faced. Another disadvantage was, that it responded to motion of the pin in two directions. These defects were removed in an instrument devised by Mr.

B. E. Eisenhour, of the Riverbank Laboratories. In this pickup the motion of the bell in one selected direction is transmitted to an entirely enclosed diaphragm, and the sound from the cavity behind the diaphragm carried to the ears by tubes, as before.

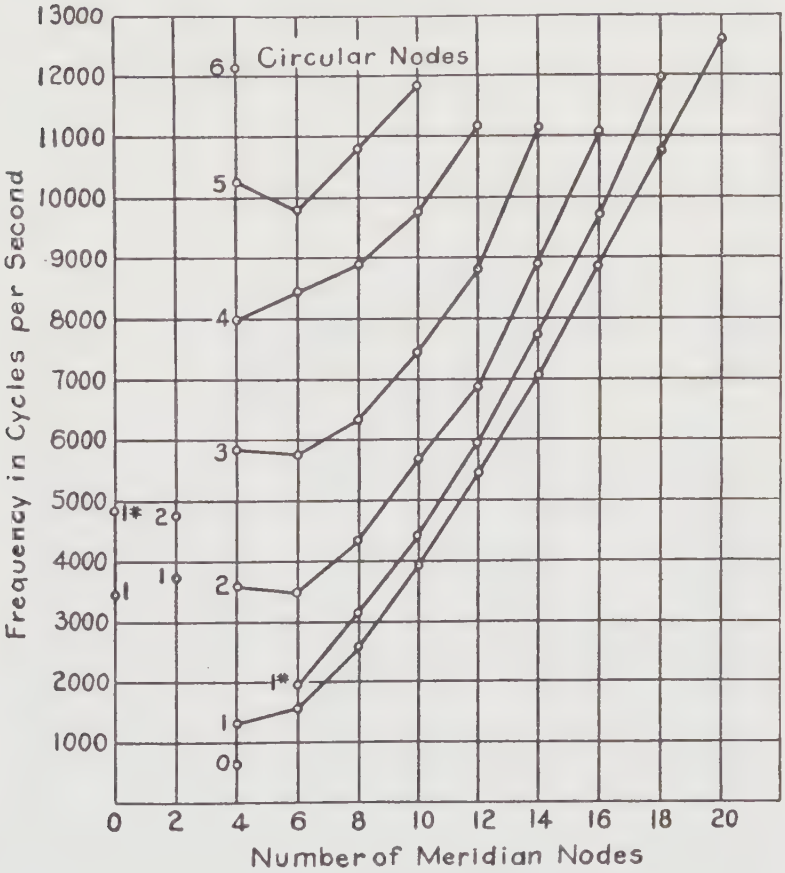
The instrument was checked for sensitivity by holding it in contact with the stem of a tuning fork, and by this means the fork could be heard even after the sound from the prongs was inaudible when held to the ear. For certain frequencies it was found that the response was cut down by resonance effects in the tubes carrying the sound to the ears. Different lengths of tubes were used for these ranges of frequencies, which were located by testing the instrument with a set of tuning forks.

The procedure of finding and classifying the vibrations was as follows: the excitor was placed in contact with the bell, and the frequency of the oscillator continuously changed until the bell was heard to respond, either directly by ear or by the use of the listening instrument. The number of meridian and circular nodes was then found by exploration over the surface of the bell with the listening device, and the frequency was determined by comparison with a set of tuning forks covering the range between 512 and 4,096 D.V. To determine frequencies higher than 4,096, a second oscillator was tuned to a known multiple of the frequency of a fork and the difference tone between this frequency and the one in question was determined by the use of the set of forks. With this arrangement it was easy to determine frequencies up to 13,000 cycles per second, which was the limit of the investigation. Frequencies below 512 were found by comparing the harmonics of the oscillator with the set of forks.

All the tones presented in the results were not discovered the first time the range of frequencies was traversed, but when the results of the first run were plotted as frequency against number of nodes, the regular series of curves obtained gave an indication of the frequencies of the missing tones. The fact that some tones were missed in the first run is due to the quite considerable differences in loudness, and to the possibility of two occurring at very nearly the same frequency. In this latter case, the two might be mistaken for one form of vibration with a slightly different position of the nodal pattern, due

to lack of symmetry in the vibrating system, as explained above. By moving the excitor around the circumference, one of the pair may be made to disappear for a number of points equal to the number of meridian nodes, thus checking the

FIG. 1.



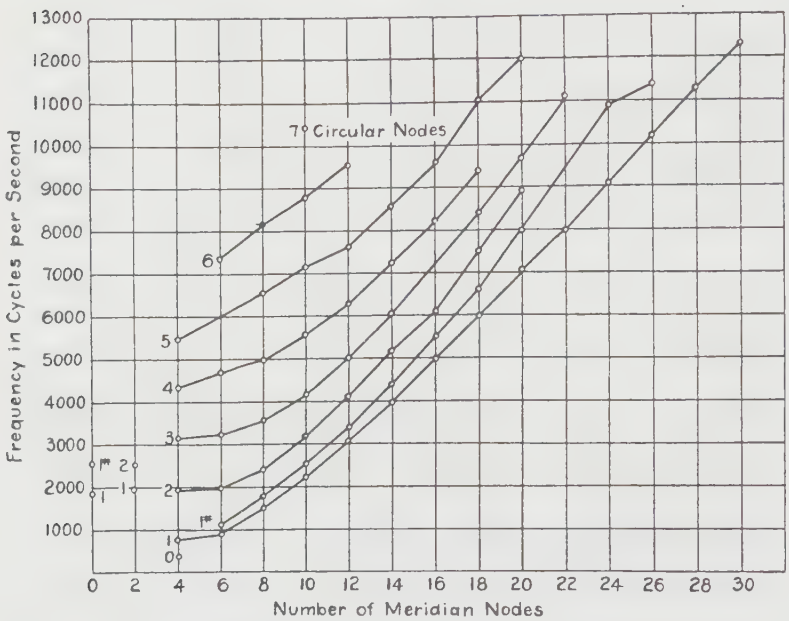
Nodal characteristics of the partials of a 35 lb. bell.

explanation of beats given by Lord Rayleigh. When the bell manufacturers pick out a certain point at which the bell is to be struck, it is in the effort to avoid beats by exciting only one of the two tones which would produce the most objectionable interference.

Three bells of different sizes were investigated, and the

tones found are presented in the form of a table. The weights of the bells were: 800, 250, and 35 lbs., and the frequencies of the hum tones were *C*-259 *F*♯ 367, and *E*♭ 650, respectively. The most complete set of partials was found for the 35 lb. bell, which gave 44 tones below the frequency limit, 13,000 cycles per second. The frequency of the tone was plotted against the number of meridian nodes, as shown in Fig. 1, giving a curve for each number of circular nodes.

FIG. 2.



Nodal characteristics of the partials of a 250 lb. bell.

The 68 tones found in the 250 lb. bell are shown in Table 1, and are plotted in Fig. 2. Since the fundamental of this bell is nearly an octave lower than the small bell, more tones were found, involving larger numbers of circular and meridian nodes, before the frequency limit was reached, but the series of tones are not as complete. The largest number of meridian nodes was 30, while 7 circular nodes were found.

The work on the 800 lb. bell was carried only far enough to be sure that the series of tones was similar to those of the two

smaller bells. The results from this bell are also shown in Table 1.

An interesting series of tones was found in which a circular node was located at the sound bow very close to the rim. To distinguish this from the tone having a circular node between the sound bow and the crown, its number of circular nodes will be designated as 1♯. The motion of the bell for this 1♯ tone is maximum about half way between the crown and sound bow, and there is almost no motion at the sound bow. This position of the node is due to the inertia concentrated at the sound bow. An analogous vibration can be produced by holding one end of a flat spring in a vise and attaching a

TABLE 1.

Tone.	Meridian Nodes.	Circular Nodes.	Frequency 250 lb. Bell.	Frequency 35 lb. Bell.	Frequency 800 lb. Bell.
1	4	0	367 D.V.	647 D.V.	269 D.V.
2	4	1	731	1,304	517
3	6	1	861	1,552	622
4	6	1♯	1,093	1,968	830
5	8	1	1,468	2,596	1,034
6	8	1♯	1,766	3,120	
7	0	1	1,850	3,430	1,308
8	4	2	1,915	3,588	1,404
9	6	2	1,934	3,482	1,413
10	2	1	1,952	3,734	1,487
11	10	1	2,200	3,920	1,540
12	8	2	2,384	4,304	1,796
13	10	1♯	2,500	4,388	2,064
14	2	2	2,522	4,635	
15	0	1♯	2,592	4,816	2,495
16	12	1	3,024	5,440	2,112
17	4	3	3,136	5,824	2,181
18	10	2	3,168	5,680	2,414
19	6	3	3,200	5,752	2,320
20	12	1♯	3,368	5,926	2,830
21	8	3	3,580	6,312	2,620
22	14	1	3,936	7,078	2,752
23	12	2	4,096	6,848	3,140
24	10	3	4,160	7,424	3,111
25	4	4	4,316	7,974	
26	14	1♯	4,381	7,712	3,688
27	6	4	4,690	8,432	
28	16	1	4,928	8,848	3,430
29	8	4	4,968	8,896	3,621
30	12	3	4,992	8,800	3,780
31	14	2	5,136	8,880	
32	4	5	5,450	10,272	
33	16	1♯	5,472	9,696	
34	10	4	5,504	9,744	4,072
35	6	5		9,792	

TABLE 1.—*Continued.*

Tone.	Meridian Nodes.	Circular Nodes.	Frequency 250 lb. Bell.	Frequency 35 lb. Bell.	Frequency 800 lb. Bell.
36	18	1	5,942	10,720	4,126
37	14	3	6,009	11,120	
38	16	2	6,016	11,072	
39	12	4	6 246	11,158	4,672
40	8	5	6,528	10,800	
41	18	1#	6,594	11,936	
42	20	1	7,024	12,592	
43	10	5	7,120	11,840	
44	4	6		12,160	
45	14	4	7,200		
46	6	6	7,332		
47	18	2	7,488		
48	12	5	7,597		
49	22	1	7,936		
50	20	1#	7,978		
51	8	6	8,120		
52	16	4	8,186		
53	18	3	8,381		
54	14	5	8,510		
55	10	6	8,736		
56	20	2	8,880		
57	24	1	9,040		
58	18	4	9,376		
59	12	6	9,504		
60	16	5	9,568		
61	20	3	9,648		
62	26	1	10,144		
63	10	7	10,400		
64	24	1#	10,830		
65	18	5	11,008		
66	22	3	11,072		
67	28	1	11,232		
68	26	2	11,332		
69	20	5	11,968		
70	30	1	12,238		

weight to the other end. When the spring is struck a sharp blow, it will vibrate transversely without moving the weight, except for a slight motion perpendicular to the motion of the spring. This system of circular nodes was found for each number of meridian nodes from six up. There seems to be no reason why a 4–1# should not occur, but it could not be found in any of the bells tested.

Two tones were found in each bell consisting of expansion and contraction without deformation of the circular shape. The number of circular nodes was 1 and 1#. This form of vibration was not found for larger numbers of circular nodes, and there was no way to tell just where to look for the more

complicated forms, as it did not follow the same sort of a curve as the vibrations having meridian nodes.

Two tones, having two meridian nodes, were found in each bell, a 2–1 and a 2–1♯. The two meridians, are merely lines of no radial motion, as the radial component of the motion on either side of the line is in the same direction. On investigating the direction of the motion, it was found that it was not radial as in all the other tones, but all parts moved parallel to each other, perpendicular to a plane passed through the two meridian lines. This is different from the transverse vibration where a deformation of the circular shape would produce a minimum of four meridian nodes.

In every case except three, the frequencies increased with the number of meridian nodes for a given number of circular nodes. These exceptions occurred in the 35 lb. bell where the 4–2, 4–3 and 4–5 had higher frequencies than the 6–2, 6–3, and 6–5, respectively. These reversals can be explained by the fact that the circular nodes did not remain in the same place as the number of meridian nodes was increased. Thus the shifting of the position of a given number of circular nodes in the unequal cross-section of the bell, counteracted the frequency rise that would be expected from an increase in the number of meridian nodes. In all cases the rise of frequency as the number of nodes increased from four to six, was small.

The simplest form of vibration with no circular nodes was found only with four meridian nodes. Excluding this series, it can be seen, by referring to the curves, that the 47 partials below 12,000 D.V. found in the 35 lb. bell are complete except for one tone, the 4–1♯, not found in any bell.

The results, here given, agree with those of Lord Rayleigh for the first five partials, except that he did not find any circular node for the fourth partial which is located at the sound bow. He noticed, however, that this tone was best excited by tapping the middle part of the bell. A. T. Jones found the number of meridian nodes for ten partials, and the number of circular nodes for the first seven of these. A comparison of these results is made in Table 2. It is believed that the improved apparatus used and the completeness of the series of tones found justify confidence in the results presented.

TABLE 2.

Comparison of the Lower Partials with Those Found by Other Investigators.

	Lord Rayleigh.	A. T. Jones.	This Investigation.
1	4–0	4–0	4–0
2	4–1	4–1	4–1
3	6–1	6–0	6–1
4	6–?	6–1	6–1#
5	8–1	8–1	8–1
6		8–2	8–1#
7			0–1
8			4–2
9			6–2
10			2–1
11		10–2	10–1
12		8–?	8–2
13		10–?	10–1#
14			2–2
15			0–1#
16		12–?	12–1

No attempt was made to provide a formula for the curves of frequency plotted against the number of circular nodes, as the shape of a bell is so irregular and bells made by different manufacturers do not, in general, have the same shape. Calculations of the frequency of the tones having two meridians were made by assuming that they were the same as the compressional waves in a rod with a length equal to one-half the circumference of the bell at the soundbow. For the 35 lb. and the 250 lb. bell the frequencies as calculated were 3,660 and 1,830 D.V. and as measured in the 2–1 form they were 3,734 and 1,952 D.V. respectively. This calculation does not take into account the varying diameter of the bell, the presence of a circular node, or the fact that the vibrating segment is not straight but curved.

In locating tones it was found useful to plot frequency against the number of circular nodes, as well as against the number of meridian nodes, as in the curves presented. This gave a set of curves one for each number of meridian nodes, but is not an independent way of plotting the results.

The quality of a bell is commonly regarded as being determined by the relative frequencies of the first five partials. These are supposed to be according to the ratios 1–2–2.4–3–4, or in musical terms; a fundamental or hum note, its octave, a

minor third, a perfect fifth, and the next octave. Thus the musical notes corresponding to the first five partials of a C bell are: C_1, C_2, E_{b2}, G_2, C_3. Some writers extend this to seven tones, including a major third and a perfect fifth in the next octave.

The better bell manufacturers produce the proper relationship of these five tones by design of the bell casting, and by grinding or turning the bell at different places. It is difficult to avoid some grinding, since, even when casting two bells of the same size, a slight difference in the composition of the metal or in the rate of cooling would prevent duplication. Tuning is rather a difficult process as the five tones are interrelated, and work must be done in certain regions where some tones are affected more than others. The tuning is done by the removal of metal in certain relations to the positions of the circular nodes, and is possible because they are located at different points in the bell.

It is considered difficult to make small bells with a pleasing tone. This is probably due to the difference tones between the partials, and to beats, both of which are more pronounced in the high frequency bells. Since the tones are in the ratios 1–2–2.4–3–4, the difference tones between the second, fourth, and fifth tones should be the same pitch, and if any of these are a little off pitch, beats will be produced. Low frequency difference tones also give more trouble in small bells since the difference tone between two tones above 2,500 D.V. usually seems louder than the primary tones.

There has been considerable discussion about the pitch and character of the so-called strike tone of a bell. When a bell is struck, a complex sound is produced which gives the sensation of a certain pitch. This predominating pitch is called the strike or striking tone. Professor Jones gives it as the octave below the fifth tone, and regards its origin as something of a mystery. In the bells here investigated the pitch seemed to be the same as the second partial, which was not far from being an octave below the fifth partial. In such a complex sound where the various components are dying out at different rates, it is hard to judge what tone or combination of tones predominates.

In conclusion I wish to express my appreciation of the

assistance rendered by the staff of the Riverbank Laboratories, especially to Mr. B. E. Eisenhour whose many valuable suggestions and improved listening device were very helpful in the investigation.

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Some Notes on the Character of Bell Tones

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(Received June 16, 1933)

THE object of this paper is to present some new data on the frequency and position of the partials of a bell. Good bells always have been described as having five to seven stronger tones or partials in a slightly inharmonic state. These partials consist of the hum note, usually an octave lower than the strike note; the strike note which is the most predominate tone; the tierce, a third above the strike note; the quint, a fifth above the strike note; the nominal, an octave above the strike note; the upper tierce or the tenth above the strike note and the upper quint or the twelfth above strike note. An idealized numerical relation of these partials would be 1, 2, 2.4, 3, 4, 5 and 6. These numbers correspond, respectively, to the names of the partials listed above. This paper will show, among other things, how these partials do not exactly conform to this relation.

To make such a test it was necessary to find a good set of bells, adequately placed for ease in analysis and representative of bells, generally. Such a set was found at the First Methodist Episcopal Church of Germantown, Philadelphia, Pa. This set consisted of forty-eight bells from the foundry of John Taylor and Company, of Loughborough, England. They cover four chromatic octaves, less the lowest semitone, tuned from *E* flat below middle *C* to *E* flat four octaves above. The upper fourteen bells are doubled, to increase the volume of the small bells, making the total number of bells sixty-two. The bells are located in a heavy stone tower eighty-eight feet above the street level. Unfortunately the church is located on one of the busiest streets in the vicinity so that care had to be taken during the tests to exclude the high street noises.

On close inspection of these bells it was noticed that they varied from octave to octave in tone quality. The most pleasing tones were found in the heavier bells. For our analysis it was advisable to test the most pleasing and harmonious bell of the set. To find this bell, a musician with considerable experience with bells and bell music and a carillonneur were asked to select what they thought was the best bell of the set. As a check on their results and to acquire the opinion of the average laymen on bell tone, several others not so familiar with bells were asked to make their selection. All agreed on the judgment of the musician and the carillonneur. Their selections consisted of several bells in the lowest octave, of which the *F* bell, pitched at 345.3 cycles per second was chosen.

The equipment used to make this study of the bells consisted of an RCA Victor portable noise meter, a two stage a.c. power amplifier and dynamic loudspeaker, a General Radio oscillator, a portable motion picture camera with projector and a set of tuning forks. The most important equipment was the noise meter which warrants a short description.

The noise meter consists of a permanent magnet ribbon microphone to pick up the sound, followed by sufficient amplification to give a reasonable deflection for low intensity sounds on a vacuum tube voltmeter. The overall frequency response is adjusted so as to correspond with the pure tone sensitivity of the ear as determined by Kingsbury.[1] Since the sensitivity of the ear as a function of frequency depends on the intensity of the sound, two positions are made available—one corresponding to a sound intensity of approximately 70 db above the threshold of audibility at 1000 cycles, and the other approximately 40 db above this threshold. The latter is used when weak sounds are encountered and the former for intense sounds. The vacuum tube voltmeter is so constructed that its response follows the

[1] B. A. Kingsbury, Phys. Rev. **29**, 588 (1927).

square law over the useful range so that the energy of the composite sound, weighted in accordance with the pure tone frequency response at either 70 db or 40 db, is what is measured.

At its maximum sensitivity, the instrument can measure sound approximately 20 db above the threshold of audibility and sufficient attenuation is supplied so that any louder sounds which are encountered can be determined. The output of the microphone is fed to a two-stage amplifier and then to an attenuator which is calibrated in decibels. The zero level is 0.0005 bar at 1000 cycles. This is followed by another tube and switching arrangement which permits transfer from 40 db to 70 db level. This is followed by another volume adjustment which permits the sensitivity of the whole system to be adjusted to some predetermined value. The final stage is followed by the vacuum tube detector and the switching arrangement which permits the amplifier output to be carried to some external binding posts instead of to the detector for frequency analysis of the sound impressed, if such is desired.

In order to calibrate the instrument, a standard sound is supplied which is placed at a predetermined distance in front of the microphone. This sound is generated by a pitch pipe actuated by constant air pressure. The constant air pressure is obtained by having a relatively heavy piston drop under the force of gravity in a cylinder in which the pitch pipe is an outlet. The pressure supplied to the pitch pipe is thus always equal to the superficial weight of the piston, since its weight is made sufficient so that friction encountered in falling can be neglected.

Along with the analysis of the bell tones, it was thought of general interest to measure the intensity of the bells in the tower and then to check the intensity at some location on the street in order to compare the outputs and to see how much of the sound was lost or absorbed in this particular tower.

An inspection of the tower was made to locate the microphone of the noise meter so as to eliminate as much as possible standing and reflected waves. The microphone was set up about four feet from one corner of the tower pointing across to the other corner (Fig. 1) and on a level with the open louvers. The only

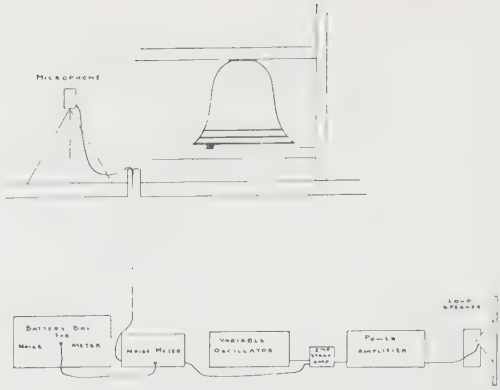

Fig. 1. Showing location of noise meter in bell tower.

equipment necessary in the tower with the bells was the microphone and stand, thus eliminating any possibility of reflection from the rest of the noise meter equipment and operators. An enclosed stairway served handily for the meter box, battery box and the operators.

The first test was to measure the intensity of the bells in decibels; at the moment of strike; ten, twenty and thirty seconds after strike; and the total time required for the putput of the bells to be attenuated to the surrounding noise level in the belfry. This test was made on two bells of each octave, namely, the *F* and *E*♭ bells. Each bell was struck with approximately the same force from the clavier with both the hand and foot. The results of this test are listed in Table I.

The second test was made with the noise meter set up outside of the church across the street at a most advantageous point to listen to the bells. The noise meter was about 160 feet

Table I.

Bell	Level at strike	Level (10 sec.)	Level (20 sec.)	Level (30 sec.)	Length of time heard	Level at strike	Level (5 sec.)	Length of time heard
	In Tower					*In Street*		
F_1	95	58	50	45	41.0	66	51	9.0
$E\flat_1$	93	59	50	44	35.0	64	45	9.0
F_2	94	52	45	—	29.0	63	42	7.0
$E\flat_2$	93	46	—	—	17.0	65.5	50	6.0
F_3	90	45	—	—	13.0	58	40	5.0
$*F_4$	76	—	—	—	5.2	45	—	2.0
$*E\flat_4$	74	—	—	—	4.2	42	—	1.2

* These are double bells.

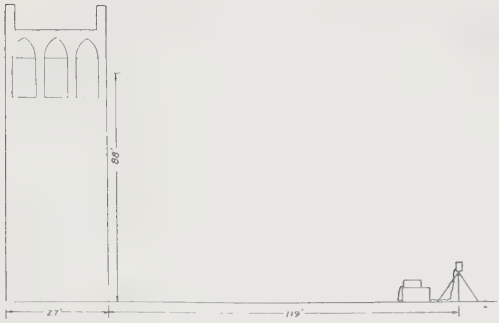

FIG. 2. Showing location of noise meter in relation to tower.

from the bells as shown by Fig. 2. The results of this test are shown in the second part of Table I.

The bells are numbered as F_1, F_2 and so forth indicating the octaves. In other words, F_1 is the first octave or the lowest octave of the set, F_2 is in the second octave and so on.

A set of curves (Figs. 3–6) has been made from the results as listed in Table I, showing the attenuation of the bells. It is rather surprising to note the amount of volume lost in the tower. Some of the bells in the upper octaves are just barely heard and if heavy traffic should be prevalent, the upper bells may not be heard distinctly. Fig. 3 shows the attenuation of the bells in the tower and Fig. 4 shows the attenuation in the street.

There is a considerable difference in the attenuation, showing a loss of volume. To show this more advantageously Table II has been made up,

TABLE II.

Bell	Intensity (db)	Sound pressure (bars)	Energy flux density (micro-watts/cm²)	Distance (feet)	Total acoustic output (watts)	Efficiency (percent)
F_2	94	25.5	1.55	6	0.326	59.5
	63	0.72	0.0013	160	0.194	
Eb_2	95	29.0	2.0	6	0.42	57.0
	64	0.81	0.0016	160	0.239	
F_3	90	16.0	0.62	6	0.131	45.8
	58	0.42	0.0004	160	0.06	
F_4	76	3.4	0.027	6	0.00567	33.0
	45	0.09	0.00002	160	0.003	
Eb_4	74	2.55	0.017	6	0.00357	41.8
	42	0.064	0.00001	160	0.00149	

giving the acoustic watts of the bells from each point. In the determination of acoustic watts, the distance the sound travels is taken into consideration, so that the acoustic watts of any bell, calculated from data collected in the tower and in the street, may be directly compared.

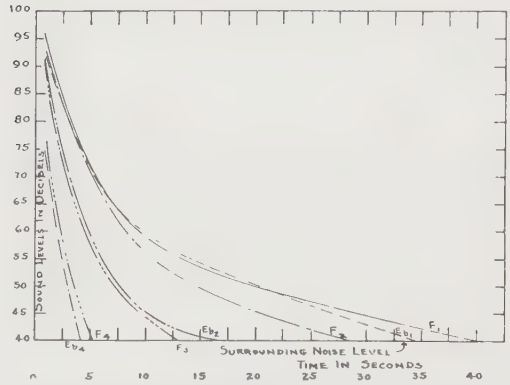

FIG. 3. Approximate attenuation of bells in tower.

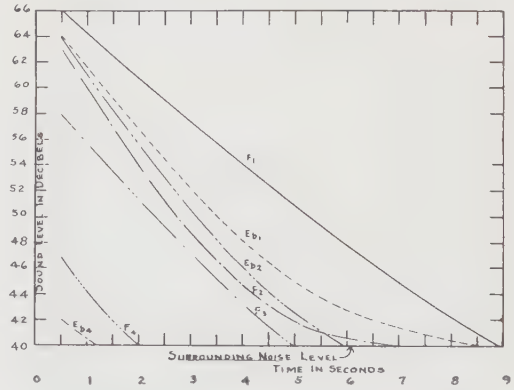

FIG. 4. Approximate attenuation of bells in street.

In the calculations made to obtain acoustic watts shown in Table II and other tables, conversions had to be made from decibels to terms which can be used numerically. Since some of the data are taken in terms of sound level above threshold of hearing for a normal ear, given in decibels, it is necessary to change this intensity in decibels to sound pressure in bars. Since the zero level in decibels equals 0.0005 bar pressure, the effective pressure corresponding to any reading is determined by the equation

$$DB = 20 \log_{10} P/0.0005$$

where P is bars pressure corresponding to intensity as expressed in decibels. The relation between pressure in bars and energy flux density per square centimeter per second is

$$P=(J\rho_0 C)^{\frac{1}{2}}$$

where P is sound pressure in bars, ρ_0 is density of air and C is the velocity of sound in air. Substituting the values of ρ_0 and C for air, the equation becomes

$$P=6.6(J)^{\frac{1}{2}} \text{ when c.g.s. units are used}$$

where J is energy flux density in ergs per square centimeter per second. This energy flux density is then changed to microwatts per square centimeter by the ratio of 10 to 1.

The equation for calculation of acoustic watts is then

Acoustical output in watts

$$=\overline{2.54\times 12\times D}^2\times 2\pi\times J/10^6$$

in which D is the distance in feet from the source of the sound to the point of reception. The (2.54×12) changes feet to c.g.s. units and J is energy in microwatts per square centimeter.[2]

The percentage efficiency as listed in column five of Table II indicates the percent of the original volume of the bells which reaches the street at the point of observation, taking into consideration the distance from the bells. It will be noticed that this efficiency has dropped considerably for the higher bells. Some of this can be accounted for with a general loss of high frequencies, but such a large percentage loss cannot all be accounted for in this manner. Therefore, the tower and its structure enters into the absorption of these higher frequencies. In the construction of the bell mountings, the upper bells are placed in a pocket above the louver level, necessitating the output of these bells to travel downward before getting out of the tower. The loss involved reduces the value of the higher bells considerably because of more rapid attenuation and lower initial volume output due to small mass of these particular bells, as compared to the heavy bells. To improve the output of these upper bells, sound directors could be used which would increase the efficiency of the higher tones in reaching the outside of the tower. More space for louvers and better location of the higher bells would also help this condition.

Many times, while bells are being played, an undertone has been noticed which is out of place and at times rather disconcerting. Sympathetic vibrations and other sources of vibration cause some of the bells to speak slightly, although they have not been struck with their clapper. The third test was for the determination of the volume output of this undertone compared to the volume output of the bells being played.

The noise meter was set up the same as for test one. Three bells were struck rather hard and then were quickly damped by holding the clappers against the bow of the bells just struck for a period of five seconds and then measurements were made on the volume of the other bells which had been set in motion by sympathetic or other vibrations. The results of this test are shown in Table III.

TABLE III.

	Intensity (db)	Sound pressure (bars)	Energy flux density (microwatts/cm²)	Distance (feet)	Total acoustic output (watts)	Percent
1	77	3.6	0.034	6	0.00714	3.3
2	52	0.2	0.0001	20	0.000235	

Part 1 of Table III is the intensity of several bells five seconds after they have been struck and allowed to attenuate freely. Part 2 is the intensity of the agitation as found. Comparing Parts 1 and 2, the "noise" created by the playing bells is 3.3 percent of the volume of the bells played five seconds after strike. The percentage is small, but the fact remains that this "noise" is present and may be strong enough in some passages of music to disturb those listening. To overcome such a possibility it may be advisable to use some damper arrangement on the bells of the lower octaves which are probably causing this agitation and which are the most sympathetic. Such a damper would work with the pedals of the clavier so that when the bell is

[2] Conversion tables and equations used taken from *Acoustical and Electrical Power Requirements for Electrical Carillon* by A. N. Curtiss and Irving Wolff, Proc. I. R. E., pp. 626–646, April (1932).

struck the damper is removed. After striking, a small time delay is used before the damper is applied to the bell in order to give the bell its natural period of attenuation.

Concluding the tests on the efficiency and undertone of the bells, which were preliminary issues to the analysis of bell tones, it was thought the first move for such an analysis and one which would prove valuable in the location of the partials, was to make a complete attenuation curve of the *F* bell from the moment of strike until its output had reached the threshold of hearing in the tower. A preliminary run of this test showed that the meters for measuring this attenuation changed too rapidly for accurate recording. Therefore, a motion picture camera was used to take pictures of the meters during the test. The camera took pictures at the rate of eight frames per second. The noise meter is used as such in this test, and was set up only a few feet from the bell to be studied. After the motion picture was taken and the film developed, the picture was run back through a projector one frame at a time. From each frame the meters were read, including the time of the picture, taken from a stopwatch which was laid on the meter panel. The results of this test are shown in Fig. 5. (The data were recorded but because of their length they have been omitted.) This curve represents the attenuation of the *F* bell (345.3 cycles). The curve has five distinct peaks and indications of several others. Each of these peaks

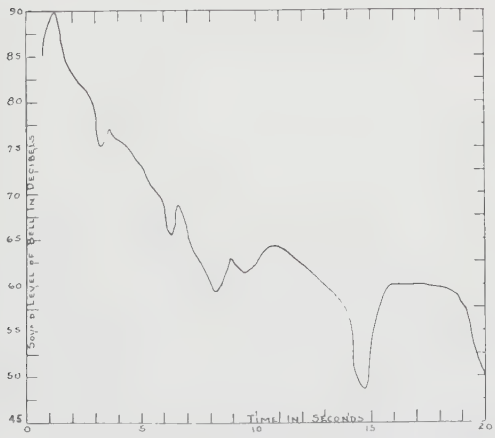

FIG. 5. Attenuation of bell after strike. *F* bell.

is caused by one of the harmonics or partials of the bell building up to maximum at different times after strike. The location and time of each of these peaks was used as a definite indication of when in the period of attenuation each of the partials might appear. Assuming that the higher partials build up and die more rapidly, which is logical, then these partials may be expected in the earlier periods of attenuation, and the lower partials expected in the latter period of attenuation.

Several runs were made on two of the heavier bells. The pressure at strike was varied from one run to the other intentionally to see how the peaks adjusted themselves. It was found that these peaks would vary in time appearance depending on the initial intensity of the bell, up to the point of overloading of the bell caused by too hard a strike.

To check these assumptions and to locate definitely each of the partials, another test was devised. For this test the noise meter is used as a microphone and amplifier, together with the General Radio oscillator and two-stage amplifier in a circuit as shown in Fig. 7. The *F* bell (345.3 cycles) was used for this analysis in order to check with the previous test. The output of the bell is amplified through the noise meter amplifier, to the two-stage amplifier to the loudspeaker. The output of the General Radio oscillator is coupled into this same amplifying system to the loudspeaker. This test for finding partials is much better than using tuning forks, because with the variable oscillator any frequency can be generated for use in searching, while tuning forks are limited to fixed commercial frequencies. Also, the intensity of the output of the variable oscillator may be varied to correspond with the sound level of the bell under test.

The variable oscillator was used to beat against the output of the *F* bell to find the various partials of the bell. A stopwatch was used to note when such a partial began to beat with the oscillator, and when it stopped. It was previously known that each partial appeared at a little different time after strike. There were a great many more partials which were weak compared to the ones which are recorded in Table IV. In this test nothing was done to determine the intensity of each of these partials because of the

TABLE IV.

Tones	Frequency in cycles	Time in seconds Begin	Maximum	End
1	130	3	5	15
2	160	2	5	10
3	187	1.5	—	8
4	205	1.25	6	15
5	215	1.	—	6
6	230	1.	—	7
7	280	0.5	—	3
8	330	.3	3	5
9	345	.25	4	5
10	365	.25	4	5
11	385	.25	2	4
12	450	.25	2	4
13	512	.25	2	4
14	570	2.0	4	5
15	640	2.0	3	4
16	675	0.25	—	2
17	700	.25	—	2
18	750	.25	—	2
19	1060	.25	1.5	3
20	1125	.25	—	1

limitations of the equipment used. Each stronger partial was merely found and then timed. The maximum noted in the table is merely a notation of the time at which that particular harmonic seemed strongest to the ear. This test was made three times on this particular bell to check the frequencies and the length of time the partials appeared. The only variation in the through tests was the time of appearance and disappearance of the listed partials. This was caused by the variation in the original intensity of the bell when struck. This variation checked the variation in the peaks of Fig. 5. The results differ from previous published work somewhat but the method of detecting these partials is new and probably gives a more complete analysis of the partials in the lower band of the scale.

Numbering the partials as is done in Table IV is merely for convenience and does not indicate an octave rating in which partials sometimes appear.

CONCLUSIONS

There has always been a controversy in the judgment of the quality of bells because of the inharmonic partials which make up the complete tone. Errors have been made by several prominent musicians and physicists of the past in the correct determination of the octave partials in particular.

The results of the last two tests of this paper emphasize the known peculiarities of bells because of the varying amplitude of the partials. At the present time bells are cast to certain definite shapes to contain the conventional partials, but no method to date has been devised

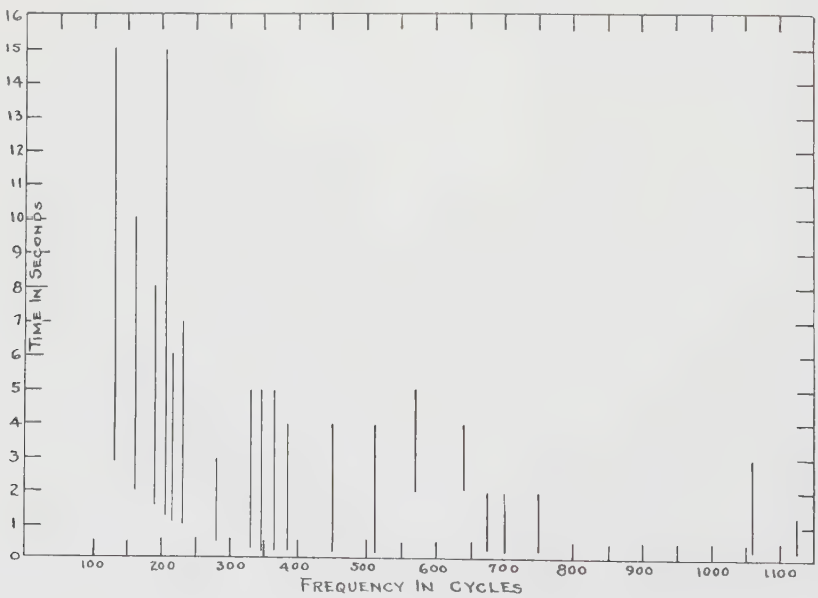

FIG. 6. Frequency analysis of bell. *F* bell tuned to 345.3 ~.

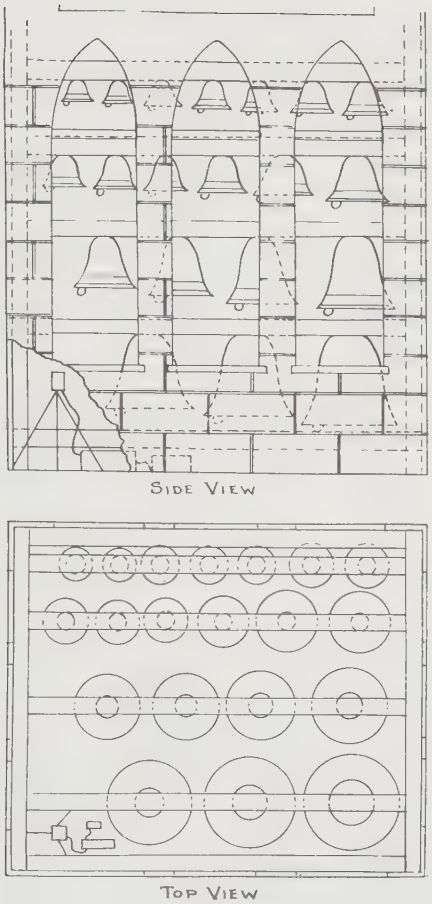

FIG. 7. Showing set-up for tone analysis.

to control the amplitude of these partials. For this reason some of the partials confuse the listener and cause the impression that the bells are out of tune. Fig. 5 shows how the overall volume of sound from a good bell in the lower octave does not decrease at any progressive rate but rather rises and falls at various stages. Each of these peaks is well defined and corresponds to the partials in the time areas of Fig. 6.

Another interesting fact brought out by Fig. 6 is that the predominate partials are not made up of one tone but several tones. An examination of the curve reveals the strike note has the following tones at 330, 345 and 365 cycles. The hum note is made up of two tones, namely at 160 and 187 cycles. The nominal is made up with two tones, namely 675 and 700 cycles. The tierce or third has two tones and is found to be wider in difference between the tones. The fifth or quint is a single tone and was about the same intensity as the third. The upper third or tenth above for this bell was in such a jumble of tones, all of about the same amplitude, that it was not recorded. The upper fifth or twelfth above is made up with one tone at 1060 cycles. The other tones not accounted for belong in the spectrum of some partial, but were not strong enough to consider seriously. A list of the partials of this bell and the tones that make up these partials might help in the discussion. The frequencies in the third column of Table V are the pitch frequencies of the notes that should appear in the bell and should be close to the ratios as shown in column 1. The strike note of the bell (345.3 cycles) was given by the John Taylor Company.

TABLE V. *Partials of F bell.*

Number	Partial	Frequency of note	Note	Tone frequency
1	Hum note	172.6	F_1	160
				187
2	Strike note	345.3	F	330
				345
				365
2.4	Third (tierce)	410.6	$G\sharp\sharp$	385
				450
3	Fifth (quint)	517.3	C	512
4	Nominal	690.5	F	675
				700
5	Upper third	870.0	A	——
6	Upper fifth	1034.6	C^1	1060

From the above table it may be seen that the bell was tuned to one of the tones in the strike note. The only partials that come close to the idealized relation mentioned in the opening paragraph are the third and fifth. These two partials were also quite strong, although they only lasted four seconds. The third partial in particular bears further discussion because of its prominent position and amplitude.

Bells in a chromatic scale have their strike notes in harmonic succession but their partials sometime make them sound out of tune. This is due to the presence of a third partial above the strike note and corresponding partials in the higher octaves. This third is usually a minor third in the bells of the middle and higher registers and gives the peculiar tone characteristic of the usual church bell and is not

strictly musical. If this third should become too strong, strict limitations are necessary in playing such bells in order to give pleasing effects.

In the bell studied (*F*, 345.3 cycles), the third partial does not stand out over the strike note and also attenuates before the strike note. In this particular bell, and in the other bells of this octave, the third partial is not out of place. All of these bells in this octave have a very pleasant tone when played alone or when played in major and minor chords with other bells.

With the bells of the upper octaves the attenuation curve does not have such peaks as that of the lower bells but has a more regular decrement of attenuation. The third partial is seldom apparent in an appreciable amount. This difference in tone is due to the change in the design of the bells. Large bells of the lower octave have a waist that is thin compared to the thickness of the sound bow, while the smaller bells have a thick waist. Another difference is the change of the ratio between the weight of the clapper to the weight of the bell. In the heavier bells, the ratio is approximately 1 to 100 and in smaller bells the ratio is approximately 1 to 10. These changes are made in an effort to keep the volume of the bells in any octave fairly constant over the chromatic scale. This means that the tones of a carillon are not the same throughout the scale, because the component tones of the various bells, even though properly tuned, differ considerably in amplitude. The poorest effect appears in the bells of the middle register where a compromise is struck in the design between that of the large and small bells. In these bells the third partial is stronger than in the higher register and is not delayed after the strike as in the bells of the lower octaves, but appears at the moment when the bell is played, which badly interferes with the tones of the other bells.

In the preliminary inspection of the bells at Germantown, it was found that the bells of the middle register were not as pleasant as those of the higher and lower octaves. The lower and higher octaves also differed in tone quality but both were pleasing. To complete the scale a compromise design is used for the bells of the middle register and they do not sound as good as either of the end octaves. The bells of the lower octave are the most pleasing, which suggests that for a better tone quality for the entire chromatic scale, it might be advisable to use the design of the heavy bells throughout. With such a design the higher bells would lack volume because of the thin waist and light clapper. Rather than use two or three similar bells to increase the volume of one note, appropriate architectural design of the tower and bell mountings together with proper electrical amplification would improve the higher bells in volume without detracting any from their tone. Such a construction, together with amplification, would keep the best characters of bell tones homogeneous throughout the scale, thus making possible the playing of every kind of music with high artistic results.

The authors of this paper wish to thank Mr. Bernard R. Mausert, Carillonneur, and Rev. Dr. J. S. Ladd Thomas, the Pastor of the First Methodist Episcopal Church of Germantown, whose cooperation and help are greatly appreciated.

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Measurements of the Tonal Characteristics of Carillon Bells

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High quality magnetic tape recordings were made of individual bells at the carillon location, including bells cast by Meneely and Company, Watervliet, New York; Van Aerschodt, Louvain, Belgium; John Taylor and Company, Loughborough, England; Franz Schilling Söhne, Apolda, Germany. Measurements made on the tapes with a sound analyzer, Stroboconn and an automatic level recorder gave the relative tuning, the relative amplitudes at the microphone position, and the decay rates of the various components in the bell tone.

THERE is a great wealth of published material on the characteristics of bell tones.[1–9] Since many different devices, ranging from the little Indian elephant bells to the multi-ton Bourdon in a large cast-bell carillon, are called bells, it is no wonder that there is some confusion as to the actual make-up of a bell tone. The extent of confusion is illustrated by very authoritative sounding statements appearing either in print or in conversation, such as "The strike tone of a bell has no physical existence but is supplied by ear." "The strike tone and the fundamental coincide," or, in some instances, "do not coincide." "There is a hum tone in an English carillon bell that is an octave below the strike tone." "The hum tone in an English bell is a sixth below the strike tone." "A Flemish bell contains a minor third whereas the English bells contain a major third," (sometimes stated just the opposite). "English bells are untuned and suitable only for change ringing while the Flemish bells are tuned and are suitable for playing harmony." "The finest tuned carillons in this country are made in England."

In view of this confusion the authors of this paper undertook an investigation of the tonal structure of a number of different carillon bells made in the United States, Belgium, England, and Germany. Since so much of the early analysis of bell tones was done by comparing the various components with the tones of tuning forks by ear, it seemed advisable to make the measurements with more reliable and repeatable instruments. The measurements reported in this paper were completed before the authors were aware of the existence of a very authoritative and scientific treatise called *Acoustical Measurements on Church Bells and Carillons* by E. W. Van Heuven.[1] This very comprehensive book includes a tremendous amount of data on bells that were recovered from carillons wrecked during World War II. In many instances, the bells analyzed by Dr. Van Heuven were taken into the laboratory and the measurements made under laboratory conditions.

The data in this paper are being presented, however, since they do provide additional information on the frequencies of the partials, their maximum amplitudes and their decay rates, as well as measurements made on some American bells that have not been reported before. It is also believed that the techniques used to make the measurements are of interest. Our interest was primarily in the bells used in tuned carillons and in the sound of the bell tone itself rather than in the modes of vibration or in the metallurgical characteristics of the bell.

PROCEDURE

Since all the bells measured were in existing carillons they could not be brought into the laboratory. Our procedure was to obtain recordings of the tones of a number of carillon bells and then analyze the recorded tone. Magnetic recording has many advantages as a means for recording bell tones for study. Some of these are rather obvious. The equipment is readily portable and easy to operate. After a recording has been made, a loop can be made of a bell tone or a part of a bell tone and the recorded tone played over and over again as often as necessary for its analysis. It may sometimes be advantageous to play back at a slower speed than that of the original recording. As will be explained later, it is also advantageous to reverse the loop so that the analysis is made with the bell tone played backwards. All measurements reported here were made on tones which had been recorded on magnetic tape.

There is room for considerable difference of opinion as to the microphone position and acoustical environment in which the tone of a bell should be recorded if the analysis of the recorded tone is to have significance. For our own purposes, any recording technique which resulted in a bell tone that was convincing to the listener when it is played back over a high-quality system would be satisfactory for analysis, since we were not attempting to report exactly the minute peculiarities

* Now at Radio Corporation of America, Camden, New Jersey.
[1] Englebert Wiegman Van Heuven, *Acoustical Measurements on Church-Bells and Carillons* (De Gebroeders van Kleef, 's-Gravenhage, The Netherlands, 1949).
[2] Frank Percival Price, *The Carillon* (Oxford University Press, London, 1933).
[3] Arthur Lynds Bigelow, *Carillon* (Princeton University Press, Princeton, 1948).
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[5] A. T. Jones, Phys. Rev. **16**, 247–259 (1920).
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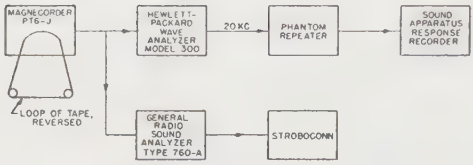

FIG. 1. Block diagram of measuring equipment used for the analysis of bell tones.

of a particular carillon. The frequency of the partials would not be influenced by the microphone location. The decay rates of the recorded partials might be influenced by standing wave systems within the tower. This would be a second-order effect, however, and would not seem to be important to the listener, since similar effects are also present when one listens to a carillon directly. The same argument applies to the maximum amplitude of the partials.

A microphone location three feet from the sound bow at the strike point was used wherever possible. In some cases this location could not be used but the recordings, when judged by ear, seem to be entirely satisfactory as bell tones. The microphone used was a Stromberg-Carlson MD-37 microphone with its transformer removed in order to convert it to the low impedance required by the Magnecorder. The microphone was calibrated by comparison with a calibrated Western Electric D-96436 condenser microphone. The equipment used to analyze the recorded bell tones is indicated in Fig. 1. The relative amplitudes and decay rates were determined as follows: a loop of tape containing the recorded bell tone was reversed and played continuously on the magnecorder. The signal from the Magnecorder was fed to the Hewlett-Packard Wave Analyzer which was then set carefully to the desired component. The 20-kc intermediate frequency signal was obtained from the wave analyzer by connecting the input of the Phantom Repeater to the grid cap of the 6F8G in the Wave Analyzer. The amplitude of the 20-kc signal is proportional to the amplitude of the component selected. The input impedance of the Phantom Repeater, which has a modified cathode follower input, is high enough that it could be connected to the grid of the 6F8G without affecting the normal operation of the Wave Analyzer. The 20-kc signal, amplified by the Phantom Repeater, was then fed to the Sound Apparatus Response Recorder which recorded on a moving strip of paper the amplitude of the component being studied. Since the paper chart of the response recorder moves at a constant rate of speed, the decay rates as well as the relative amplitude could be determined from the chart. The writing speed of the Sound Apparatus Response Recorder was of the order of 50 db per second, hence it could not be relied upon to measure components having decay rates in the vicinity of 50 db per second or greater. Most of the components of the lower bells had decay rates smaller than 50 db per second; hence this was not a serious limitation. A possible technique for components with decay rates faster than 50 db per second would be to play back the recorded tone at a slower speed than that used in recording, or use a response recorder with a much faster writing speed. Although the writing speed of the response recorder was adequate for the decay rates of the components, it was not fast enough to follow the rise of the component at the instant of strike. Consequently, if the bell tone had been played in the normal way, there would have been considerable error in the maximum amplitude of the components, especially for those having large decay rates. For this reason, the tape was always reversed so that the bell tone was played back backwards. The pen of the recorder was thus able to follow the rise of a component to its maximum value. Since in measuring decay rates we were not concerned with buildup time, the time for the pen of the recorder to return to its lowest level was not important. The frequency of each component was determined with the Stroboconn, using the General Radio Sound Analyzer as a filter to select the particular component measured. The Sound Analyzer is not a frequency changing device, and its output has the same frequency as the input. It is selective enough so the Stroboconn could give a clear indication of the frequency of the selected components.

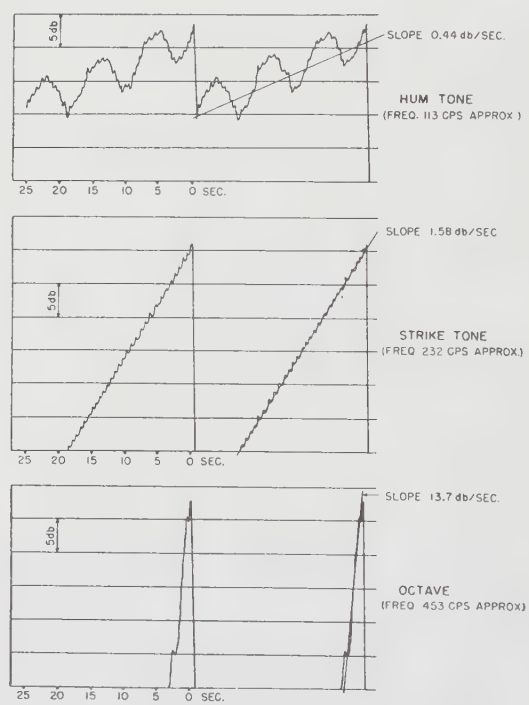

FIG. 2. Automatic level recorder graph of the decay of three components in a bell tone. Note that the tape was running backwards.

In making the measurements, searching for components was done with both the Wave Analyzer and with the Sound Analyzer and headphones. Since many components change rapidly, and last only a very short time, it is easily possible to miss a component. However, the searching was done very carefully and it is believed that all the important components are included. The decay rates were determined graphically from the response recorder chart. The response recorder has a logarithmic scale so exponential decay results in a straight line graph. Most components showed exponential decay with small irregularities, but some components showed very large dips and peaks, due, apparently, to beats between two or more components having nearly identical frequencies.

In all cases a line which appeared to represent the average decay rate was drawn and the decay rate determined from this line. Figure 2 shows a typical set of decay curves taken with the tape recording loop running backwards.

Use of a loop of tape on the Magnecorder rather than the normal reeling arrangement results in a small change in the velocity of the tape because of different amounts of drag, etc. For that reason, the frequencies measured are not the exact frequencies of the bell whose tone was recorded. The exact frequency could have been determined had a test tone from tuning fork been recorded at the same time as the bell recording, but this precaution was not taken. A change in speed merely shifts the frequency of all components by the same ratio. The results obtained represent the tuning of the individual components of a particular bell, relative to the strike note, but do not represent the tuning of different bells relative to each other. The difference in speed is estimated to be less than three percent, hence can be neglected for purposes of tone analysis except when comparing tuning of different bells within a carillon.

RESULTS

The tones of ten bells have been analyzed. For all of the tones analyzed, the bell was struck with medium force. The Germantown bells were recorded with the microphone three feet from the sound bow at the strike point. The Valley Forge bells were recorded with the microphone in a fixed position slightly below the

Carillon	Approx. pitch of bell	Maker	Date installed	Type
Germantown	$A_3\#$ $A_4\#$ $A_5\#$ and $A_6\#$	Taylor	1926	English
Valley Forge	$A_3\#$ $A_4\#$ $A_5\#$ and $A_6\#$	Meneely	1931	American
Holy Trinity, Philadelphia	D_4	Van Aerschodt	1882	Flemish
St. Michael and Zion, Philadelphia	C_4	Schilling	1930	German

TABLE I. Germantown $A_3\#$ bell.

Component	Measured frequency on loop	Maximum amplitude	Decay rate	Time for 60 db drop
$A_2\#-8$ cents	116.0 cps	6 db	1.0 db/sec	60 sec
$A_3\#-5$	232.4	0	2.1	28.6
$C_4\#-11$	275.4	−5	1.7	35.3
F_4-3	348.6	−22	14.0	4.3
$A_4\#-10$	437.5	10	8.5	7.1
D_5-7	585.0	14	8.4	7.1
F_5-8	695.2	11	4.5	13.3
G_5-33	769.2	3	11.0	5.45
$G_5\#+22$	841.7	−11	12.0	5.0
B_5-40	965.2	5	7.6	7.9
C_6+2	1048	−9	13.0	4.62
$C_6\#+23$	1124	−4	16.0	3.75
$D_6\#+20$	1259	17	12.0	5.0
F_6+24	1416	−10	18.0	3.33
G_6+13	1580	3	9.4	6.38
$A_6\#-49$	1813	−18	17.0	3.53
B_6-7	1923	−23	17.0	3.53
$C_7\#+43$	2273	−18	14.0	4.3
E_7+0	2637	−9	12.0	5.0

TABLE II. Germantown $A_4\#$ bell.

Component	Measured frequency on loop	Maximum amplitude	Decay rate	Time for 60 db drop
$A_3\#-30$ cents	229.1 cps	−10 db	1.4 db/sec	42.8 sec
$A_4\#-35$	456.8	0	4.1	14.6
$C_5\#-42$	541.1	3	4.8	12.5
F_5-17	691.6	−25	14.0	4.3
$A_5\#-36$	913.1	16	19.0	3.2
D_6-43	1146	−12	15.0	4.0
F_6-47	1360	0	12.0	5.0
$G_6\#-27$	1636	−15	22.0	2.73
$A_6\#+17$	1883	2	16.0	3.77
C_7+47	2151	−2	39.0	1.54
$D_7\#-23$	2456	1	9.4	6.39
E_7+15	2660	−2	33.0	1.82
G_7-39	3066	4	17.0	3.53

TABLE III. Germantown $A_5\#$ bell.

Component	Measured frequency on loop	Maximum amplitude	Decay rate	Time for 60 db drop
$A_4\#-13$ cents	462.7 cps	−1 db	3.1 db/sec	19.5 sec
$A_5\#-20$	921.6	0	7.8	7.7
$C_6\#-10$	1102	5	10.0	6.0
F_6-17	1383	−29	18.0	3.3
$A_6\#-19$	1844	12	44.0	1.4
$C_7\#+32$	2177	−4	25.0	2.4
$F_7\#-40$	2730	16	30.0	2.0
G_7-9	3305	3	49.0	1.22
$A_7\#+26$	3786	4	33.0	1.82

TABLE IV. Germantown $A_6\#$ bell.

Component	Measured frequency on loop	Maximum amplitude	Decay rate	Time for 60 db drop
$A_5\#-6$ cents	929.1 cps	−5 db	9.6 db/sec	6.2 sec
$A_6\#-7$	1857	0	22	2.7
$C_7\#-7$	2226	9	33	1.8
F_7-16	2768	−7	36	1.7
$A_7\#-17$	3693	0	56	1.1

sound bow, except for the lowest bell in which the microphone was three feet from the sound bow at the strike point.

TABLE V. Valley Forge $A_3\#$ bell.

Component	Measured frequency on loop	Maximum amplitude	Decay rate	Time for 60 db drop
$A_2\#+49$ cents	113.2 cps	−6 db	0.44 db/sec	136.0 sec
$A_3\#-11$	231.6	0	1.6	37.5
$C_4\#-33$	272.0	0	1.5	40.0
E_4 +30	335.4	−15	12.0	5.0
A_4 +50	452.9	0	14.0	4.3
$D_5\#-30$	611.6	−4	6.5	9.2
F_5 −33	685.3	7	7.0	8.6
$A_5\#-33$	950.3	5	14.0	4.29
$D_6\#-5$	1241	5	8.3	7.23
G_6 −18	1552	−10	8.9	6.75
A_6 −39	1721	−13	8.5	7.06
$A_6\#+8$	1873	−9	9.1	6.6
$C_7\#-2$	2215	−14	12.0	5.0
$D_7\#+48$	2559	−21	17.0	3.53
$F_7\#-35$	2901	−20	15.0	4.0
$G_7\#-35$	3256	−16	19.0	3.16

TABLE VI. Valley Forge $A_4\#$ bell.

Component	Measured frequency on loop	Maximum amplitude	Decay rate	Time for 60 db drop
$A_3\#-4$ cents	232.5 cps	3 db	1.4 db/sec	42.9 sec
$A_4\#-13$	462.7	0	5.1	11.8
$C_5\#-3$	553.4	2	3.8	15.8
F_5 −8	695.2	−16	11.0	5.5
$A_5\#-8$	928.0	11	19.0	3.2
$C_6\#+30$	1128	13	96.0	0.63
F_6 +0	1397	16	12.0	5.0
$F_6\#+48$	1522	1	20.0	3.0
B_6 −39	1932	6	15.0	4.0
$D_7\#+20$	2518	8	16.0	3.75
G_7 +5	3145	6	17.0	3.53
$A_7\#+34$	3803	2	21.0	2.86

TABLE VII. Valley Forge $A_5\#$ bell.

Component	Measured frequency on loop	Maximum amplitude	Decay rate	Time for 60 db drop
$A_4\#-2$ cents	465.6 cps	−1 db	2.6 db/sec	23.1 sec
$A_5\#-18$	922.7	0	7.4	8.1
$C_6\#-8$	1104	3	7.2	8.3
F_6 −30	1373	−8	22.0	2.7
$A_6\#-15$	1849	8	44.0	1.4
$C_7\#+45$	2161	−20	66.0	0.91
F_7 −14	2771	15	23.0	2.6
$G_7\#+18$	3288	−5	66.0	0.91

TABLE VIII. Valley Forge $A_6\#$ bell.

Component	Measured frequency on loop	Maximum amplitude	Decay rate	Time for 60 db drop
$A_5\#-15$ cents	924.3 cps	−7 db	8.3 db/sec	7.2 sec
$A_6\#-12$	1852	0	17.0	3.5
$C_7\#+25$	2250	−7	26.0	2.3
E_7 −5	2629	−34	55.0	1.1
$F_7\#-31$	2907	−9	59.0	1.0
B_7 −37	3868	−2	45.0	1.3

TABLE IX. Holy Trinity D_4 bell.

Component	Measured frequency on loop	Maximum amplitude	Decay rate	Time for 60 db drop
$D_3\#-47$ cents	151.4 cps	0 db	1.6 db/sec	37.5 sec
D_4 −26	289.3	0	2.7	22.2
F_4 +46	358.6	−5	4.0	15.0
A_4 −35	431.2	−22	18.0	3.3
D_5 +13	591.8	5	16.0	3.8
$F_5\#+0$	740.0	−2	14.0	4.3
A_5 +2	881.0	−1	7.6	7.9
$D_6\#-47$	1211	−5	11.0	5.45
F_6 −49	1358	−7	12.0	5.0
G_6 +8	1578	5	15.0	4.0
B_6 −10	1964	−1	15.0	4.0
D_7 +18	2374	−7	22.0	2.73
F_7 +5	2802	−17	33.0	1.82
G_7 +45	3038	−30	33.0	1.82
$A_7\#-27$	3672	−19	23.0	2.61

TABLE X. St. Michael and Zion C_4 bell.

Component	Measured frequency on loop	Maximum amplitude	Decay rate	Time for 60 db drop
B_2 +22 cents	125.0 cps	−13 db	0.74 db/sec	81.2 sec
C_4 −50	254.2	0	2.0	30.0
$D_4\#+0$	308.1	−13	1.8	33.3
$F_4\#+17$	373.6	−10	7.0	8.6
C_5 −29	514.6	6	10.0	6.0
$D_5\#+44$	638.3	−6	15.0	4.0
G_5 −47	763.0	−6	4.9	12.2
A_5 +48	904.7	−16	9.1	6.6
C_6 −2	1045	−1	8.6	7.0
E_6 +44	1352	3	8.9	6.75
$G_6\#+15$	1676	2	11.0	5.45
B_6 +35	2016	0	12.0	5.0
D_7 +6	2225	−5	13.0	4.62
F_7 −50	2714	−7	14.0	4.28
G_7 −40	3064	−25	23.0	2.61
$A_7\#+16$	3764	−22	18.0	3.33

The results of the measurements are shown in Tables I through X. These tables show for each component the measured deviation in cents from the indicated note of the equally tempered scale, the corresponding frequency, the maximum amplitude on a logarithmic scale relative to the strike tone, the decay rate and

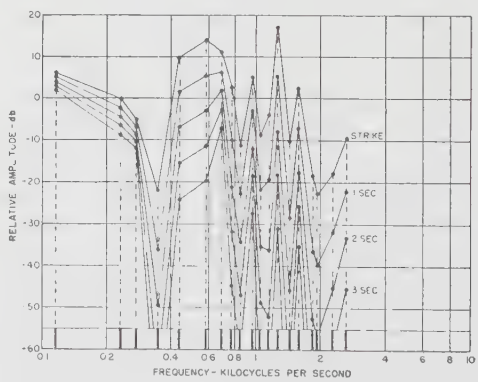

FIG. 3. Frequency spectrum and relative amplitudes of components at one second intervals of the Germantown $A_3\#$ bell (Taylor 1926).

TABLE XI. Relative tuning of components in cents. See text.

Bell	Note	Hum	Strike	Minor Third	Fifth	Octave	Major Third	Fifth
Taylor	$A_3\sharp$	−3	0	−6	+2	−5	−2	−3
Taylor	$A_4\sharp$	+5	0	−7	+18	−1	−8	−12
Taylor	$A_5\sharp$	+7	0	+10	+3	+1	+12	−20
Taylor	$A_6\sharp$	+1	0	+14	−9	−10	...	...
Meneely	$A_3\sharp$	+50	0	−22	−59	−39	+81	−22
Meneely	$A_4\sharp$	+9	0	+10	+5	+5	−57	+13
Meneely	$A_5\sharp$	+16	0	+10	−12	+3	−37	+4
Meneely	$A_6\sharp$	−3	0	+37	{−105 + 69	+63	...	...
Van Aerschodt	D_4	+79	0	+72	−9	+39	+26	+28
Schilling	C_4	−28	0	+50	−33	+21	−6	+3

corresponding time to drop to 60 db. A summary of the tuning of the first seven components relative to the strike tone is shown in Table XI. This table was compiled by taking the tuning of each strike tone for reference and assuming each component tuned to the appropriate note of the equally tempered scale. The "note" listed in Table XI is the nearest one in the equally tempered scale. Refer, for example, to Table I where the strike is listed as $A_3\sharp-5$ cents and the fifth as F_4-3 cents. Now if the nominal interval be assumed that of equal temperament from $A_3\sharp$ to F_4 (or the same thing, $A_3\sharp-5$ to F_4-5 cents) then the experimentally observed fifth is relatively 2 cents sharp. Although a different assumed internal tuning would change by a constant amount the values listed in any column of Table XI the variations would be the same as now evident.

The relative amplitude of each component is plotted at one second intervals in Figs. 3 through 12.

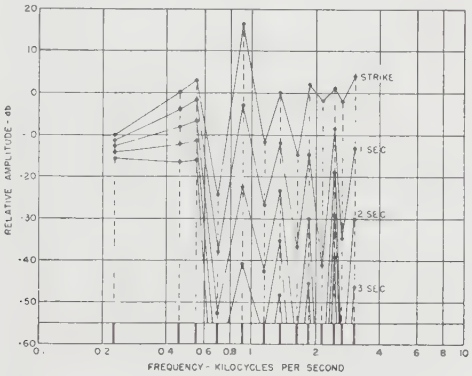

FIG. 4. Frequency spectrum and relative amplitudes of components at one second intervals of the Germantown $A_4\sharp$ bell (Taylor 1926).

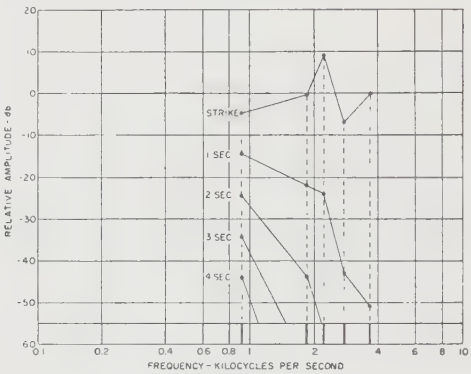

FIG. 6. Frequency spectrum and relative amplitudes of components at one second intervals of the Germantown $A_6\sharp$ bell (Taylor 1926).

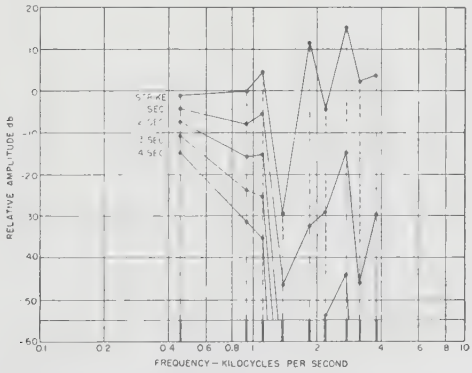

FIG. 5. Frequency spectrum and relative amplitudes of components at one second intervals of the Germantown $A_5\sharp$ bell (Taylor 1926).

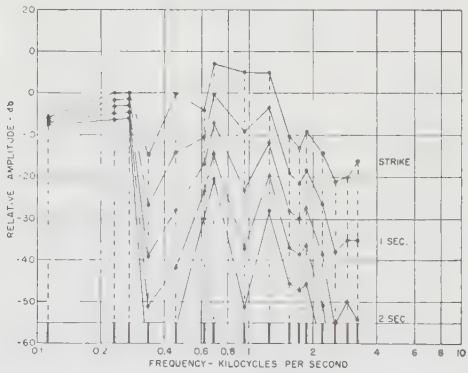

FIG. 7. Frequency spectrum and relative amplitudes of components at one second intervals of the Valley Forge $A_3\sharp$ bell (Meneely 1931).

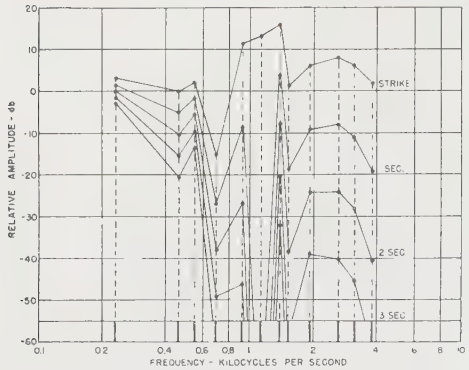

FIG. 8. Frequency spectrum and relative amplitudes of components at one second intervals of the Valley Forge $A_4\#$ bell (Meneely 1931).

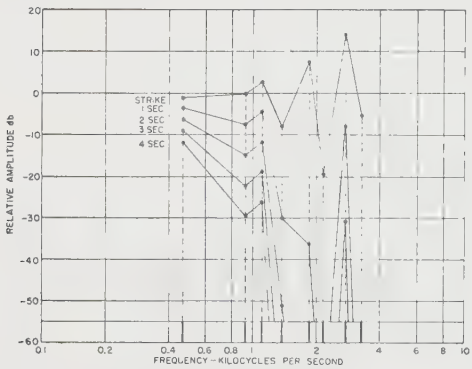

FIG. 9. Frequency spectrum and relative amplitudes of components at one second intervals of the Valley Forge $A_5\#$ bell (Meneely 1931).

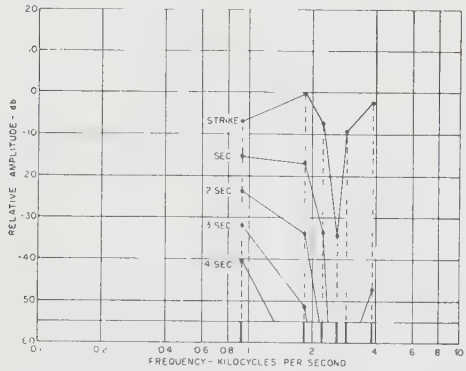

FIG. 10. Frequency spectrum and relative amplitudes of components at one second intervals of the Valley Forge $A_6\#$ bell (Meneely 1931).

There is not much that need be said about the data presented. The bells measured were those that were readily accessible and the number was not large enough to indicate very definite trends among the bells manufactured in different countries. The most complete sampling was on bells made by John Taylor and Company of Loughborough, England, and the Meneely Bell Company of Watervliet, New York. In both the Taylor bells (English) and the Meneely bells (American) the hum tone was an octave below the strike tone and both contained a component a minor third above the strike tone. The Taylor bells were better tuned, particularly the top two named components. The Van Aerschodt bell was poorly tuned but since it was the only low country bell measured, no statements can be made about Flemish *vs* English bells based entirely on the authors' measurements. Also, it was cast so long ago that it could not be considered representative of present-day trends in the low countries.

From a comparison of van Heuven's data[1] and the data reported in this paper, the best tuned bells of the low country bell foundries (i.e., "Flemish") and those of the English bell foundries are identical in their components from the hum tone to the component two octaves above the hum tone. Mr. van Heuven, however, reports measurements only on the first five components. The data reported in this paper include two more components, which in the Taylor bells seem to be held fairly accurately. By confining our effort to tuned carillon bells the question of the physical presence or absence of the strike tone did not arise. In a tuned carillon bell there is a component which coincides with the apparent pitch of the bell. We chose to call

TABLE XII. Distribution of deviations from ideal pattern, with reference to the fundamental, for 171 carillon bells and 98 swinging bells measured by van Heuven.

Deviation from eq. tempered interval (Semitones)		Fundam. —hum n. swing. (%)	Fundam. —hum n. car. (%)	Minor th. —fundam. swing. (%)	Minor th. —fundam. car. (%)	Nominal —fundam. swing. (%)	Nominal —fundam. car. (%)
Less than	−2.00	9.4	2.1	..	...	2.1	...
−2.00	−1.60	6.4	1.1	..	...	4.3	...
−1.60	−1.35	3.2	...	...	...	...	...
−1.35	−1.15	5.3	..	1.1	...	...	..
−1.15	−0.95	4.3	4.3	1.1	...	...	...
−0.95	−0.85	1.1	1.1	...	2.2	2.1	...
−0.85	−0.75	3.2	3.2	...	1.1	...	...
−0.75	−0.65	2.1	1.1	1.1	...	2.1	...
−0.65	−0.55	6.4	1.1	...	...	...	...
−0.55	−0.45	3.2	...	3.2	.	...	...
−0.45	−0.35	1.1	...	3.2	...	2.1	1.4
−0.35	−0.25	2.1	...	1.1	1.1	2.1	2.7
−0.25	−0.15	5.3	3.2	2.1	6.6	2.1	2.7
−0.15	−0.05	4.3	7.5	5.3	5.5	12.8	9.6
−0.05	+0.05	9.6	55.9	4.3	14.3	21.3	34.2
+0.05	+0.15	6.4	7.5	11.7	30.8	10.6	11.0
+0.15	+0.25	2.1	6.5	6.4	12.1	4.3	2.7
+0.25	+0.35	5.3	2.1	6.4	5.5	...	2.7
+0.35	+0.45	3.2	1.1	5.3	4.4	2.1	1.4
+0.45	+0.55	1.1	1.1	2.1	5.5	6.4	2.7
+0.55	+0.65	1.1	...	4.3	3.3	4.3	2.7
+0.65	+0.75	2.1	...	4.3	...	6.4	1.4
+0.75	+0.85	1.1	...	2.1	1.1	...	4.1
+0.85	+0.95	4.3	...	3.2	1.1	4.3	...
+0.95	+1.15	1.1	1.1	7.4	1.1	...	...
+1.15	+1.35	2.1	...	4.3	1.1	4.3	...
+1.35	+1.60	1.1	...	5.3	2.2	2.1	1.1
+1.60	+2.00	1.1	...	5.3	...	2.1	4.4
exceeding	+2.00	1.1	...	9.6	1.1	2.1	6.8

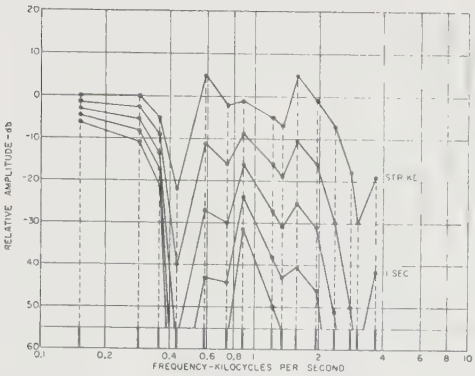

FIG. 11. Frequency spectrum and relative amplitudes of components at one second intervals of the Holy Trinity Philadelphia D_4 bell (Van Aerschodt 1882).

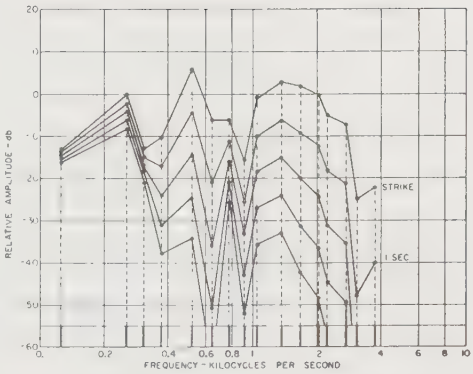

FIG. 12. Frequency spectrum and relative amplitudes of components at one second intervals of the St. Michael and Zion C_4 bell (Franz Schilling Söhne 1930).

this component the strike tone even though some authors choose to distinguish between the rather "clangy" impression of the tone of the bell the moment it is struck and the single sinusoidal component producing the same apparent pitch. There is a strong minor third component in the tone of any good carillon bell while the fifth is likely to be weak and also to have a very rapid decay.

Dr. van Heuven points out that when a bell is struck on the sound bow it is likely that the fifth will not be excited at all. The hum tone, in general, is weaker than many of the other components at the instant of strike but it decays so slowly that it becomes the strongest component in a relatively short time.

ACKNOWLEDGMENT

The authors wish to express their thanks to Professor Robert B. Kleinschmidt of Lehigh University† and member of the Guild of Carillonneurs for the invaluable assistance in making the recordings of the bells which were analyzed in this paper.

APPENDIX

Tables I–XV are taken from van Heuven's thesis[10] and are included for purposes of comparison. Table XII gives a statistical compilation of the tuning of the hum tone, minor third and the nominal in relation to the fundamental for both carillon bells and swinging bells. The ideal pattern mentioned in the caption of Table XII is that of equal temperment. It should be

TABLE XIII. Tuning of the carillon at Zierkzee Town Hall (Taylor 1927) measured by van Heuven. Deviations in semitones from note indicated.

No.	*M* kb	R_0 meters	Hum tone	Fundam.	Minor th.	Fifth	Nominal
23	270	0.39	C 3−0.27	C 4−0.18	E♭4−0.12	G 4−0.17	C 5−0.21
22	195	...	D 3−0.22	D 4−0.21	F 4−0.09	A 4−0.17	...
21	145	0.31	E 3−0.16	E 4−0.20	G 4−0.07	B 4−0.13	E 5−0.19
20	123	0.28	F 3−0.21	F 4−0.19	A♭4−0.06	C 5−0.15	F 5−0.24
19	121	0.28	F♯3−0.16	F♯4−0.20	A 4+0.04	...	F♯5−0.36
18	113	0.27	G 3−0.18	G 4−0.17	B♭4−0.04	...	G 5−0.02
17	95	...	G♯3−0.26	G♯4−0.22	B 4−0.14	...	...
16	85	0.23	A 3−0.14	A 4−0.16	C 5−0.07	...	A 5−0.21
15	72	0.23	A♯3−0.19	A♯4−0.21	D♭5−0.20	...	A♯5−0.53
14	60	0.21	B 3−0.19	B 4−0.15	D 5−0.20	...	B 5−0.08
13	50	0.21	C 4−0.17	C 5−0.18	E♭5−0.30	...	C 6−0.17
12	39	0.20	C♯4−0.15	C♯5−0.10	E 5−0.04	...	C♯6−0.03
11	33	0.18	D 4−0.17	D 5−0.17	F 5−0.55	...	D 6−0.14
10	26	0.17	D♯4−0.19	D♯5−0.16	G♭5−0.15	...	...
9	24	0.16	E 4−0.17	E 5−0.19	G 5−0.04	...	E 6−0.14
8	22	0.16	F 4−0.19	F 5−0.13	A♭5−0.19	...	F 6−0.15
7	19	0.15	F♯4−0.18	F♯5−0.13	A 5−0.48	...	...
6	17	0.14	G 4−0.18	G 5−0.18	B♭5−0.09	...	...
5	15	0.13	G♯4−0.18	G♯5−0.13	B 5−0.13	...	...
4	13	0.13	A 4−0.21	A 5−0.12	C 6+0.10	...	...
3	11	0.12	A♯4−0.19	A♯5−0.17	...	...	...
2	10	0.12	B 4−0.19	B 5−0.26	...	...	...
1	lost						

† Now at the University of Buffalo.
[10] Reference 1, pp. 46, 59, 62, 64.

TABLE XIV. Tuning of the carillon at Den Helder Monument (Van Bergen 1935) in semitones. Deviations measured by van Heuven.

No.	M kg	R_0 meters	Hum tone	Fundam.	Minor th.	Fifth	Nomial
30	130	0.30	E 3−0.16	E 4−0.22	G 4−0.26	B 4−0.19	E 5−0.22
29	109	0.28	F 3−0.21	F 4−0.21	A♭4−0.21	C 5−0.33	F 5−0.21
28	87.5	0.265	F♯3−0.23	F♯4−0.20	A 4−0.09	...	F♯5−0.24
27	80	0.26	G 3−0.31	G 4−0.28	B♭4+0.05	...	F♯5+0.39
26	61	0.23	G♯3−0.22	G♯4−0.19	B 4−0.12	...	G♯5−0.12
25	58	0.225	A 3−0.25	A 4−0.23	C 5+0.04	...	...
24	51	0.215	A♯3−0.25	A♯4−0.23	D♭5−0.09	G 5+0.07	...
22	39.5	0.20	B 3−0.22	B 4−0.19	D 5+0.00	...	B 5+0.16
23	41	0.19	C 4−0.19	C 5−0.22	E♭5−0.40	A 5−0.30	...
21	35	0.18	C♯4−0.17	C♯5−0.20	E 5+0.22	...	C♯6+0.43
20	29	0.175	D 4−0.22	D 5−0.23	F 5−0.02	...	D♯6+0.47
19	26	0.165	D♯4−0.16	D♯5−0.15	G♭5+0.22	A♯5−0.12	...
18	24	0.158	E 4−0.22	E 5−0.24	G 5+0.25	B 5−0.03	E 6+0.57
17	16.5	0.145	F 4−0.21	F 5−0.01	A♭5+0.47	B 5+0.29	F♯6−0.43
15	15.5	0.14	F♯4−0.25	F♯5−0.08	A 5+0.35	...	...
16	16	0.14	G 4−0.16	G 5−0.16	B 5−0.13	D 6−0.27	A 6−0.30
14	14.5	0.13	G♯4−0.21	G♯5−0.22	B 5−0.03	D♯6−0.23	G♯6+0.01
11	10.5	0.123	A 4−0.20	A 5−0.23	D♭6−0.26	E 6−0.28	A 6+1.17
13	12	0.12	A♯4−0.23	A♯5−0.24	D 6−0.33	E 6+0.41	D 7+0.01
12	11	0.115	B 4−0.21	B 5−0.19	D 6+0.43	F 6+0.10	C 7−0.40
10	9.5	0.11	C 5−0.23	C 6−0.08	E♭6+0.09	G 6−0.32	C 7+0.3
8	8	0.108	C♯5−0.19	C♯6−0.26	E 6+0.22	G 6+0.40	C♯7+0.50
4	7.5	0.10	D 5−0.21	D 6−0.22	G♭6−0.05	...	E 7−0.48
5	7.5	0.10	D♯5−0.20	D♯6−0.20	G♭6−0.45	...	...
7	7.5	0.10	E 5−0.24	E 6−0.21	G 6−0.43	C 7−0.29	...
2	7	0.095	F 5−0.22	F 6−0.21	A♭6−0.21	C♯7+0.12	A♯7−0.03
6	6.5	0.09	F♯5−0.18	F♯6−0.24	A♭6+0.40	D♯7+0.34	...
3	7	0.09	G 5−0.23	G 6−0.27	B♭6−0.05	...	A 7+0.25
1	6.5	0.09	G♯5−0.18	G♯6−0.42	B 5+0.10	E 7−0.35	G♯7+0.13
9	8	0.108	A 5−0.20	A 6−0.42	...	...	G♯7+0.20

TABLE XV. Tuning of the carillon at Gouda (Hemony 1675–1676) measured by van Heuven. Deviations in semitones.

Nr.	Hum note	Fundam.	Minor th.	Fifth	Nominal
7	F♯2−0.37	F♯3−0.14	A 3−0.16	C♯4+0.08	F♯4−0.26
4	G 2−0.53	G 3−0.49	B♭3+0.47	D 4−0.43	A♯4+0.49
2	G♯2−0.30	G♯3−0.31	C 4+0.03	D♯4−0.3	...
1	A 2+0.08	A 3−0.16	C 4+0.16	E 4−0.19	A 4−0.14
3	A♯2−0.30	A♯3−0.34	D♭4−0.26	...	A♯4−0.32
12	B 2−0.07	B 3−0.21	D 4−0.01	...	B 4−0.21
8	C 3−0.51	C 4−0.47	E♭4−0.26	F 4+0.48	C 5−0.44
5	C♯3−0.39	C♯4−0.26	E 4−0.19	G♯4−0.31	C♯5−0.28
6	D 3−0.51	D 4−0.52	F 4−0.41	...	D 5−0.46
10	D♯3−0.45	D♯4−0.26	G♭4−0.21	...	D♯5−0.37
19	E 3−0.35	E 4−0.19	G 4−0.06	...	...
11	F 3−0.31	F 4−0.39	A♭4−0.22	...	F 5−0.40
9	F♯3−0.42	F♯4−0.16	A 4−0.12	...	F♯5−0.31
16	G 3−0.63	G 4−0.46	B♭4−0.36	...	...
15	G♯3−0.18	G♯4−0.39	B 4−0.22	...	.
18	A 3−0.20	A 4−0.22	C 5−0.07	...	..
17	A♯3−0.49	A♯4−0.42	...	.	..
13	B 3−0.21	B 4−0.05	D 5−0.13	..	..
14	C 4−0.55	C 5−0.36	E♭5−0.30	..	...
27	C♯4−0.39	C♯5−0.29	E 5−0.20	...	...

remembered that 0.01 semitone is equal to 1 cent. The corresponding names of the components are as follows:

Slaymaker-Meeker	Van Heuven
Hum tone	Hum tone
Strike tone	Fundamental
Minor third	Minor Third
Fifth	Fifth
Octave	Nominal
Octave and third (Also referred to as major third in Table XI)	...
Octave and fifth (Also referred to as fifth in Table XI)	...

[11] See reference 1, p. 4.

In van Heuven's[11] notation A3=440 cps, while the authors of this paper have followed the Stroboconn calibration in which A_4=440 cps.

Table XIII gives the bell number, mass, radius at the sound bow end, and the note name of a set of Taylor bells cast in 1927 and located in the Zierkzee Town Hall. Table XIV gives the same data for a set of Van Bergen bells cast in 1935, located in Den Helder Monument. Table XV gives the same data for a set of Hemony bells cast in 1675–1676 located at Gouda. It is included because of the historical position of Hemony bells.

7

ACOUSTICAL INVESTIGATIONS ON CHURCH BELLS

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SUMMARY

In this paper a report on a series of measurements on Church bells is given. First the forms of the vibrations of singly excited partial tones are determined and their amplitude distribution measured on the surface of the bells. Then the frequencies of single vibrations are ascertained and set out schematically in a plan, arranged according to the number of node-circles and node meridians. Further investigations are concerned with the dependence of sound pattern on mass, material, form and strike of the clapper. Thereby an interdependence of the amplitude-distribution of the partial (tone) and the duration of contact of the clapper on the side of the bell is found. Finally the damping of the partials is reported. It appears that there is a frequency range of maximum radiation damping, which depends on the adjustment of the length of the bending wave of the bell to the length of the airborne wave.

1. INTRODUCTION

The acoustical properties of bells were first described by Lord Rayleigh [1], and since then have been examined by several authors. Van Heuven [2] gives a thorough treatment of the most important problems of bell investigations with the aid of electroacoustical measuring methods. In this report, we summarize our acoustical measurements on bells, which were carried out over a series of years in cooperation with the Bochumer Company for cast steel manufacture and which concern problems of the partials of bell vibrations.

The requirements that are placed on the sound of a bell stem from its application as a liturgical musical instrument, which should be audible throughout the parish. The sound must be musically pleasant, recognizable as a bell sound, and at the same time, long-ranging. These requirements concern, in the first place, the formation of the sound field, the creation of the sound, and the location and arrangement of the bell chamber.

In order to describe a bell sound physically, the following information is required:

(1) Frequencies of the partials
(2) Relative amplitudes of the partial tones at the moment of strike
(3) Damping of the partials
(4) Sound field and sound diffusion.

The frequencies of the partials are determined by the material and the shape of the bell. The amplitude relationship of the partial tones depends, first, on the place of the strike and the shape and mass of the clapper, and then, on the duration of the individual modes. The damping consists of 3 components: internal damping of the material, friction damping, and acoustical damping. The latter also depends on the shape and material of the bell through the relationship of the bending wavelength to the sound wavelength. The sound field and the diffusion of the sound are determined by the arrangement and location of the bell chamber, as well as by the construction, the atmospheric conditions, and the features of the landscape surrounding the belfry. The following report deals only with the first three influential quantities.

2. APPARATUS FOR MEASURING FREQUENCIES, DAMPING, AND VIBRATION FORM OF THE PARTIAL TONES OF A BELL

Figure 1 shows the apparatus for measuring frequency, damping, and vibration form of the partials of a bell as it was used in the experiments described here. The bell hung in a frame constructed expressly for this purpose, or else it stood with the crown on the floor, which was covered with a sound-isolating mat. On the left side of the figure is the stimulation, on the right, the reception of the bell vibrations. An electromagnetic transformer, fed by a continuously-variable RC-generator, served as stimulator. The choice of the reception system was governed by the partial-tone properties that were to be determined each time. In order to measure the frequency and damping, the sound that radiated from the

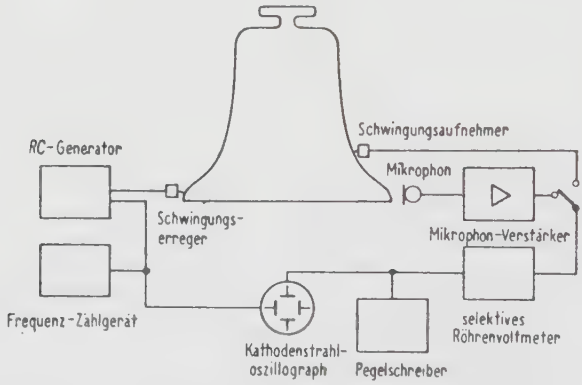

Figure 1. Measuring apparatus for determining frequency, damping, and waveform of the partials of a bell.

bell was received by a microphone. A continuously-variable bandpass filter, tuned to the frequency of the partials (selective VTVM), was attached to the microphone amplifier. Because of it, the effective signal could be largely separated from interference in the room.

The agreement of stimulated and radiated frequency was observed with a cathode-ray oscilloscope; the generator voltage supplied the horizontal deflection, and the microphone voltage the vertical deflection. If both frequencies are the same, the screen displays an ellipse whose vertical amplitude decreases continually without wobbling when the vibration stimulation is removed from the bell (i.e., while the stimulated partial is dying away). The generator frequency was measured with an electronic frequency calculator.

If the generator frequency is finely adjustable and steady, the frequencies of the partials can be determined to an exactness of $\pm$ 2% by this method. This is sufficient to indicate with certainty the deviations in 1/16 semitones, as is customary with bell acoustics (1/16 semitone $\simeq$ 4%).

The damping D of partial tones in dB/s and the sound decay time T could be calculated from the tracings on the strip-chart recorder during the decay. Additionally, to compare the damping of partial tones of various bells, the logarithmic damping decrement was determined. In order to determine the vibrational mode of a partial tone, the course of the nodal lines on the bell surface must be traced. For this purpose, an electromagnetic transformer is used again, as in the case of the stimulation. On the cathode-ray oscilloscope, a vibration node is shown by a minimum in the vertical amplitude of the ellipse, and a change of phase through crossing a nodal line as a turning of the ellipse.

For determination of the amplitude relationships of the vibration components perpendicular to the bell surface, a scale in the form of a longitudinal line with markings at intervals of 5 and 2.5 cm, respectively, was drawn between two nodal meridians of the stimulated vibrations. From the measured voltage on a piezoelectric vibration detector set on one of these markings, the vibration amplitude perpendicular to the surface was determined.

The selective VTVM was also operated as an amplifier and bandpass filter in measurements with the body-sound receivers. At the same time, it served as a voltmeter in determining the amplitude distribution along the bell wall.

3. THE VIBRATION QUALITIES OF THE BELL.

The bell, by nature of its shape, is a circular shell with variable diameter and variable wall strength. The vibration theory of such a shell is extraordinarily complex. Its meridian shape and the variable moment of inertia make the solution of the equations of motion and the calculations of the intrinsic frequencies more difficult, so that all bell shapes have been developed up to now only on the basis of experimental values. However, in certain cases the rules of behavior of vibrations of geometrically simpler

shells can be applied to the bell shape, and they show the tendency of the change of vibration properties which would result from the change in shape of the bells. Besides, through these analogies, the possible vibration modal shapes can be better grasped and ordered.

3.1. Single vibration modes

The bell, on account of its shape, should vibrate in a manner similar to a circular cylindrical shell (i.e., it is to be compared with a short pipe, whose cross section is expanded approximately fourfold from the center out).

It is customary in bell research to specify, for each partial tone, a mode of vibration by specifying the number k of its nodal circles and the number n of its nodal meridians. In hollow cylinders, the vibrational modes are described by the longitudinal half-wave number i and the peripheral wave number m. By analogy in the two shell forms, the number k corresponds to the nodal circles of longitudinal half-wave number i and the number n to the nodal meridians of circumferential wave number m.

The size and number of areas on the bell surface that vibrate in contraphase are determined by the nodal lines. The vibration amplitudes of side-by-side areas lying between equal nodal circles are, for reasons of symmetry, approximately the same; however, the amplitudes of the stacked-up fields depend on the shape of the bell (i.e., the respective curvature and wall thickness).

The vibrations of the bell wall were measured perpendicular to the surface. Therefore, in the following figures illustrating the modal forms, the outer skin of the bell on the left is sketched as if it were extended to a straight line. On the right and left of this line, the vibration amplitude is diagrammed. The vibration phase is sketched in the one end position with a solid line, in the other end position with a dotted one.

To examine the measured bell vibration modes in their entirety, it is expedient to view them in accordance with the order scheme for hollow cylinder vibrations. In the following, pairs of k and n values are discussed briefly and compared with the measured results.

3.1.1. 0 nodal circles, 0 nodal meridians

The shell expands and contracts under changes in its volume. The circular cross section remains preserved while the shell "breathes." This vibration mode was not observed up to now on the bell.

3.1.2. 1, 2, 3 . . . nodal circles, 0 nodal meridians

The shell conducts axially symmetric vibrations during which the circular cross section is preserved. In longitudinal section, the vibration phase changes. The areas between the nodal circles vibrate alternately, outward and inward.

In examining four bells, four axially symmetric vibrations were found. They had 2, 3, 4, and 5 nodal rings, and are indicated from now on with

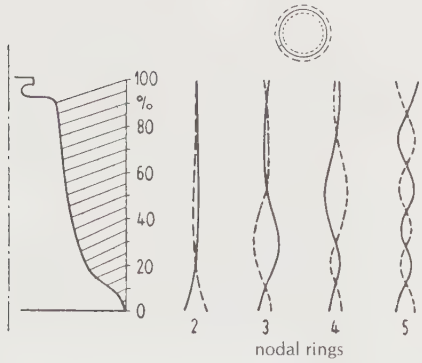

Figure 2. Amplitude distribution of bell vibration along the rib for 2 to 5 nodal circles and 0 nodal meridians.

the symbols 2-0, 3-0, 4-0, and 5-0. These amplitude conditions of vibration are presented in Figure 2. If we follow the vibration amplitude along the rib of the bell, it is easy to determine that the frequency is most probably determined by the region with the greatest amplitude. The lowest vibration (2-0) is that of the sound bow (stroke ring), then follows the vibration (3-0) of a region at approximately 25% of the height, then the (4-0) mode at 50%, and finally the (5-0) mode at the shoulder. The frequencies of the four modes of vibration lie approximately in the range of an octave, since the smallest and the largest diameters of the bell are in the ratio of approximately 1 : 2, and frequency is inversely proportional to diameter. With decreasing bell diameter the width of the main zones increases; that is, compared to the main zone, the zone at the sound bow is the narrowest, because here the diameter changes very rapidly, and the shoulder zone is the widest, because here the diameter remains almost constant over a greater longitudinal cut.

3.1.3. 0 nodal circles, 2 nodal meridians

The circular cross section and the diameter of the shell remain preserved, it vibrates back and forth in the direction of a diameter. This vibration was not found in the bells.

3.1.4. 1, 2, 3 nodal circles, 2 nodal meridians

The shell performs bending vibrations in the longitudinal direction. The zero positions of this vibration form parallel nodal circles outside of which every circular cross section vibrates back and forth unchanged in the direction of a diameter. Vibrations of this group (2-2, 3-2, 4-2 . . .) with the exception of 1-2 were measured on the bell. Figure 3 shows the amplitude distribution along the rib.

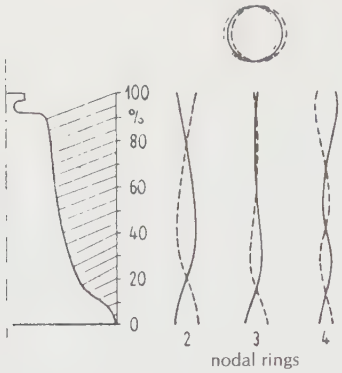

Figure 3. Amplitude distribution of bell vibrations along the rib for 2 to 4 nodal circles and 2 nodal meridians.

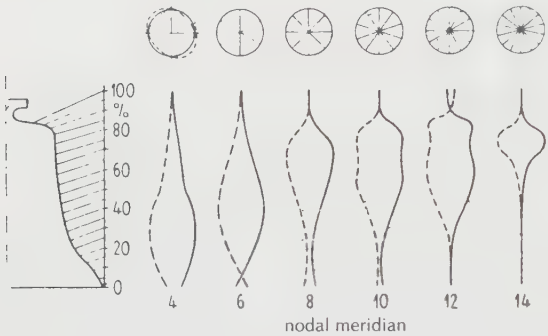

Figure 4. Amplitude distribution of bell vibrations along the rib for 0 nodal circles and 4, 6, 8, 10, 12, and 14 nodal meridians.

3.1.5. 0 nodal circles, 4, 6, 8 . . . nodal meridians

The shell vibrates radially. In the process, its cross section changes so that the points that have been cut from one longitudinal plane always have the same phase of vibration. The zero points of the peripheral vibration create nodal lines in the direction of the rotation axis. The spacing of the nodal points on a circular cross section are the same; the nodes lie in pairs opposite each other. Vibrations of this kind with 4, 6, and 8 nodal meridians can be easily stimulated and measured in a bell. As Figure 4 shows, the amplitude decreases toward the crown and the sound bow. The sound bow acts here as a stiffener, so that the bell vibrates like a cylindrical shell supported at both ends.

3.1.6. 1, 2, 3 . . . nodal circles, 4, 6, 8 nodal meridians

The shell undergoes bending vibrations in both the peripheral and longitudinal directions, and thus it is divided by nodal lines into regions that alternately bend inward and outward. The values k and n can theo-

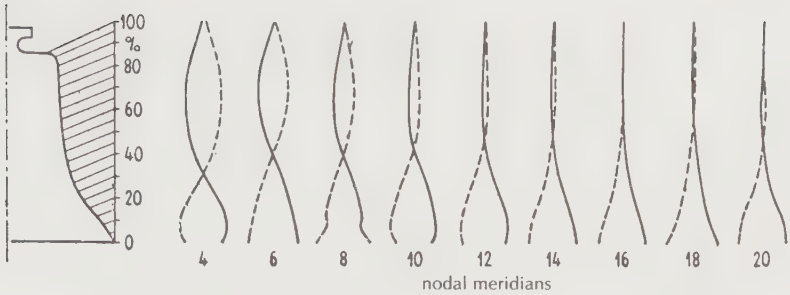

Figure 5. Amplitude distribution of bell vibrations along the rib for 1 nodal circle and 4, 6, 8, 10, 12, 14, 16, 18, and 20 nodal meridians.

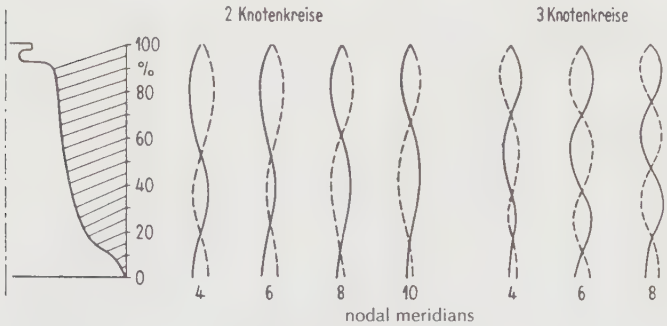

Figure 6. Amplitude distribution of bell vibrations along the rib for 2 and 3 nodal circles and several nodal meridians.

retically increase at will, and the number of possible vibration modes is indefinitely great.

In the case of the bell, these vibrations can be measured in any number. We combine the modes with the same nodal circle number *k* and increasing meridian number *n* into one group or series, and designate them with the symbols 1-4, 1-6, 1-8 . . . , 2-4, 2-6, 2-8 . . . and so forth. In Figures 5 and 6, the amplitude relationships of some of these vibration modes are presented.

3.2 Twin tone creation

The number of frequencies that fall in the range of hearing is controlled by the size of the bell, and ranges from 50 to 60 in a medium-sized bell. All bending vibration modes appear twice (twin tones), and the nodal meridians of the one twin lie exactly on the maximum vibration of the other. The reason for the splitting of partial tones is to be found in the asymmetrical mass of the bell. Through the distribution of mass in the casting, the position of the nodal lines is determined. In the case of perfect symmetry, the nodal configuration would depend only on the

stroke point; there would not be any twin tones. However, such a pouring is not practically attainable. The degree of symmetry in distributing the mass throughout the individual cross sections of the bell determines the symmetry of the vibration modes, as well as the frequency spacing of the twin tones. In the case of bells with good mass symmetry, the difference amounts to only a few Hz and is audible as a beat. In the case of more asymmetrical bells, the difference becomes greater, and one has the impression of impure partial tones.

Especially sensitive to asymmetry are the twin tones of the vibrations with two nodal meridians ($n=2$). They depend not only upon the wall strength asymmetry, but also on deviations of the cross sections from circular, which have little influence on the remaining modes of vibration. This is due to the fact that a tube of elliptical cross section exhibits quite pronounced differences in the frequencies of its longitudinal bending vibrations having the same order in the directions of both main axes of the ellipse [3]. The frequency interval is equal to the relationship between the ellipse axes. The interval between twin tones can thus be used as a measure of the asymmetry of a bell.

3.3 Frequencies of vibration modes

The shape and material of a bell determine the frequencies of the individual vibration modes. Bells of the same shape from different materials have frequencies proportional to their sound velocities, because the sound velocity c (the square root of the elasticity module divided by the density) contributes a constant factor in the frequency equation.

The frequency relationship of the individual modes, the inner harmony of the bell, depends on the rib shape and the Poisson constant of the material. The Poisson constant contributes in a complicated way to the frequency equation. Differences in material, then, also play a role in the frequency relationship of vibration modes, in the sense that the differences in the intervals become greater the higher the order of the vibrations. Unfortunately, statements about the size of these interval displacements cannot be made, since investigations into bells of the same rib shape but different material are not available.

According to their nature, bending vibrations of shells are not harmonic. It is therefore surprising that already in the eighth century a bell could be developed with bending vibrations whose frequencies created a sound suitable to the musical sensitivity of the West. It would not be possible, either with circular rings or with hollow cylinders, to attain such a sound.

The longitudinal section of the bell shows that the bending stiffeners that determine the frequency vary continuously in a different manner. The wall thickness determines the stiffness of the material, and the curve of the rib determines the stiffness of the shape. Therefore, the bell is rigid in terms of material mainly in the area of the sound bow (i.e., the thickest point of the casing or shell); on the other hand, shape stiffness predominates near the shoulder and underneath the conical part, where the frame bends outward and the upper surface creates a saddle plane.

The exact knowledge of the nodal configuration of the intrinsic modes of vibrations of the bell makes possible the systematic formation of the sound. The frequency of a mode can be raised or lowered most effectively if the rib is strengthened where it has the greatest frequency-determining stiffness for this vibration. Van Heuven [2] described systematic investigations of the influence of the change in shape on the frequency relationships of vibration modes. Similar experiments were also conducted at the PTB in cooperation with the Bochumer Company.

3.4. Inner harmony

The frequency spectrum of the individual vibrations of the bell always begins with the so-called undertone. The interval to the next highest frequency is, according to bell type, that of a sixth, a seventh, an octave, or a ninth. In the following discussion, we restrict ourselves to the description of the partial tone intervals of an octave bell, since it represents the best known and most common type.

It is desirable that a good octave bell have as its five lowest partial frequencies the intervals of a lower octave, prime, third (minor or major), fifth, and upper octave. This means that the intervals should stand in the following relationship: 0.5 : 1 : 1.2(or 1.25) : 1.5 : 2. The higher partials that belong in the so-called mixture territory should, if possible, form focal points in the octave interval to these principal points (for example: tenth, twelfth, and double octave). In good bells we can also find above the upper octave a dividing of the spectrum into octaves whose intervals are frequently stretched. The density of the partial tones increases rapidly with increasing frequency. If there are two partial tones between the first and upper octave, we will find approximately ten partial tones between the upper octave and double octave, and approximately twenty partial tones between the double octave and triple octave. Some of these partials have frequencies that are closely related to each other; that is, they create focal points, so that even in mixture territory approximately harmonic relationships are preserved: next to the third, fifth, and octave intervals, one finds here also a fourth, a sixth, and a seventh. The intervals in the mixture territory are different from bell to bell because the vibrations of higher order react more sensitively than the simpler ones to the changes in shape that result from inexact pouring; also, they are not tuned separately.

3.5. Relationship between vibration form and frequency

The relationship between vibration form and frequency is shown in Figure 7 with the example of an as_1 (A_4) bell. The undertone (lower octave or hum) has the frequency of the basic vibration of the group with 0 nodal circles (0-4). The next highest vibration in this series, with six nodal meridians, has an interval of a fifth to the prime; the next, with 8 meridians, lies above the upper octave in the range of the tenth. This and all vibrations of higher meridian number in this series are insignificant in the

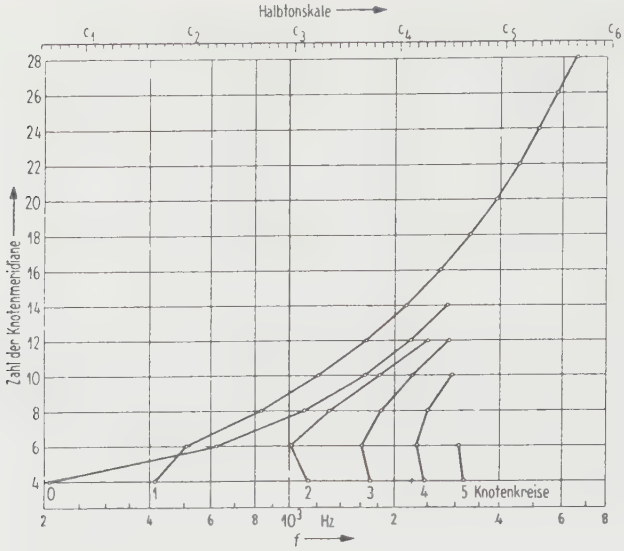

Figure 7. Distribution of eigenvalues of a bell as a function of the number of nodal circles and nodal meridians.

sound spectrum, since they have their maximum amplitudes outside the sound bow in the area of the bell casing and are only minimally excited by the stroke of the clapper (Fig. 4).

For the total sound spectrum, the series of frequencies with one nodal ring above the sound bow and increasing meridian number is of decisive importance, since all these vibrations have their maximum amplitude in the vicinity of the sound bow and are disturbed strongly by the clapper (Fig. 5). Three of the principal tones belong to this series, the prime 1-4, the third 1-6, and the upper octave 1-9. In the mixture territory, the twelfth has the form 1-10, and the double octave the form 1-12. The intervals of the vibrations 1-14, 1-16, and 1-18 correspond in most instances to those of an upper fourth, an upper sixth, and a triple octave. The bell founder must, therefore, give his greatest attention to this series and must examine the influence of every change in shape of the bell on the intervals of its partial tones.

The lowest frequency of the series with two nodal circles and increasing meridian number (Fig. 6) belongs to the vibration form 2-6 and has the partial tone interval of a tenth. The frequency of the vibration mode 2-4 is somewhat higher; its interval varies between a third and a fourth above the upper octave. Closely related is the frequency of the 2-8 mode. It still lies below the upper octave in the territory of the sixth, while the next highest modes, 2-10 and 2-12, are found as a second and a fifth above the double octave.

The frequencies of the 3-6 and 3-4 vibration modes lie in the range of the double octave, closely followed by 3-8 and, with greater interval, 3-10.

Examples of the amplitude distributions of these vibration modes along the bell rib are also shown in Figure 6.

The vibrations of higher order with 4, 5 and more nodal circles have less importance for the sound, since their amplitudes are small in relation to those in the series with one nodal ring.

The lowest frequency of the vibrations with two nodal meridians (Fig. 3) appears in the proximity of the tenth (or undezime); the highest ones lie in decreasing upward intervals of fifths, fourths, and thirds.

The frequencies of the four axially symmetrical vibrations of the bells (Fig. 2) cover, as already mentioned, the territory of an octave. The lowest one lies between the undezime and upper seventh; the three highest follow in intervals of a fifth and two thirds. The relative position of the axially symmetrical vibrations in relation to the prime is dependent on the weight of the rib of the bell and on its wall thickness in relation to its diameter. In the case of light ribs, the absolute tone position of the bell is lower (because of the smaller bending stiffness) than in heavier ribs of the same diameter. Compared with the other vibrations, the frequencies of the axially symmetrical vibrations are less dependent on the wall thickness than on diameter. Thus the lighter the rib, the greater the interval to the prime.

3.6. Pitch

With a given rib shape, every desirable pitch of the bell can be attained, without disturbing its inner harmony, through accurate enlargement or reduction. If we start with a bell of known pitch and rib shape, the pitch of a second bell made from the same material and similar in shape is f/a, where f is the pitch and a is the dimension or similarity factor which creates the linear measurements of both bells.

This rule can be deduced in the case of cylindrical shells, for which the intrinsic frequencies can be given in closed form, through similarity observations. According to Flügge [4] the following equation is true for such shells:

$$\omega^2 = \frac{D}{\mu r^2} F(\nu, n, \lambda, k).$$

where:

$F(\nu,n,\lambda,k) =$	explicit function of parameters ν,n,λ,k
$\omega =$	circular frequency
$D = \frac{Ed}{1-\nu^2} =$	stretch stiffness
$E =$	modulus of elasticity
$d =$	wall thickness
$\nu =$	Poisson constant
$\mu = \rho d =$	mass of the shell per unit area
$\rho =$	density

r = middle radius of shell
n = peripheral wave number
$\lambda = \frac{2\pi r}{2l/m} = \frac{m\pi r}{l} = \frac{\text{middle circumference}}{\text{axial wavelength}}$
m = longitudinal wave number
l = length of the shell
k = $d^2/12r^2$

If one shell has the measurements r, d, l, and a similar shell the measurements ar, ad, al, we have the following equation for the first shell:

$$\omega^2(r,d,l) = \frac{E}{\varrho(1-\nu^2)}\frac{d}{d\,r^2}F\left(\nu, n, m\pi\frac{r}{l}, \frac{1}{12}\frac{d^2}{r^2}\right)$$

and for the second:

$$\omega^2(a\,r,\ a\,d,\ a\,l) = \frac{E}{\varrho(1-\nu^2)}\frac{a\,d}{a\,d\,a^2\,r^2}\times$$
$$\times F\left(\nu, n, m\pi\frac{a\,r}{a\,l}, \frac{1}{12}\frac{a^2\,d^2}{a^2\,r^2}\right)$$

thus:

$$\omega(a\,r,\ a\,d,\ a\,l) = \frac{1}{a}\,\omega(r,d,l).$$

If the pitches of the bells are, for example, calculated according to tempered tuning, the frequencies of the tempered scale follow a geometrical series with the factor: $i = \sqrt[12]{2} = 1.0596$. The measurements of the bell must then be changed by the factor $1/i$ with increasing tonal scale. The weight of the bells changes with the cube of this number, that is, with the reciprocal value of the factor $i_{\text{Gew}} = \sqrt[12]{2^3} = 1.18921$.

Diatonic scales in a pure tuning have unequal tonal steps. The similarity factor for bells is thus different from note to note. As an example, Table 1 presents the seven-step diatonic minor scale.

Table 1. Seven-step diatonic minor scale

Tone step	*Vibration frequency relative to the fundamental*	*Interval between steps*	
		as a ratio	*as a decimal*
Prime	1		
Second	9 : 8	9 : 8	1.12500
Minor third	6 : 5	16 : 15	1.06667
Fourth	4 : 3	10 : 9	1.11111
Fifth	3 : 2	9 : 8	1.12500
Minor sixth	8 : 5	16 : 15	1.06667
Minor seventh	9 : 5	9 : 8	1.12500
Octave	2	10 : 9	1.11111

4. INVESTIGATIONS ON THE INFLUENCE OF MASS, MATERIAL, AND CLAPPER SHAPE ON THE SOUND OF THE BELL

The sound color of a bell can be influenced through the shape and material of the clapper, as well as through the manner in which the stroke is administered. For example, bells that are struck with a hammer, as in the case of our bells, or bells in a glockenspiel, sound differently than do bells in ringing. The following experiments show how the duration of contact of the clapper on the bell wall, and thus the sound spectrum of the bell stroke, can be changed.

4.1. Experimental conditions

The experimental bell stood in a sound-absorbing room with its crown toward the bottom of the mat-covered floor. Over the bell a frame for hanging the clapper was positioned so that the clapper reached the bell wall at the height of the sound bow shortly after the pendulum swung vertically. The starting point from which it could be set in motion with a releasing device could be adjusted to the desired height.

Figure 8 shows schematically the arrangement with which the contact duration of the clapper on the bell wall was measured. Bell and clapper were furnished with contacts to which a d.c. was applied. During the stroke the circuit was closed. The electrical charge $Q = \int i dt$ that flowed during the time of contact was measured with a flux meter. With constant current, $Q = i\,t$, and the contact time is $t = Q/i$. The scale of the flux meter was calibrated so that the contact time could be read directly.

To examine the sound spectrum of the stroke, the bell was tape recorded with a microphone at the height of the sound bow at a distance of 3.30 m, and later analyzed with a search-tone analyzer. The sound level of the stroke was also measured.

For the experiments, we used two spherical steel clappers (10 and 20 kg) and three cylindrical ones, each weighing 10 kg, in whose front

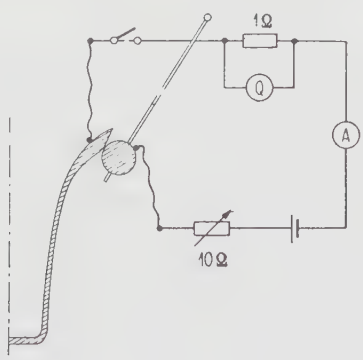

Figure 8. Procedure for measuring the contact duration of the clapper on the bell wall.

surfaces had been imbedded pins of bronze, brass, and heat-resistant wood. The cylinder with the brass pin was 20 cm long, the other two were 13.5 cm long.

4.2. Results of the experiments

4.2.1. Measurement of the contact time

In Figure 9 the measured contact times are shown for different fall heights. The experimental results can be summarized as follows:

(a) Doubling the mass (curves a and b) approximately doubled the contact time.
(b) With decreasing fall height, the contact time increased in the case of all clappers; however, the dependence on the elasticity module of the clapper material was greater than the dependence on the fall height (curves a, c, and d).
(c) In the case of two cylindrical clappers, the one with the greater length in the direction of the stroke had the longer contact time. The ratio of the lengths was 1 : 1.5; the ratio of the contacts times was 1 : 1.25 (curves c and d).

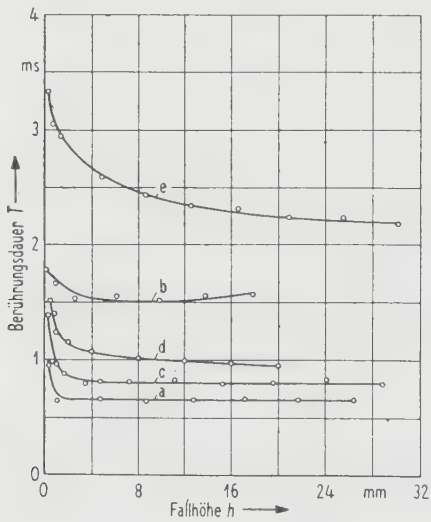

Figure 9. Duration T of the clapper on the bell wall as it depends on the height of fall h.
a) Steel clapper, spherical pin, 10 kg;
b) Steel clapper, spherical pin, 20 kg;
c) Clapper with cylindrical bronze pin, 13.5 cm long, 10 kg;
d) Clapper with cylindrical brass pin, 20 cm long, 10 kg;
e) Clapper with cylindrical pin of heat-resistant wood, 13.5 cm long, 10 kg.

(d) The contact time decreased with increasing elasticity module of the clapper material (compare curves e, c, and a). The measured times for bronze and steel were in the ratio 1.23 : 1. Since the clappers with wooden pins have no practical use and since the elasticity module of the wood is not known, curve e will not be discussed any further. The wood served to create an extremely long contact time.

These results on contact time measurement tend to agree with Hertz' theory of the pressure of two bodies against each other [9], because Hertz' formulae say, among other things, the following:

(a) The thrust duration increases with the mass of the thrusting body, approximately proportional to it.
(b) It increases with the length of the thrusting body.
(c) It decreases when the fall height (that is, the speed of the thrust) increases.
(d) For bodies with varying elastic moduli, the thrust time is approximately inversely proportional to the square root of the elastic modulus.
(e) The thrust time decreases markedly when the bending radius of the striking surface increases.

If we calculate the thrust-time relationship of steel to bronze, according to (d), from the ratio of 1.2 : 2 in elasticity moduli, we obtain a ratio of 1.29 : 1. The contact time of the bronze clapper should have been 1.29 times as long as that of the steel clapper; the experimental result was 1.23 times.

There were no clapper experiments to test (e).

4.2.2. Sound spectra

Figures 10-13 show the relationships of the partial tone amplitudes in the sound of the bell to the mass, fall height, length, and material of the clapper. The sound pressure of the individual partials is recorded linearly as a function of the frequency. From the four graphs, it can be deduced that procedures which lengthen the contact time result in a change in the amplitude relationship in this way: the higher partials are weakened, and the lower ones are strengthened. This effect becomes especially pronounced if one compares the sound spectra of the strokes with the steel clapper and the clapper with the wooden pins (Figs. 10a and c). The contact times have a ratio of 1 : 4.

Also, doubling the mass (Fig. 11), which at constant kinetic energy L resulted in a contact time ratio of 1 : 2.5, changed the energy distribution in the spectrum markedly in favor of the low frequencies.

If the fall height h was reduced from 7 mm to 0.4 mm (Fig. 12), the kinetic energy L changed in the same ratio because of the relationships $L = \frac{m}{2} v^2$ and $v = \sqrt{2gh}$. The bell was therefore stimulated more weakly, and the sound level sank from 107 dB to 100 dB. The contact time

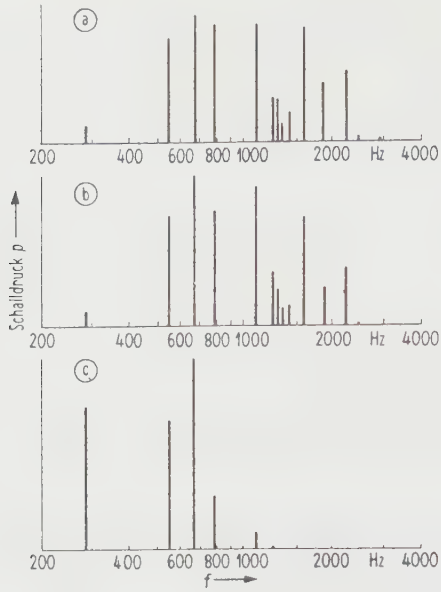

Figure 10. Partial tone spectrum of a bell as struck by clappers of different material: (a) Steel clapper, 10 kg, T = 0.7 ms, 108.5 dB; (b) Clapper with bronze pin, 10 kg, T = 0.98 ms, 107 dB; (c) Clapper with wooden pin, 10 kg, T = 3.04 ms, 97.5 dB.

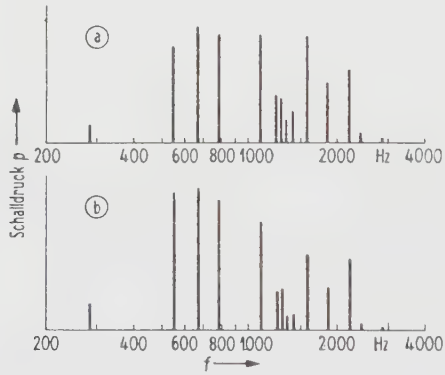

Figure 11. Partial tone spectrum of a bell as struck by clappers of the same material but with different masses: (a) Steel clapper, 10 kg, T = 0.73 ms, 108.5 dB; (b) Steel clapper, 10 kg, T = 1.74 ms, 108.5 dB.

increased 1.7 times. In order to better illustrate the change in the amplitude relationships in the spectrum, the level of the sound in Figure 12b was raised until the amplitude of the fifth partial attained the same size as in Figure 12a. The conversion is marked by dotted lines.

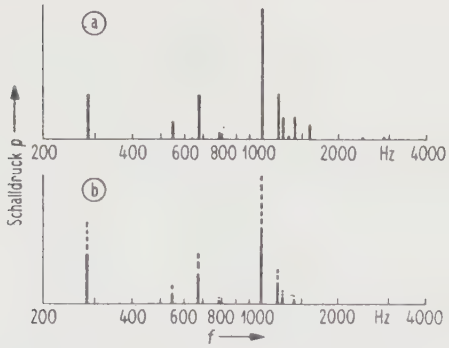

Figure 12. Partial tone spectrum of a bell as struck by the same clapper from different fall heights: (a) Clapper with bronze pin, 10 kg, h = 7 mm, T = 0.8 ms, 107 dB; (b) Clapper with bronze pin, 10 kg, h = 0.4 mm, T = 1.38 ms, 100 dB.

(b) Klöppel mit Bronzezapfen, 10 kg, $h = 0{,}4$ mm, $T = 1{,}38$ ms, 100 dB.

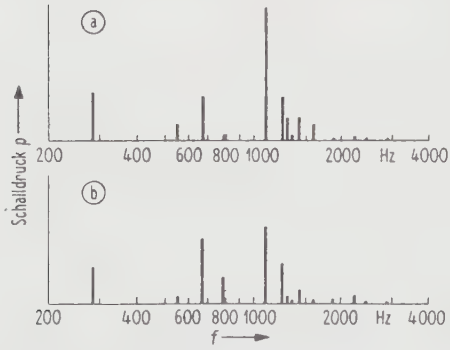

Figure 13. Partial tone spectrum of a bell as struck by clappers of different pin lengths: (a) Clapper with bronze pin, 10 kg, 13.5 cm long, T = 0.98 ms, 107 dB; (b) Clapper with brass pin, 10 kg, 20 cm long, T = 1.40 ms, 107 dB.

The lengthening of the clapper by 1.5 times in the direction of the stroke (Fig. 13), keeping the kinetic energy and sound level constant, resulted in a lengthening of the contact time by 1.4 times. This factor is relatively small, as were the amplitude changes in the sound.

4.3. Discussion of the measured results

The four possibilities that were here examined for changing the sound color of the bell through changes in the clapper and its fall height show unequivocally the dependence of the relationships of partial tone ampli-

tudes on contact time of the clapper. Since this time is identical with the duration of the impulse that the clapper transmits to the bell, the relationship to the stimulation spectrum can be described as follows: If the stroke has a very short contact time, the stimulating impulse is also very short, and the stimulation spectrum extends up to high frequencies. It follows that the higher intrinsic tones of the bell are strongly stimulated to radiate. On the other hand, in the case of longer impulses, the stimulation spectrum falls off as the function $(\sin x)/x$, so that the higher intrinsic frequencies of the bell are stimulated more weakly.

5. DAMPING OF THE PARTIAL TONES

The total damping of a vibration consists of three parts [5]:

(1) Internal damping, which is predominantly dependent on the plasticity of the material. Kneser and Voelz [6], on the basis of measurements on metal rods in vacuum, have related the inner damping to thrust and pressure viscosity. The damping of the longitudinal bending vibrations is thus:

$$\vartheta_L = \vartheta_B = \frac{\pi \omega_0}{E} \left[\frac{4}{3} (1+\nu)^2 \eta_S + (1-2\nu)^2 \zeta \right],$$

where E represents the elasticity module, ν the Poisson constant, n_s the thrust viscosity, and ζ the pressure viscosity.

For our observations it is important that the logarithmic damping decrement θ of the bending vibrations does not become larger than 12×10^{-5} in steel or in the alloys of copper throughout the audible range of frequency; that is, that this damping portion is small compared to the total damping. Van Heuven [2] reports that while the inner damping of steel is smaller than that of bronze, steel bells have a higher total damping than bronze bells.

(2) Friction damping, which results from the friction of the bell surface with the surrounding atmosphere. According to Voelz [7], it is proportional to the square root of the density of the surrounding medium. Thus, for example, the reduced friction damping of a cylinder is:

$$\Theta_{Rbg} = \vartheta_{Rbg} \frac{\varrho'}{\varrho} = 2 \sqrt{\frac{\pi \eta_L}{\varrho f}} \frac{1}{R} .$$

Here we designate ρ' as the density of the material, ρ the density of the surrounding medium, η_L the dynamic viscosity, f the frequency, and R the radius.

The densities of the various wall materials differ from each other by a negligible amount (7.85 : 8.5 for steel to bronze, for example). The portion of the total damping due to friction damping is therefore not greatly changed through differences in material.

(3) Radiation damping, caused by the loss of energy to the radiated

sound waves. It depends on the radiation resistance of the air and on the ratio of bending wavelength to sound wavelength. A qualitative examination of the radiation damping of bells is found in van Heuven's dissertation [2], paragraphs 38 and 43.

Calculation of the radiation damping is extremely difficult because of the complicated shape of the bell. This becomes clear when we observe the sound fields of individual partial tones of the bell (Fig. 14). Schroeder [8], from whose work these illustrations were taken, gives an optical presentation of phase and amplitude in the sound fields of a model bell. The pictures show cuts through the spatial sound field of the bell, parallel to the axis of symmetry and vertical to the mouth of the bell. The phase is characterized by the black and white rings that run concentrically, and the amplitude by club-shaped lines of equal sound pressure. The clubs correspond to the vibration bulges of the bell. The vibration nodes run closely together with the isobars, since here sound level has its minimum.

If we want to compare damping in bells, it is advisable to relate the sound wavelength to a specific bell size. If we begin from the stroke point of the clapper, where the preferred partials are stimulated with great amplitude, the greatest diameter of the bell suggests itself as a reference for relative measurements. In Figure 15, the logarithmic damping decrement of the lower octave and partials with one nodal ring is shown as a function of the ratio of the maximum peripheral wavelength to the sound wavelength in four bells of different material. However, the maximum peripheral wavelength does not correspond to the bending wavelength as it does in ring-shaped bodies, for example. The values of the lower octave (0-4), the prime (1-4), the third (1-6), the upper octave (1-8) are designated with the symbols for their vibrational modes.

As can be deduced from the amplitude presentations (Figs. 4 and 5), the cylindrical part of the bell also vibrates noticeably in these four modes of vibration. Thus, they also conduct longitudinal bending waves whose wavelengths have to be taken into consideration (see Fig. 14). In the case of higher partials, beginning with 1-12, only the region of the sound bow participates in the vibration (Fig. 5), that is, the relative peripheral wavelength corresponds approximately to the bending wavelength.

In comparing the figures, it should be noted the four bells have different rib shapes; therefore the relative bending wavelengths of the modes are also different.

The damping of higher order partials decreases steadily with increasing meridian number when s is larger than 2.4. The damping of the upper octave is greatest when its abscissa is $s = 2$, that is, when its maximum peripheral wavelength is double the sound wavelength. If this value is exceeded, as in the iron bell, the damping decreases very fast. In the two bronze bells the upper octave is damped much more strongly than in the remaining partials listed here. Its abscissa values, however, do not reach $s = 2$, and the damping is smaller for smaller s. Also the damping decrement of the prime increases with increasing s. In the third and lower octave there is, in this case, no direct relationship between θ and s.

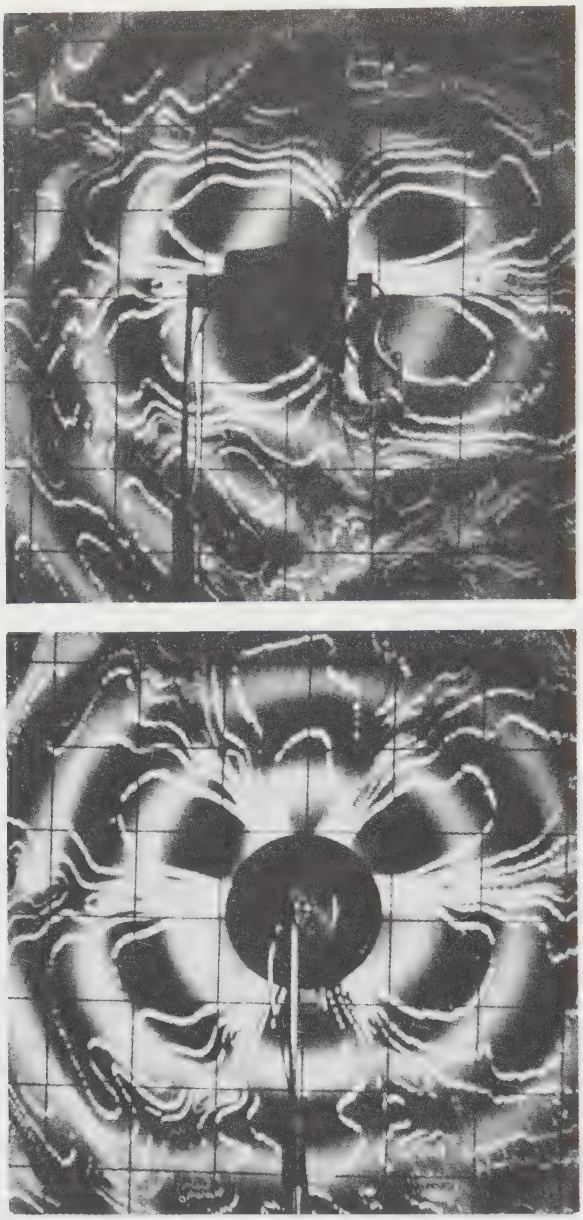

Figure 14. Sound field of a model bell combined phase and amplitude representation (after Schroeder [8]). (a) f = 3.35 kHz, viewed from the side; (b) f = 3.35 kHz, viewed from above.

The data show that there is obviously a critical area of maximum radiation damping of the partials of the bell. As the example of the iron bell shows, this area can be avoided even in a material with high sound speed if, for example, its abscissa reaches s = 2, that is, when its diameter is

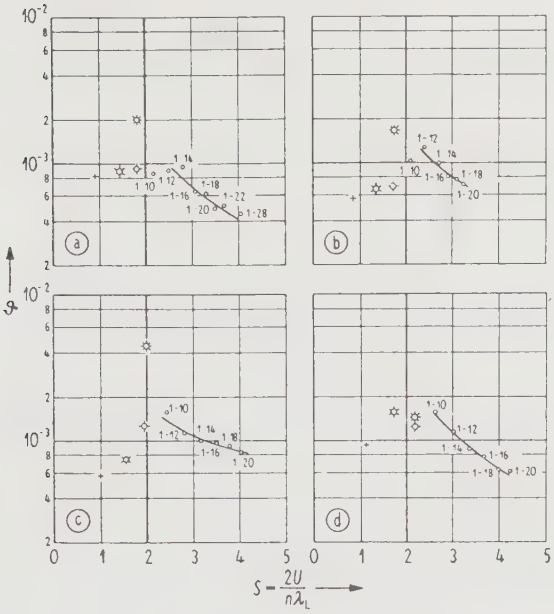

Figure 15. Dependence of logarithmic damping decrement θ on the ratio s of the maximum peripheral wavelength to the wavelength of sound in four bells of different material. (a) tin bronze; (b) silicon bronze; (c) cast steel; (d) cast iron.

relatively greater than that of a steel bell. In any case, even these shapes do not present an ideal solution to the problem of the iron bell, since the damping of the remaining principal tones is relatively high.

In the remaining partial tone groups of the bell, the vibrations with two nodal meridians ($n = 2$) exhibit a very high damping, and the axially symmetrical vibrations, an unusually low damping ($\theta < 10^{-3}$). The vibrations with two nodal meridians are stimulated strongly with ringing; since the crown of the bell, however, is not a nodal point for this vibration group, there is friction between the crown wood and the sound axis, and thus a high energy loss. On the other hand, the axially symmetrical vibrations radiate sound mainly in a narrow zone and have, because of their favorable circumstances, only a small radiation damping. Since their frequencies are relatively high and sound radiation is minimal, these partial tones are insignificant in the tonal picture despite their small damping.

The authors would like to thank the Bochumer Verein for steel castings, and especially their bell expert Mr. Bröcker for aiding in the experiments. Also, they thank Mr. H. Stappenbeck who participated in some of the experiments.

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NORMAL MODES OF THE MODERN ENGLISH CHURCH BELL

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Experimental measurements of the frequencies and nodal patterns of all the partials of a good quality 214 kg English church bell up to about 9 kHz have been made. By matching these with the results of finite element calculations an understanding of the physical mechanisms generating the various partials has been achieved. This has made possible the production, for the first time, of a classification scheme for the partials with a firm physical basis, and has given considerable new insight into church bell design. In particular it is now clear just how crucial to the production of the bell's characteristic timbre is the thick ring near its rim.

1. INTRODUCTION

Group theoretical arguments show [1] that the nodal patterns of normal modes of bells consist of $2m$ meridians at equally spaced azimuths and n circles parallel to the rim where $n, m = 0, 1, 2, \ldots$. Modes with $m > 0$ all belong to degenerate doublets, for a bell with perfect axial symmetry, whose members have identical nodal patterns apart from the meridians of one lying mid-way between those of the other. Such a pair of modes constitute a campanologist's "partial". The absolute locations of the meridians are indeterminate because the axial symmetry makes all azimuths dynamically equivalent. For $m = 0$ the modes are singlets and each one on its own constitutes a partial. There are two distinct types of these singlets, namely pure "breathing" modes and pure "twisting" modes. This degeneracy structure of bell partials has been confirmed experimentally [2].

In a real bell there are always imperfections in geometry and metallurgy. The results of this for doublet partials are [3] to cause (1) the fixing of the absolute locations of the nodal meridians such that there is no obvious connection between those from different partials, (2) slight distortions of the nodal patterns, and (3) slight splittings between the frequencies of the doublet components.

With the notable exceptions of the work by Tyzzer [4] and by Grützmacher *et al.* [5] there has been little detailed study of church bell partials beyond the five lowest in frequency. This is probably because these five "musical" partials are the only ones deliberately tuned by English founders [6]. The characteristics of these for a good quality bell always approximate closely to those listed in Table 1. Some German founders claim

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TABLE 1

Characteristics of the "musical" partials of a typical good quality church bell

Name	Frequency ratio†	Nodal meridians	Nodal circles
Hum	1	4	0
Fundamental	2	4	1 (near rim)
Tierce	2·378	6	1 (in waist)
Quint	2·997	6	1 (near rim)
Nominal	4	8	1 (in waist)

† These frequency ratios are based on the requirement that the bell be well tempered. It is common practice in the literature to round up the theoretical ratios and quote those of Tierce and Quint as 2·4 and 3, respectively, and some founders tune to this.

to tune specifically as many as nine partials, while English founders say that if the first five are correct then the others fall into line automatically. Two of the present authors (R.P. and T.C.) have confirmed that this is indeed the case with some good quality English bells [6].

The fact that physically distinct modes, with very different frequencies, can have equal values of $2m$ and n but with the nodal circles located in different places has caused considerable confusion over notation in the literature. The traditional *ad hoc* method for classifying the partials has been to define families as having a given number of circles at roughly specified positions, e.g. "one circle near the rim" and to designate different members of a given family by quoting the number of nodal meridians $2m$. This scheme has never been satisfactory because of the anomalous position of the Hum, being the *only* partial with no circles. The present writers have also found problems with the classification of a number of higher partials, probably unknown to earlier workers, with a variety of numbers and positions of circles. Clearly a better understanding of the mechanisms responsible for the various modes is needed in order to replace the old *ad hoc* scheme by one with a firm physical basis.

We have taken a good quality modern English church bell and measured the frequencies and nodal patterns of all the partials up to about 9 kHz as accurately as our equipment would allow. Accurate measurements of the geometry of the bell were then made and used as the basis for a finite element calculation of the normal modes. The hope was that we would be able to match up experimental and theoretical modes and so decide on the physical nature of each experimentally found partial by studying the finite element solution for its form. Families of partials should then be identifiable on a firm physical basis. Hopefully this should also give some insight into bell design.

2. PRACTICAL ARRANGEMENT

The bell used in the investigation had a diameter 70·2 cm, height 56·6 cm and weighed 214 kg. It was one of a peal cast by Messrs John Taylor of Loughborough and destined for St Margaret's Church, Leigh-on-Sea, Essex. Its musical note was D. It was suspended from a substantial angle-iron tripod. The bell rested on four diesel engine valve springs spaced around a plate which could be rotated inside the tripod on a central support rod through the hole destined for the clapper assembly. This was very different from the usual practical situation where the bell is rigidly bolted to a massive cast-iron headstock with a mass comparable to its own. Such normal hanging would probably eliminate some of the modes found in the present study, especially those with large amplitudes in the crown.

3. THE VIBRATION SPECTRUM

The first stage in the investigation was to discover as many partials as possible. A mode will remain undetected if either the driver or the pick-up is located on a nodal meridian or circle. The bell was therefore excited acoustically by a set of four small loudspeakers driven by an oscillator (Brüel and Kjaer (B and K) Type 1022). To minimize damping, pick-up was with a capacity transducer (B and K Type MM0004) feeding a measuring amplifier (B and K Type 2606) and a slave filter (B and K Type 2020) frequency locked to the oscillator. Output was to an oscilloscope and recorder (B and K Type 2305).

The oscillator frequency was swept very slowly—increasing at about 0·1 Hz/s—over the range 290 Hz to 20 kHz, the lower limit being selected to be just below the Hum frequency found in preliminary investigations. The frequencies of peaks on the chart were noted and the whole procedure repeated to give spectra at five different pick-up points chosen so that it was unlikely that pick-up would be on nodes of any given partial in all cases. We were therefore confident that each partial of any significance would appear on at least one of the five traces. A total of 134 partials were found in the range up to 9300 Hz and attention was concentrated on these, details of which are listed in Table 2.

4. NATURES OF THE PARTIALS

The second stage in the investigation was to take each one of these partials in turn and, in addition to measuring the frequency of each component accurately, attempt to identify it in terms of degeneracy structure, number of nodal meridians, and number and positions of nodal circles. To do this magnetic drive near an antinode was used. A small disc of radiometal was fixed to the bell with Picein wax using a large soldering iron, and driven by a magnetic transducer (B and K Type MM0002) from the same oscillator as before. Detection was with two accelerometers (B and K Type 4344) each feeding into a preamplifier (B and K Type 2625) and a measuring amplifier/slave filter pair. Output was to a double-beam oscilloscope. One accelerometer was glued firmly to the bell either near the drive point, or diametrically opposite to it, and used as a phase reference. The other was held against the bell by hand. The component modes of the great majority of partials were successfully excited from at least one of two positions located on the rim and in the waist respectively. A few of the more feeble partials required driving from positions selected by successive approximation. In several cases it was necessary to drive deliberately on a nodal circle of one partial in order not to mask the patterns of the components of a weaker partial close in frequency to it.

For practical purposes it is convenient to divide partials with doublet structures into three categories depending on the amount of splitting Δf of the pair, which usually have roughly equal half-power bandwidths $\delta f_{1/2}$. The categories are (1) $\Delta f \geqslant 3\delta f_{1/2}$, (2) $\Delta f \leqslant \delta f_{1/2}$ and (3) $\delta f_{1/2} \leqslant \Delta f \leqslant 3\delta f_{1/2}$. It is important to realize that the distinction between these categories is not sharp and that a specific partial as defined by numbers of nodal meridians and circles (plus positions) may differ in the category into which it fits as one changes from one bell to another.

4.1. WELL-SEPARATED PAIRS

The first category, that of well-separated pairs, consists of partial components so well separated in frequency that they appeared as completely separate peaks in the spectrum. Such partials in fact nearly all had two meridians. The relative height of the two peaks seen depended upon the positions of both pick-up and driver. In the simplest, but

TABLE 2

Experimental details of partials for the 70 cm Taylor bell

Reference number	Frequency (Hz)	Splitting (Hz)	Nodal meridians	Nodal circle positions (cm)	Classification (proposed scheme)	Comments
1	292·72	0	4	None	RIR	Hum
2	585·92	0	4	16	$R=1$	Fundamental
3	692·94	0	6	29	RIR	Tierce
4	882·53	0·32	6	10	$R=1$	Quint
5	1172·0	0·6	8	29	RIR	Nominal
6	1199·7	0	2	13, 41, 66	$R=1$	
7	1393·8	—	0	16, 38, 54	β?	Minor 17th
8	1470·1	—	0	21	$T=1, \alpha$?	Twister
9	1472·8	0·3	8	10	$R=1$	Major 17th
10	1525·6	0	6	11, 26	$R=2$	
11	1559·8	0·6	4	10, 28	$R=2$	18th
12	1619·8	13·1	2	11	RA	
13	1764·3	3·2	10	29, 57	RIR	Superquint
14	1776·2	—	0	11, 54	$R=1$	
15	1948·9	0	8	11, 29	$R=2$	
16	2040·3	0	2	9, 26, 47, 58, 64	$R=2$	
17	2146·8	1·2	10	9, 60	$R=1$	
18	2155·9	—	0	3, 41, 59	$R=2$	
19	2441·2	4·3	12	15, 53	RIR	
20	2443	4	2	52	β	
21	2485·7	0	4	7, 21, 38, 66	$R=3$	
22	2540·3	2·7	4	8, 39, 53, 64	RA	
23	2550·4	4·6	6	8, 21, 37	$R=3$	
24	2617·8	1·5	10	9, 32, 63	$R=2$	
25	2718·1	1·5	2	37, 55	α	
26	2804·5	—	0	8, 20, 50, 55	$R=3$	
27	2832·5	3·9	8	9, 22, 37, 66, 72	$R=3$	
28	2858·5	0·3	2	4, 20, 36, 51, 56, 63, 66, 68	$R=3$	
29	2921·6	0·9	12	9	$R=1$	
30	3028·2	—	0	8, 20, 35, 42, 63	$R=4$	
31	3172·2	—	0	5, 18, 32, 54, 63	γ?	
32	3183·1	7·1	14	31, 54	RIR	
33	3233·5	8·3	4	7, 20, 30, 44	$R=4$	
34	3369·6	4·6	2	6, 19, 30, 42, 59	$R=4$	
35	3376·8	0	10	9, 24, 38, 62	$R=3$	
36	3403·5	0·5	6	8, 29, 39, 57, 62	RA	
37	3433·4	2·1	12	9, 32	$R=2$	
38	3514·7	2·6	4	8, 12, 30, 40, 54	$R=5$	
39	3607·1	3·5	6	8, 18, 30, 40, 56	$R=4$	
40	3678·5	—	0	13, 46	$T=2, \delta$?	Twister
41	3698·1	—	0	6, 17, 28, 40, 55, 64	$R=5$	
42	3797·6	2·0	14	10, 59	$R=1$	
43	3867·0	3·5	8	8, 18, 30, 41	$R=4$	
44	3934·8	19·4	2	4, 17, 29, 39, 49, 59	$R=5$	
45	3982·0	3·6	16	27	RIR	
46	4000·0	?	4	5, 18, 28, 38, 50, 66	α	
47	4126·8	1·1	12	7, 24, 36	$R=3$	a(48)
48	4130·3	3·5	4	10, 18, 26, 42, 55	β	a(47)
49	4269	0	10	6, 18, 32, 43, 56	$R=4$	
50	4291	?	14	10, 29	?	Rogue mode

TABLE 2—*continued*

Reference number	Frequency (Hz)	Splitting (Hz)	Nodal meridians	Nodal circle positions (cm)	Classification (proposed scheme)	Comments
51	4340	0	14	7, 28	$R=2$	
52	4357	3	6	6, 17, 28, 38, 50, 73	$R=5$	
53	4360	38	2	7, 17, 28, 35, 52, 64, 66	γ	
54	4401	33	8	7, 20, 30, 42, 67	RA	
55	4433·6	—	0	5, 14, 25, 35, 45, 56	$R=6$	
56	4543	4	4	4, 15, 25, 35, 44, 58	$R=6$	
57	4765·3	5·9	16	9, 48, 57	$R=1$	
58	4813·5	0	18	20, 26, 36, 40, 50, 61	RIR	a(59)
59	4837·9	24·4	2	7, 15, 26, 34, 42, 57, 64	$R=6$	a(58)
60	4860·3	0	6	7, 17, 26, 35, 44, 56	$R=6$	
61	5004	7	8	4, 16, 26, 35, 45, 59	$R=5$	
62	5005·5	3·0	14	8, 24, 36, 55	$R=3$	
63	5076·0	11·5	12	8, 18, 30, 41, 61	$R=4$	
64	5092	23	2	5, 15, 43, 54, 62	δ	a(65)
65	5185	—	0	7, 16, 23, 31, 39, 46, 56, 59	$R=7$	a(64)
66	5255	14	4	5, 14, 23, 31, 40, 49, 60	$R=7$	
67	5298	3	10	8, 24, 34, 43, 59	RA	
68	5346	4	16	7, 28	$R=2$	
69	5523	—	0	36, 45	ε?	
70	5563	7	10	7, 17, 27, 34, 43, 59	$R=5$	
71	5599	7	6	23, 32, 37, 46	α	
72	5676	2	20	23, 30, 37, 49, 50	RIR	
73	5721	?	2	5, 15, 23, 31, 38, 44	$R=7$	a(74)
74	5721	?	4	5, 15, 23, 31, 38, 44	γ	a(73)
75	5769	0	8	5, 16, 27, 32, 40, 49, 64	$R=6$	
76	5839	47	2	6, 14, 21, 29, 38, 43, 52, 64	ε	
77	5815	6	18	9, 54	$R=1$	
78	5878	1	14	8, 18, 29, 38, 59	$R=4$	
79	5888	5	6	4, 13, 23, 30, 38, 46, 58, 70	β	
80	5989	8	4	6, 15, 24, 32, 37, 48, 60	?	Rogue mode
81	6006	4	16	8, 22, 35, 58	$R=3$	
82	6070	?	6	7, 13, 22, 32, 38, 46, 57	$R=7$	
83	6105	4	12	7, 15, 25, 37, 42, 64	$R=5$	
84	6326	—	0	6, 13, 21, 29, 36, 42, 49, 61, 70	$R=8$	
85	6365	15	4	6, 14, 22, 29, 36, 41, 52, 61	$R=8$	
86	6402	8	12	7, 20, 27, 36, 44, 59	RA	
87	6416	2	18	9, 27, 54	$R=2$	
88	6520	4	8	4, 17, 24, 32, 38, 45, 52	$R=7$	
89	6559	4	22	?	RIR	
90	6657	2	10	6, 13, 24, 31, 37, 44, 57	$R=6$	
91	6777	36	2	4, 13, 21, 28, 35, 41, 48, 54, 62	$R=8$	
92	6817	28	16	8, 18, 29, 38, 59	$R=4$	
93	6850	12	4	4, 19, 41, 51, 62	δ	
94	6921	2	14	6, 15, 25, 34, 43, 61	$R=5$	
95	6933	72	20	11, 47, 58	$R=1$	
96	7085	0	18	7, 22, 33, 58	$R=3$	
97	7125	7	8	32, 37, 42, 49, 61	α	
98	7131	?	6	3, 12, 22, 28, 35, 41, 47, 57, 66	$R=8$	
99	7159	20	2	5, 15, 20, 27, 35, 40, 45, 59, 66	ζ	a(100)

TABLE 2—*continued*

Reference number	Frequency (Hz)	Splitting (Hz)	Nodal meridians	Nodal circle positions (cm)	Classification (proposed scheme)	Comments
100	7159	15	6	5, 12, 19, 27, 34, 40, 46, 58, 66	γ	a(99)
101	7329	0	12	6, 13, 22, 30, 38, 45, 57	$R=6$	
102	7377	4	14	7, 22, 29, 37, 46	RA	
103	7441	—	0	?	$R=9$	a(104)
104	7448	7	24	?	RIR	a(103)
105	7499	0	10	5, 13, 22, 30, 36, 44, 52	$R=7$	
106	7550	3	20	6, 27, 49, 56, 62	$R=2$	
107	7559	20	4	5, 13, 20, 26, 33, 39, 45, 55, 62	$R=9$	
108	7683	0	8	5, 12, 21, 27, 34, 40, 47, 56	$R=8$	
109	7812	7	16	4, 15, 26, 33, 42	$R=5$	
110	7824	5	8	5, 12, 23, 28, 35, 39, 46, 56	β	
111	7874	4	18	6, 18, 27, 36, 60	$R=4$	
112	8104	31	2	4, 11, 20, 25, 31, 36, 42, 49, 58, 65	$R=9$	a(113)
113	8112	8	22	?	$R=1$	a(112)
114	8142	3	14	5, 15, 22, 30, 38, 45, 63, 66, 70	$R=6$	
115	8207	5	6	4, 12, 21, 27, 32, 38, 45, 51, 60, 70	$R=9$	
116	8236	23	20	7, 22, 32	$R=3$	
117	8343	5	26	?	RIR	
118	8363	15	16	6, 13, 21, 25, 34, 41, 47, 60	RA	
119	8380	3	12	6, 15, 21, 27, 34, 40, 44, 49, 63	$R=7$	
120	8405	?	2	?	η	
121	8412	?	10	6, 14, 22, 27, 35, 40, 47, 59	$R=8$	
122	8521	12	4	4, 12, 20, 26, 32, 39, 45, 63	ε	
123	8573	6	10	6, 17, 22, 28, 34, 40, 46	α	
124	8584	—	0	5, 12, 18, 24, 30, 35, 48, 54, 61, 70	$R=10$	
125	8731	2	22	5, 26, 53	$R=2$	
126	8790	35	2	5, 13, 18, 24, 30, 36, 41, 48, 56, 60, 69	θ	
127	8812	5	18	4, 16, 24, 32, 44	$R=5$	
128	8892	?	4	1, 10, 18, 25, 29, 35, 41, 47, 52, 62, 73	$R=10$	
129	8974	6	8	5, 13, 20, 25, 31, 38, 45, 49, 58	$R=9$	
130	8981	4	20	?	$R=4$	
131	9034	5	16	6, 15, 23, 30, 37, 42	$R=6$	
132	9153	2	6	6, 16, 25, 30, 36, 41, 51, 58, 66	ε	
133	9254	5	28	?	RIR	
134	9285	1	14	4, 12, 22, 27, 34, 41, 46	$R=7$	

unlikely, case where the driver was exactly at a nodal meridian of one component, and therefore approximately at an antinodal meridian of the other, movement of the mobile pick-up around the bell showed the nodal meridians of the second of these components as lines of zero amplitude with a phase change of π across them. Selecting a point

mid-way between such a pair of nodal meridians and moving up and down the bell with the mobile pick-up gave similar changes at the nodal circles. With the driver in such a position it was, obviously, very difficult to produce much response from the other component. However, the pattern for this component could be obtained by retuning to its frequency and moving the driver, with a corresponding change of reference position, to a nodal meridian of the component already investigated. It was found, as predicted by group theoretical arguments, that the nodal circles of both components always lay at the same height on the bell, while the nodal meridians of one component always lay approximately mid-way between those of the other.

In cases where the initial position of the driver proved not to be at a nodal meridian for either component the located nodal positions were no longer zero-amplitude points but rather gave amplitude minima. Once the locations of these were found it was a simple matter to place the driver at an optimum position and proceed as before.

Singlet partials, i.e., those with zero meridians, could, obviously, be treated like one component of a well-separated pair.

4.2. DEGENERATE PAIRS

The second category can be called "degenerate pairs". When the frequencies of the doublet components differed by less than about the half-power bandwidth of either member, a single peak appeared in the spectrum regardless of the positions of driver and pick-up. For *any* driver position a nodal pattern was found which had an antinode at that position. The nodal positions of both meridians and circles were well defined but the entire pattern moved around the bell with the driver position as it was changed. This is precisely what is to be expected in the case of true degeneracy, where the relative positions of the nodal patterns of the components are fixed but their absolute locations are indeterminate until some external agency, in the present case the driver, is introduced. For the bell used in the present study the Hum provided a particularly striking example of a truly degenerate pair.

4.3. INTERMEDIATE CASE

The vast majority of partials in the present study fell into an intermediate (third) category which can be defined as having differences between component frequencies of between roughly 1 and 3 half-power bandwidths. Two close peaks separated by a shallow dip appeared in the spectrum except in the unlikely event of the driver being placed exactly on a nodal meridian of one of the pair. In this case only the other member could be "tuned in" and behaviour was then exactly as in the well-separated case.

If the driver happened to be placed midway between an antinodal meridian of one component and an antinodal meridian of the other then tuning showed two equal peaks with a minimum midway between them, i.e., an "equal-amplitude point" of the type previously described by the authors in reference [7]. When driven at this mid-frequency both components were excited equally but with a phase difference of $\pi/2$ between them. Thus as the pick-up was moved around the bell at a fixed distance from the rim the appearance was that of a travelling wave, i.e., a constant amplitude with phase changing at a constant rate per unit distance. The number of meridians attributed to the partial was determined by counting the number of times that the phase relative to the reference was either 0 or π as the pick-up was moved around the bell. Nodal circles were located as before.

In general the driver did not happen to be at either a nodal meridian or an "equal amplitude point" so that one peak of the pair was stronger than the other. Tuning to the stronger peak and moving the pick-up around the bell then showed a constant rate

of phase change as before but this time with a series of maxima and non-zero minima rather than a constant amplitude. The number of nodal meridians, but not their locations, was again most readily determined by counting the 0 and π phase positions. When meridian locations were required the position of the driver had to be altered to make the minima zero.

4.4. NODAL CIRCLES

By making measurements going around a suitable antinodal circle it was always possible to obtain an unambiguous value for the number of nodal meridians as described above. In principle it should have been a simple matter to locate and count nodal circles by moving the pick-up along an antinodal meridian. In practice this was true for most partials below the "shoulder" of the bell. However, the amplitude near the top of the bell was so small for many partials that it was often impossible to determine whether nodal circles existed there or not. Another problem was that, when circle numbers became large, they tended to appear in pairs close together. If these were separated by less than the order of size of the detecting accelerometer then it obviously became very easy to miss both of them during a meridian-wise sweep. This meant that for numerous partials the total number of experimentally found nodal circles could not be used with any confidence as a descriptive parameter and was therefore of only limited value in seeking a classification scheme. Nevertheless, the *positions* of those circles which could be observed proved to be of great value for the latter purpose.

4.5. ACCIDENTAL DEGENERACY

To the group theorist accidental degeneracy in the context of vibrations means degeneracy between modes which is not due to the symmetries of the system. In the case of a bell it would mean degeneracy between different partials, i.e., over and above those from the components of the partials. Although one might intuitively expect that such degeneracies would be rare in a bell a number were found in the present study and usually manifested themselves by a mode appearing to have different numbers of nodal meridians at say the top and bottom of the bell with a region of total confusion in the waist. In some cases this led to ambiguities over which nodal circles belonged to which partial.

4.6. EXPERIMENTAL RESULTS

Details of the experimental results are given in Table 2. The numbers of meridians quoted are accurate although, for reasons already explained, care was needed in interpreting cases with accidental degeneracy. These are emphasized in the table by comments of the type "$a(x)$" where x is the reference number of a partial with one or both of whose components the given partial exhibits accidental degeneracy. The frequencies quoted are accurate to ± 1 in the last digit. In those cases with $m > 0$, where the partials are known to be doublets for group theoretical reasons, the frequencies of both components were measured whenever this was possible. In these cases the frequency quoted is the higher of the two and the frequency difference Δf is listed.

The positions of nodal circles are quoted in the table as the distance in cm upwards from the rim along the outer bell surface at a fixed azimuth. It is difficult to estimate errors on these values because there was not only the error in detecting the phase-flip position along a given azimuth to be considered but also, due to pattern distortion, the fact that the location of a given circle for a given partial component varied somewhat as one moved circumferentially around the bell. The values quoted are averages with this variation taken into account and also, in the cases with $m > 0$, with allowance made

for any small variations between the two partial components. A realistic estimate for the error on a typical circle position would be about ± 2 cm. For comparison with results for other bells it would be desirable to convert these locations into, say, fractions of the distance from rim to shoulder. This can easily be done given that this distance was 54 cm for our Taylor bell.

5. THE FINITE ELEMENT APPROACH

In the finite element method one considers a structure of complex shape as being made up of elements of simpler shape. Expressions for the dynamic behaviour of each element are obtained, and enforcing displacement continuity across the element boundaries results in an expression for the behaviour of the whole structure observed at certain sampling points.† From this expression one obtains an eigenvalue equation for the natural frequencies which is then simplified to a manageable size by a technique known as "eigenvalue economisation". By selecting the q most suitable sampling points one can expect the first $\frac{1}{3}q$ normal modes to be reliably and accurately calculated.

Thanks to the axial symmetry of the bell one can anticipate the degeneracy structure of the normal modes, restrict attention to one member of each pair and take its amplitude to be of the form

$$\mathbf{u}(r, \theta, z') = a_x(r, z') \cos m\theta\, \hat{\mathbf{i}} + a_y(r, z') \cos m\theta\, \hat{\mathbf{j}} + a_z(r, z') \sin m\theta\, \hat{\mathbf{k}}, \tag{1}$$

where (r, θ, z') are cylindrical polar co-ordinates, with the symmetry axis of the bell as z'-axis. The unit vectors $(\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}})$ are a right-handed orthogonal Cartesian set with $\hat{\mathbf{i}}$ lying along the symmetry axis in the sense from rim to crown and $\hat{\mathbf{j}}$ normal to this axis in some fixed reference plane. Because the axial symmetry enables one to fix the θ variation of the amplitude one is able, for finite element purposes, to model the bell as a two-dimensional structure in this reference plane. Once the geometry of the Taylor bell had been measured it was found that the set of quadrilateral elements shown in Figure 1 was appropriate. The amplitude of the other member of each degenerate pair is also given by equation (1) except that the sine and cosine functions must be interchanged.

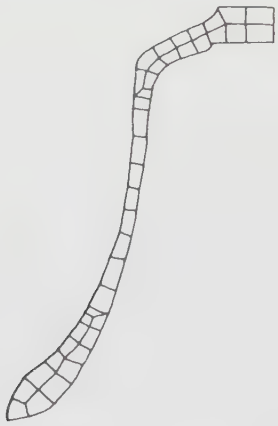


Figure 1. The finite elements used in the calculations.

† In the literature on the finite element method these points are described as "nodes". Such jargon is avoided in the present paper in order to avoid confusion with the more conventional use of the term in vibration work.

In the present work we used the finite element package known as PAFEC (Program for Automatic Finite Element Calculations) with which a separate computer run is needed for each value of m. The value of q was selected so that about the first 20 partials were expected to be calculated accurately for each m value. The values of Young's modulus, Poisson's ratio and the density for bell metal input as data were 103 GPa, 0·38 and $8{\cdot}85 \times 10^3$ kg m^{-3}, respectively. In addition to estimating the frequencies of the normal modes PAFEC was used to produce graphical displays of the amplitudes in the x–y plane. Examples are shown in Figures 2–4.

The package also produces the z-components but these could not easily be incorporated into the figures, nor would this have been particularly helpful for present purposes.

Figure 2. The six-meridian inextensional radial ring driven (RIR) mode; $2m = 6$. (Actually the Tierce.)

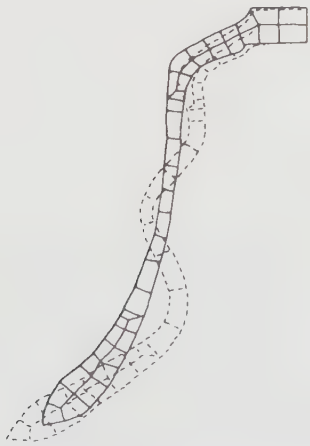
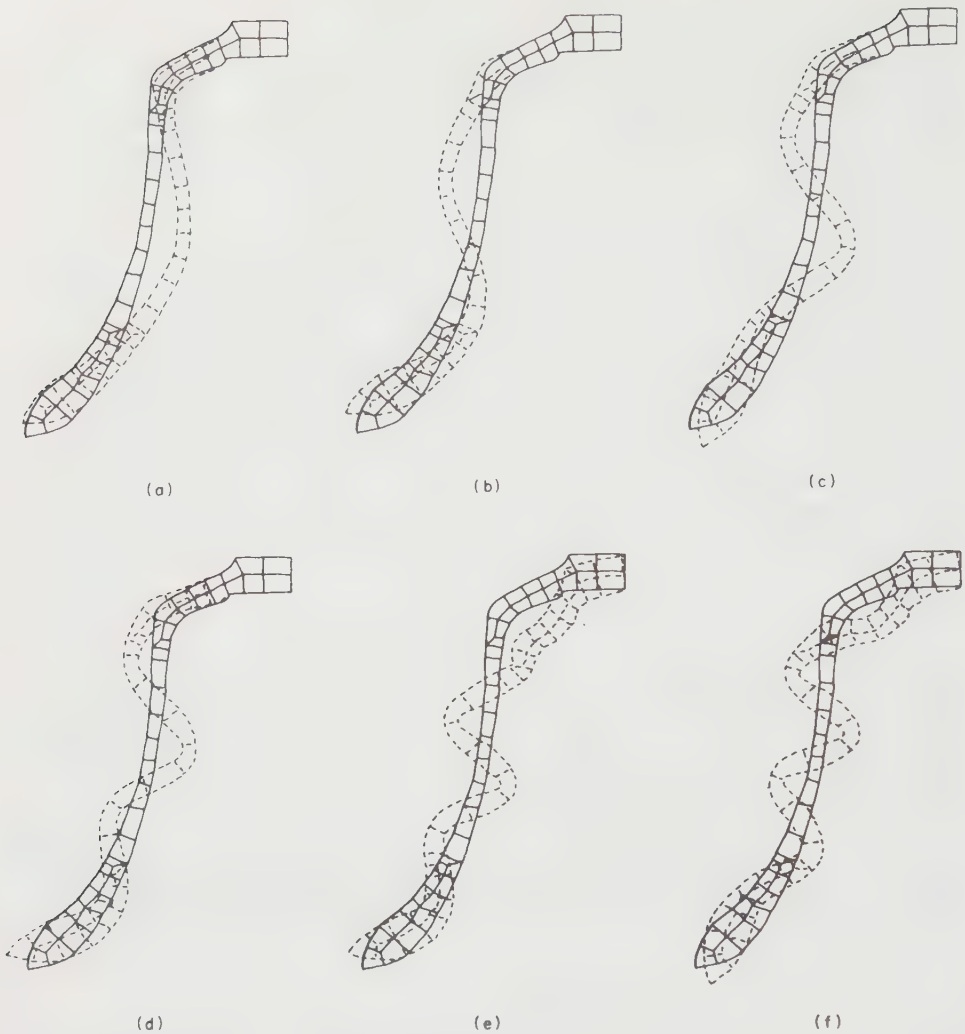


Figure 3. The six-meridian axial ring driven (RA) mode; $2m = 6$.



Figures 4(a)–(f). The six-meridian "shell driven" modes with $R = 1$–6. $2m = 6$; $R =$ (a)1, (b)2, (c)3, (d)4, (e)5, (f)6.

6. COMPARISON OF EXPERIMENTAL AND FINITE ELEMENT RESULTS

Since the numbers of nodal meridans $2m$ were known reliably for both the experimental and finite element results, a comparison of these two sets of information for each value of $2m$ could be made with some confidence. Optimistically one might expect to be able to write down two lists of partials for a given $2m$ in order of increasing frequency, pair them off in sequence and so achieve agreement between both frequencies and nodal circle details. In practice things were more difficult for a number of reasons. (1) Sometimes a particular partial wrongly appeared twice in the experimental list because it had not been realized that what was present was a well-split doublet rather than single members of two doublets whose partners it had not been possible to excite. A re-identification of which modes were components of which partials was then necessary. (2) Sometimes a

partial was missing from the experimental list, although none should have been missing from the theoretical one. In many such cases it was possible to produce these missing (weak) partials experimentally in a subsidiary experiment by driving at a location and in a manner which the finite element solution indicated was likely to excite the modes. These "extra" partials have been included in Table 2. (3) Experimental nodal circle numbers and positions were often unreliable, especially above the shoulder. Although more unusual it was also sometimes the case that theoretical circle information was unreliable in regions where the amplitude became small—again usually above the shoulder. (4) The theoretical frequency predictions could be expected to be reasonably accurate early in the sequence but to become increasingly too high as one moved up it. (5) Although, after allowing for missing experimental items, the order of corresponding partials should have been the same on both lists there were some cases where two consecutive theoretical partials came out the wrong way around. It was usually nodal circle data which made it evident that this had happened, and it was always in situations where the two partials concerned were very close in frequency but of different physical types, i.e., just where one might expect rounding errors to produce such a situation.

The matching procedure was in fact carried out from $2m = 0$ up to $2m = 10$ inclusive. In order to give some idea of how it was done and how good the agreement was a listing of the frequencies for the eight-meridian case is given in Table 3. This was a particularly comprehensive example. The theoretical partial numbers 1 to 5 and 7 to 9 were well matched, in the context of the preceding comments, by the corresponding experimental items in terms of both frequency and nodal circles. The items corresponding to partial number 6 had been badly split and realization of this caused us to replace what had been listed as two separate items with missing partners in the preliminary version of Table 2 by a single badly split item. Partial numbers 10 and 13 were missing from the initial experimental list. However, a subsidiary experiment with the bell driven tangentially near the rim, as suggested by the PAFEC solution, succeeded in exciting number

TABLE 3

Comparison of finite element predictions and experimental measurements for the eight meridian partials

Mode number (PAFEC)	Frequency (Hz) Theory	Frequency (Hz) Experiment	Classification (proposed scheme)
1	1166	1172	RIR
2	1465	1473	$R = 1$
3	1962	1949	$R = 2$
4	2840	2833	$R = 3$
5	3909	3867	$R = 4$
6	4367	{4368, 4401}	RA
7	5081	4997	$R = 5$
8	6067	5769	$R = 6$
9	6858	6516	$R = 7$
10	7133	7127†	α
11	7989	7824	β
12	8201	7683	$R = 8$
13	9008	—	γ
14	9782	8968	$R = 9$

† Mode excited only after subsidiary experiment with suitably chosen drive arrangement.

10 which is therefore incorporated in the final version of Table 2 but has been given a dagger in Table 3. We were unable to excite number 13 even with what appeared to be a suitable driving arrangement in a subsidiary experiment. The PAFEC items 11 and 12 have come out the wrong way around, a fact which was initially ascertained by making a careful comparison of nodal circle locations, and were found to have very different physical mechanisms, as will be explained in section 7. It should be noticed that by the time PAFEC item 14 is reached the theoretical frequency has become very high compared with the experimental one, as anticipated, although the nodal circle patterns match well.

7. MODE CLASSIFICATION FOR $2m \geqslant 4$

The distinction between modes and partials having been drawn in section 1 one can, for purposes of producing a classification scheme, treat them as being synonymous. Also one can initially restrict discussion to modes with four meridians or more. This is because, although a scheme has been produced which is very satisfactory for $2m \geqslant 4$, the incorporation of modes with zero or two meridians into it has proved problematical. The reason for this is that the effective boundary conditions at the crown differ for $m = 0$ and $m = 1$ both from all cases with $m \geqslant 2$ and from each other. They will be more fully discussed in section 8.

The finite element solutions show that there are two broad categories of mode: (1) those primarily in-azimuthal-plane but with some out-of-plane† motion; (2) those primarily out-of-plane but with some in-plane motion. The primarily in-plane modes divide into two types which may be described as "ring driven" and "shell driven". For a given number of nodal meridians there is always one partial (provided $2m \geqslant 4$) which corresponds to the heavy ring at the rim of the bell going into its inextensional radial partial [8] and driving the rest of the bell. Consequently the number of nodal circles will vary depending upon the natures of the nearest modes of the rest of the bell. This family, which includes the formerly anomalous Hum as well as the Tierce and the Nominal, i.e. three of the five important musical partials, one may designate as RIR. The PAFEC diagram for the Tierce is shown in Figure 2 as a typical example. In classical ring vibration theory the inextensibility condition is written in the form $u + \partial v/\partial\theta = 0$ where $\mathbf{u} = u\hat{\mathbf{j}} + v\hat{\mathbf{k}} + w\hat{\mathbf{i}}$, which, in the notation used in equation (1), becomes $a_y = ma_z$. Thus as m increases the radial component becomes increasingly large compared with the tangential one—a feature which is particularly easy to check against the PAFEC solutions.

A second "ring driven" partial also arises for each value of $2m$ which corresponds to the rim ring going into its axial‡ mode [8]. These one may designate RA and again they do not correspond to a fixed number of nodal circles, and for the same reason. An example is shown in Figure 3. The fact that neither RIR nor RA partials correspond to a fixed number of nodal circles, so that the quantum number n is not a "good" one, explains the anomalous position of the Hum and some of the higher partials in the old *ad hoc* classification schemes.

The remaining primarily-in-plane partials, which actually form a very large majority of all partials in the region up to 9 kHz, are all essentially driven by the "bell minus rim ring" system which one may describe as the "shell". These partials are thus "shell driven" with the heavy rim ring remaining roughly at rest and supplying a nodal circle at or near the rim. For a given value of $2m$ there is a sequence of these partials which, according

† It is important to understand that the "plane" referred to here and subsequently is one, such as the reference plane introduced in section 5, which contains the symmetry axis of the bell.

‡ It is not entirely clear whether these modes are better described as "axial" or "torsional" ring driven. This point will be discussed in detail elsewhere. The important point is that they are certainly *ring* driven.

to PAFEC, have 1, 2, 3, . . . nodal circles and which one may designate as forming $R = 1, 2, 3, \ldots$ families. The first six partials in the sequence for $2m = 6$ are shown in Figures 4(a)–(f). The good quantum number R is clearly related to the number n of nodal circles found experimentally in each case. However, there are frequent discrepancies between the two mainly because of extra experimental circles in the crown. This is no doubt due in large measure to the experimental problems associated with the low amplitudes in the crown region. However, it seems reasonable to suppose that these low amplitudes themselves may permit otherwise insignificant "crown driven" modes to interfere to a measurable degree with the "shell driven" ones in this region. In any event it is significant that there is seldom any disagreement concerning either the numbers or positions of nodal circles in the region below the shoulder.

From the sequence of diagrams for "shell driven" modes shown in Figures 4(a)–(f) for increasing values of R with $2m$ held fixed at 6, this being selected as a typical case, it can be seen that for low values of R the crown is essentially at rest, playing a similar role at the top of the bell to that played by the rim ring at the bottom. However, as R increases, and so the wavelength of the standing waves in the meridional direction decreases, a point is reached where the crown behaves as an integral part of the "shell" subject only to the constraint (for $2m > 2$) of a node at the very top. Further increases in R produce modes where the crown continues to participate. The value of R at which the crown "takes off" in this way increases as $2m$ increases: empirically for a given value of $2m$ the change occurs, approximately, when $R = m + 1$. The nodal circles predicted by PAFEC rise above the shoulder only after this take-off point has been passed.

Further evidence for the existence of a family relationship amongst these "shell driven" modes comes from three different ways of looking at the experimental data. (1) Plotting the positions of the nodal circles l_R against $2m$ for fixed R shows R smooth curves in each case. In Figure 5 the case of $R = 3$ is shown as a typical example. (2) Plotting l_R

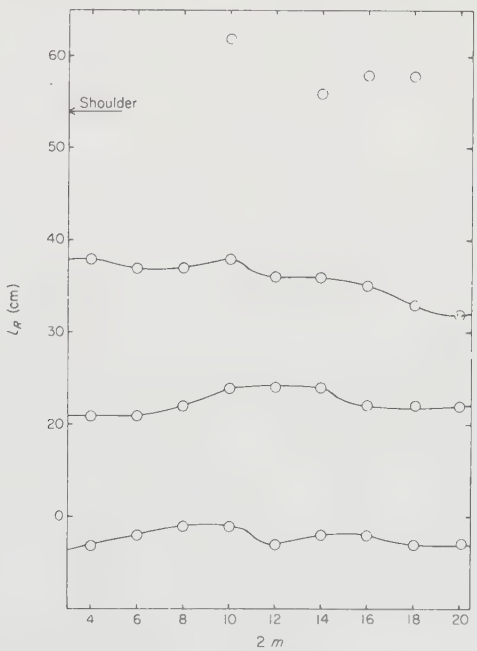


Figure 5. Graph of the ring positions against number of meridians with fixed R (= 3).

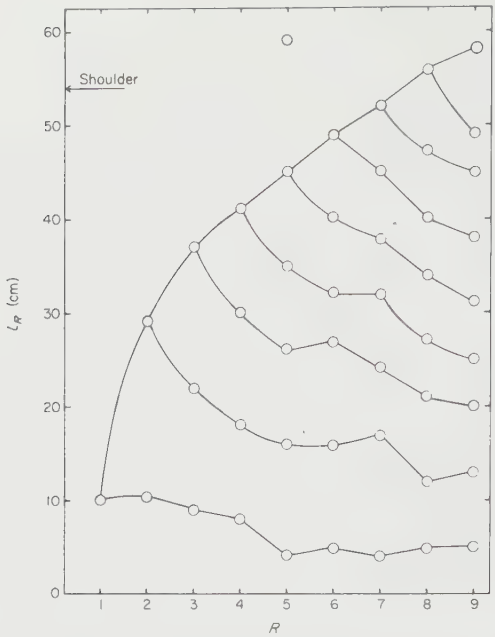


Figure 6. Graph of ring positions against R value with fixed $2\,m$ $(=8)$.

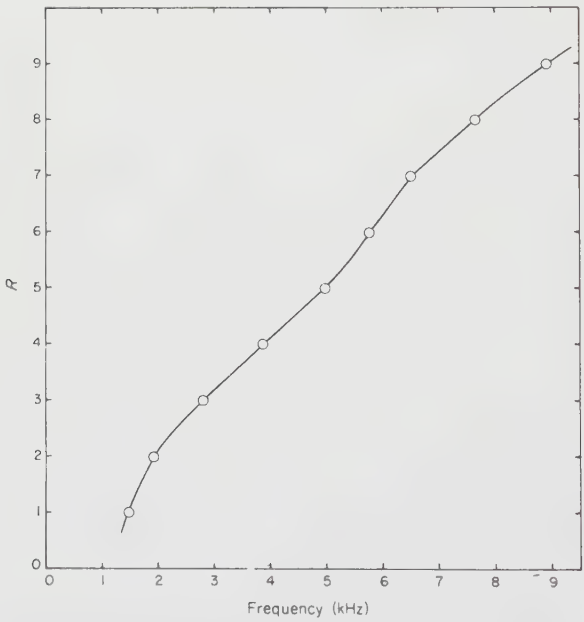


Figure 7. Graph of frequency against R value with fixed $2\,m$ $(=8)$.

against R for fixed $2m$ shows a characteristic pattern which varies very little as $2m$ is changed. The case for $2m = 8$ is shown in Figure 6. (3) Plotting the frequency f against R for a fixed value of $2m$ yields a smooth curve, which is better than can be achieved with any other ordering or combination of modes with that value of $2m$, with a kink in the region corresponding to the crown take-off. The eight meridian case is shown in Figure 7. If one plots $\log f$ *vs.* R then the curve for fixed $2m$ is replaced, to a surprisingly good degree of approximation, by two straight lines which intersect at the point previously occupied by the kink. If the value of $2m$ is changed then a new pair of straight lines is produced. With the exception of $2m = 4$ where the situation is rather ill-defined, the position of the point of intersection is at a value of R which, approximately, increases by unity each time m increases by unity. The cases from $2m = 6$ up to $2m = 22$ inclusive are shown in Figure 8(a) while, for the sake of clarity, that for $2m = 4$ is given in Figure 8(b).

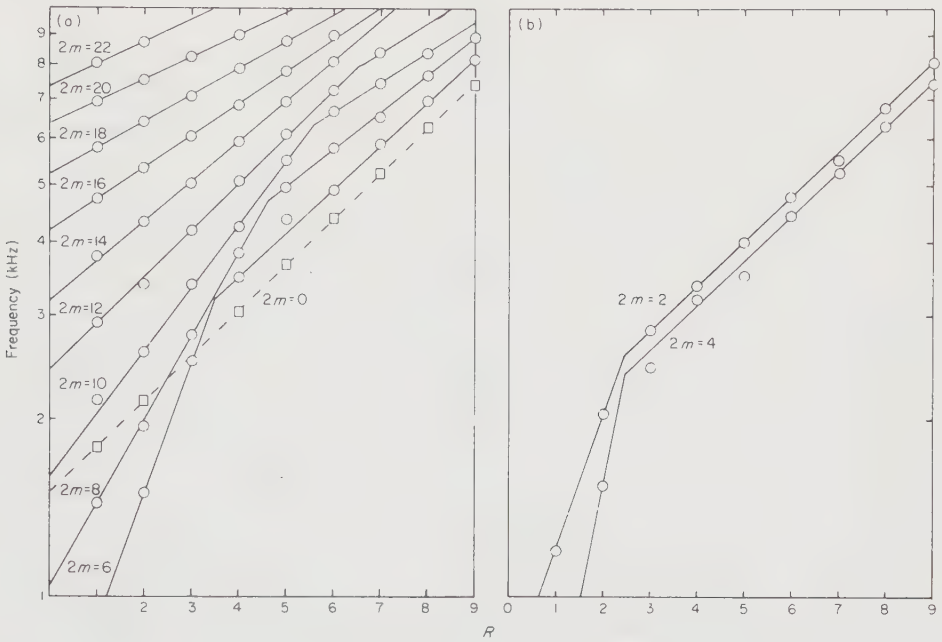


Figure 8. Graph of $\log f$ against R value with fixed $2m$: (a) $2m = 0$ and 6, 8, 10, . . . , 22, and (b) $2m = 2, 4$.

The modes which are primarily out-of-plane are relatively high in frequency and difficult to excite, the latter fact being responsible for a relatively high proportion of them being missing from the experimental list initially and indeed even after supplementary experiments with tangential drive. For a fixed value of $2m$ there is a sequence of these modes having an increasing number of nodal circles for out-of-plane motion. One may designate these as α, β, γ, ... The first of these is due to the heavy rim ring going into its extensional radial mode [8] and driving the shell with it, so one may give this family the alternative designation RER. The others seem to be due to various other ring sections of the bell going into their extensional radial modes in a similar way but the situation is extremely complicated. The modes were allocated to the various families mainly by examination of the out-of-plane part of the PAFEC solutions and by requiring smooth interpolation curves in the frequency against meridian number diagram to be

discussed below. Since these modes are of little acoustical importance we shall not pursue the matter in detail here. In Figure 9 the PAFEC diagram for the δ mode for $2m = 6$ is shown as a typical example but clearly, since the most important part of the motion is out-of-plane, these diagrams are not particularly helpful in these cases.

If one plots all the partials on a $2m$ versus frequency diagram then at first glance the situation is chaotic. Even when the framework of the classification scheme, which covers

Figure 9. The six-meridian δ mode.

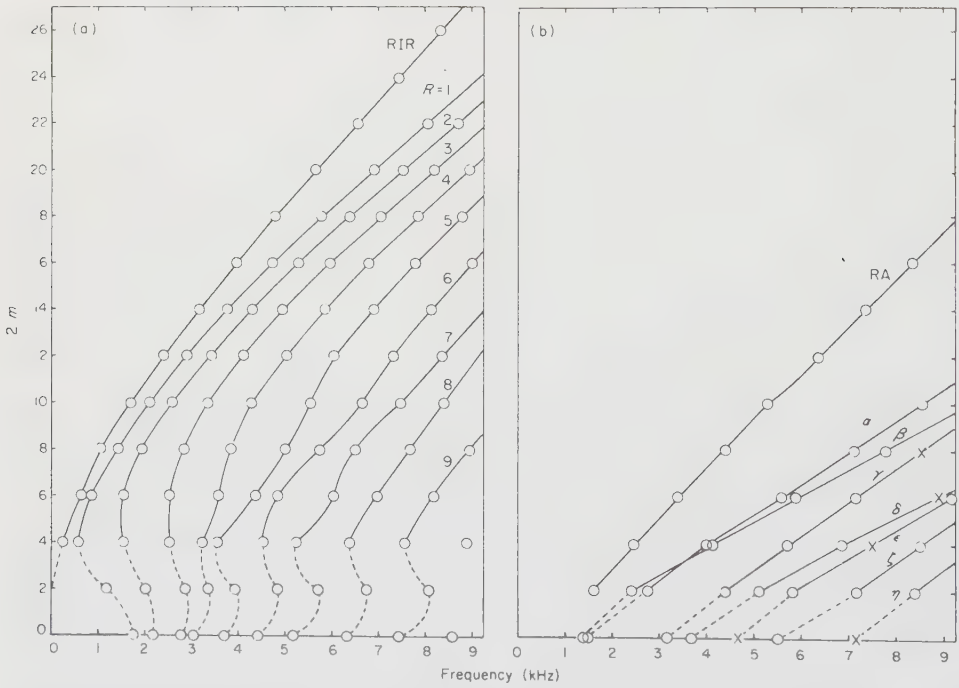


Figure 10. Graphs of meridian number against frequency for (a) the shell driven and the RIR families and (b) the families RA and $\alpha, \beta, \gamma, \ldots$.

almost every partial with $2m \geqslant 4$, is superimposed the situation is still very confusing. We have therefore split it up and show the various "shell driven" modes in Figure 10(a) together with the RIR family and in Figure 10(b) the primarily out-of-plane modes together with the RA family. Family relationships have been indicated in these figures by smooth interpolation curves. For completeness modes with $2m < 4$ have also been incorporated: their classification is discussed in the following section. A number of primarily out-of-plane modes were "missing", i.e., predicted by PAFEC but not detected experimentally even after subsidiary experiments. These have been incorporated into Figure 10(b), with their status clearly indicated by the use of crosses rather than circles, using their predicted frequencies. Their presence is required in order to make sense of the low population γ, δ and ε families. The approximately linear natures of all the family curves in Figure 10(b) is worthy of note: the significance of this will be discussed more fully in a further paper.

8. MODE CLASSIFICATION FOR $2m < 4$

For modes with two nodal meridians there again occurs a sequence of "shell driven" type with the rim ring essentially at rest, forcing the system to have a nodal circle at or near its location. However, there being no necessity for a node at the crown, the sequence has 2, 3, 4, ... nodal circles. These modes have been described in the literature as "swingers" [2]. The PAFEC diagram for the first member is shown in Figure 11(a). For "shell driven" modes with $2m \geqslant 4$ the node at the top of the crown was not counted for the purpose of deciding R values. Thus in the present case it seems reasonable to define the members of the sequence with 2, 3, 4, ... nodal circles as $R = 1, 2, 3, \ldots$ and this is the procedure that was used in drawing up Table 2. Although this approach is unambiguous and helpful it is not entirely clear whether these R designations place the modes into genuine physically meaningful family relationships with the corresponding modes with $2m > 2$. It is for this reason that the interpolation curves in Figure 10(a) are shown as broken below $2m = 4$. The tenuous nature of the family relationships is emphasized by the fact that, on the whole, these $2m = 2$ modes do not fit well onto the l_R *vs.* $2m$ curves for fixed R. However the pattern produced in the l_R *vs.* R at fixed $2m$ diagram is reasonably satisfactory apart from the appearance of an extra curve in the "above shoulder" region due to the changed effective boundary conditions in the crown. Also the log f *vs.* R diagram consists of two intersecting straight lines, shown in Figure 8(b), as in the cases with $2m \geqslant 4$.

Two-meridian modes, analogous to the primarily out-of-plane cases with $2m \geqslant 4$ in the sense that they can be classified readily by looking at the out-of-plane parts of the PAFEC solutions, also occur. These correspond to the two-meridian extensional radial modes of cylinders. The kind of problems encountered with the two-meridian "shell driven" modes do not arise. This is reflected in the way that the linear family curves in Figure 10(b) can be continued down to include the two meridian cases without difficulty. It is because we are convinced of the validity of our family allocations for these out-of-plane two meridian modes that these interpolation curves are shown in solid lines in Figure 10(b) unlike their counterparts in Figure 10(a). There is little doubt that this difference in status is connected with the fact that extensional ring modes with two meridians do occur, whereas inextensional ones do not. For interest the in-plane PAFEC diagram for the α mode with two meridians is shown in Figure 11(b).

Clearly there can be no RIR mode with two meridians because classical ring theory predicts zero frequency for inextensional radial modes with $2m = 0$ or 2. Indeed the interpolation curve for the RIR family extrapolates smoothly down to zero frequency

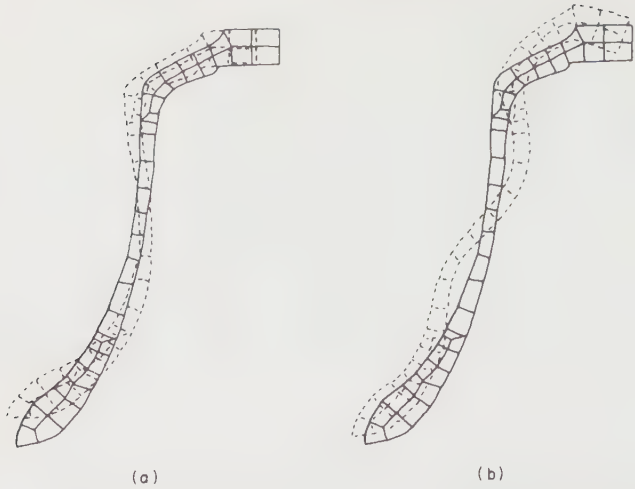


Figure 11. (a) The two-meridian shell driven mode with $R = 1$; (b) the two-meridian mode α.

at $2\,m = 2$ as the classical theory implies that it should. The occurrence of an RA mode with two meridians is, however, a surprise since, on the basis of classical ring theory, no such mode should occur. This is one reason why it may be more meaningful physically to regard the entire family of RA modes as torsional ring driven since a two-meridian member would then be expected.

Modes with zero meridians differ from all others in that they are not members of degenerate doublets. A consequence of this is that such modes must be either purely in-plane (e.g., "breathing" modes) or purely out-of-plane (i.e., torsional modes twisting about the bell's axis of symmetry). Also, as with the two-meridian cases, there is no necessity for a node at the top of the crown in the purely in-plane cases. The torsional modes form a sequence with 1, 2, 3, . . . nodal circles for tangential motion which is designated as $T = 1, 2, 3, \ldots$ in Table 2: such modes were in fact very difficult to excite and only the first two were not "missing". There are, as expected, neither RIR nor RA modes with $2\,m = 0$.

From amongst the purely in-plane modes it is straightforward to select a "shell driven" set having 2, 3, 4, . . . nodal circles and no node at the top of the crown which, in exactly the same way as for $2\,m = 2$, one can designate as $R = 1, 2, 3, \ldots$. The PAFEC diagram for the $R = 5$ case is shown in Figure 12. Like their two-meridian counterparts these "shell driven" zero-meridian modes do not fit smoothly onto the l_R *vs.* $2\,m$ at fixed R curves and give a l_R *vs.* R at fixed $2\,m$ pattern which is satisfactory apart from an extra curve in the above shoulder region. The log f *vs.* R curve is a good *single* straight line as shown in Figure 8(a): there is no crown "take-off" effect here because the crown participates fully from the start of the sequence. In Figure 10(a) the family interpolation curves can only be continued down to zero meridians in a similarly tenuous way to the two meridian cases, and for the same reasons. Nevertheless the fact that the right number of modes occur in something like reasonable places is in itself fairly impressive in view of the undoubted fundamental differences between $2\,m = 0$, 2 modes and the cases with $2\,m > 2$.

The mechanisms responsible for producing the remaining purely in-plane zero-meridian modes are complicated but seem to be due to various geometrical ring sections of the bell going into their individual breathing modes and driving the rest of the bell

Figure 12. The zero meridian shell driven mode with $R = 5$.

in much the same way as the α, β, γ, . . . families are produced for higher values of $2m$ by the extensional radial modes of such ring sections. It is perhaps therefore not too surprising to find that when these modes, plus the torsional modes and any "missing" ones, are plotted on Figure 10(b) they supply the right number in more or less the right places to fit onto extrapolations of the α, β, γ, . . . family curves. We have therefore allocated the corresponding modes to these families as listed in Table 2 (with the somewhat tenuous nature of these allocations clearly indicated). However since these zero meridian modes, like their slightly less difficult two-meridian counterparts, are usually considered to be of little importance to the musical quality of the bell we shall not pursue the matter further in this article.

9. DISCUSSION

Of the five important musical partials (i.e., those specifically tuned by English founders) three are of the RIR type (the Hum, Tierce and Nominal) and the other two are of the $R = 1$ variety. Now that an understanding of the mechanisms responsible for these two families of partials has been achieved, the truly crucial role of the thick rim ring in the production of the church bell's characteristic timbre has become clear: the RIR modes are directly driven by this ring while the $R = 1$ modes, although shell driven, have their effective boundary conditions and thus their frequencies massively influenced by it. The very obvious difference in timbre between church bells and handbells is almost certainly due to the fact that no such ring is present in the latter case, rather than to any subtle differences between the remainder of their profiles (after suitable scaling). It has sometimes been suggested that the thick rim ring, being located at the clapper's strike point, serves the sole purpose of strengthening the bell there in order to avoid cracking, especially during change ringing. While the ring certainly does achieve this purpose and this could perhaps have been the original reason for its introduction there can now be little doubt that, once tested, it was retained as much for its remarkable effect upon the timbre as for any reasons of mechanical strengthening. Clearly the rim ring is the optimum clapper strike point if one wishes to maximize the relative contributions of the various RIR modes.

It is interesting that some German founders claim to tune individually not only the five musical partials of the English founders but also the musical 17th, 18th and 19th (the Superquint) relative to the Hum. The required theoretical ratios for these to be well tempered are 4·756, 5·038, 5·340 and 5·993, respectively, for the Minor 17th, Major 17th, 18th and 19th. These are to be compared with the experimental ratios for our Taylor bell of 4·762, 5·032, 5·327 and 6·016, respectively. Once the five "musical" partials have been tuned then the higher ones come out correctly "in the wash" for the Taylor bell. This implies that either the claimed teutonic thoroughness is unnecessary or else the basic shape of their bells is inferior.

The phenomenon of crown "take-off" reported in section 7 is not only extremely interesting in itself but is likely to be of importance in systems other than bells. A theoretical explanation of the empirical result that partials with $R \geqslant m$ exhibit crown participation will be reported in a subsequent paper. It seems likely that a similar effect could occur whenever the system under study contains relatively stiff and physically readily identifiable sub-systems. Within the field of percussion instruments the cymbal seems a likely candidate because of its dome-like central region. Experimental data on cymbals which seems to support this idea has been reported by Rossing [9] who, when plotting $\log f$ *vs.* $\log m$ for $n = 0$, found that his data fell on two intersecting straight lines. We would expect these to correspond to the participation and non-participation, respectively, of the central dome in the overall motion of the system. A detailed investigation of this is proceeding.

ACKNOWLEDGMENTS

The authors are indebted to Messrs John Taylor and Company, Bell Founders of Loughborough, for the loan of the bell used in this study and for access to their archives and technical expertise. In the latter respect special mention must be made of Mr M. J. Milsom and of the late Mr P. L. Taylor. Part of the work described here was undertaken while one of us (R.P.) was working as an N.R.A.C. Senior Research Fellow with the Auckland Industrial Development Division of the D.S.I.R. in New Zealand. The financial support of the N.R.A.C. during this period is gratefully acknowledged, as is the hospitality of Mr W. R. Beasley, the Director of A.I.D.D., and his staff.

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Part III

TUNING OF BELLS

Editor's Comments on Papers 9, 10, and 11

9A VAN HEUVEN
Chapter III. The Tuning of Carillon Bells Incorporated in the Measurements

9B VAN HEUVEN
Chapter IV. The Process of Tuning

9C VAN HEUVEN
Excerpt from *Chapter V. The Decay of Sound*

10 LEHR
The System of the Hemony-Carillons Tuning

11 LEHR
Contemporary Dutch Bell-Founding Art

The most extensive work written on bell acoustics is the dissertation of E. W. van Heuven entitled *Acoustical Measurements on Church Bells and Carillons.* This dissertation reports, among other things, the results obtained from the author's extensive studies on bell acoustics for the Department of Preservation of Arts and Monuments in The Netherlands following World War II. Excerpts from this dissertation appear as Paper 9.

Bells have traditionally been pillaged by advancing armies in time of war in order to be used in the manufacture of cannon and other weapons of destruction. At no time in history was this destruction more extensive than during World War II. An extensive report on the condition of the carillons of Europe made by Price (*Campanology, Europe 1945,* University of Michigan Press, 1948). Approximately 70% of the bells in Holland were removed from their towers, and after the war less than half of them were recovered intact. This sad situation presented a unique opportunity to make measurements on many historic bells, however, that would not have been possible with bells hanging in their tower. The results reported in Paper 9 include discussions of modes of vibration, tunings of some important Dutch carillons, the process of tuning, decay of the sound, and the relative

sound intensities of the partials.

Papers 10 and 11 are by André Lehr, curator of the Dutch National Carillon Museum and a well-known authority on bells. Paper 10 discusses the tuning of three seventeenth century carillons by the brothers Pieter and Francois Hemony. The author concludes that the Hemony brothers employed meantone tuning which is tonally superior to equal temperament and yet permits playing in different keys.

Paper 11, translated from Dutch, features a discussion of bell tuning that complements that of van Heuven in Paper 9.

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9A

Reprinted from pages 54–73 of *Acoustical Measurements on Church Bells and Carillons*, DeGebroeders van Cleef, The Hague, Netherlands, 1949, 148p.

THE TUNING OF CARILLON BELLS INCORPORATED IN THE MEASUREMENTS

E. W. Van Heuven

§ 20. The Intonation of Carillon Bells

Among the bells of which the sound was documentated, were those of which the carillons of Zierikzee (Town Hall), Den Helder (Monument), Den Briel (Catharina Church), Goes (Maria Magdalena Church), Tholen (Town Hall) and St. Maartensdijk (Dutch Reformed Church) are composed. In addition a series of measurements were made on the carillon of the Main Church at Gouda.

Of the carillons at Tholen and St. Maartensdijk only a few bells have been saved; of the bells of the other carillons none, or only a few have been lost at the sequestration.

If bells of various sizes are used in combination, the latter will either be called a *Chime* or a *Carillon*, depending upon the number of bells in it and on the musical notes of them. The following definitions are given by the U.S.A. Carillon Guild at its sixth congress, Princeton 1946[1]) and by Prof. P. Price of the University at Michigan (by personal letter):

A *Chime* is a combination of bells playing a single melody. The range is 10 bells approximately. The bells are meant to be struck successively. There may be a keyboard for manual playing. (The Dutch word is "voorslag").

A *Carillon* is an instrument comprising at least two octaves in chromatic series, of which the lower two semitones may be missing (23 bells). If more bells are sounded together, the resulting sound shall be concordant. It is normally played from a keyboard, where the expression and volume of the strokes on the bells can be controlled through variations of the touch of the keys.

In accordance with these definitions, before the second world war, in the Netherlands there were 66 carillons, comprising from 23 to 49 bells; in addition to these there were 7 incomplete "carillons" with a diatonic series of between 16 and 23 bells which should bear

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[1]) Cited by Bigelow, A. L.: Carillon. Princetown Univ. Press 1948.

the name of "collossal" chimes, and 6 normal size chimes of at least 4 bells.

In the composition of a carillon special attention should be paid to the partial frequencies of each bell to be used. Not only the internal tuning of each bell needs to be considered, but also the chromatic scales of the corresponding partials of different bells (external tuning).

As to the internal tuning of bells the ideal pattern that is pursued by modern bell casters has already been stated in § 2. In normal swinging bells the internal tuning must be right. For carillon bells on the contrary, the external tuning is of primary importance. It is even possible that a carillon consists of bells that all show the same discrepancies from the ideal values in the internal tuning, but yet is very well acceptable from the musical aspect. All musical scales of the corresponding partials are identical in this case so that the melody that is played can easily be recognised.

In § 13 a historical survey is given of the various musical scales that have existed or are still in use. In this connection it must be noted that the scale of equal temperament has especially been sponsored by the use of instruments with a fixed tuning. A carillon is a special example of such an instrument; in fact it is almost the only instrument that keeps its original tuning through centuries, provided that no excessive wear or corrosion takes place.

On the other hand a carillon is never used together with other instruments and the music played on a carillon is mostly of a simple character. The use of complicated chords is impossible due to physical limitations in the handling of a carillon keyboard. Further, many good carillons exist dating from the time before the scale of equal temperament was used. There are therefore no intrinsic reasons why carillons should have a scale of equal temperament. Even so the latter is always preferred in modern carillons, for the following reasons:

a) The cost of a carillon is largely dependent on the size and pitch of the largest bell (comp. fig. 15). The older carillons nearly always started with an arbitrary note, labelled as a C-note in the carillon scale. The keyboard normally starts with a C-key, so that often the whole instrument is transposing. If later new bells are added to the bass side of the carillon, usually the corresponding number of keys are added to the treble side of the keyboard, so that the rate of transposition is altered and the whole scheme of the just tempered scale will be spoilt. In a scale of equal temperament this addition can always take place without objection.

b) The modern mechanical improvements of the transmission system between the keybord and the clappers make the playing less strenuous, so that there is nowadays a tendency to use the carillon for less simple compositions. In this case the same difficulties arise as have been found with other fixed-tuned instruments when music had to be played in different keys. The solution is the use of a scale of equal temperament.

c) The scale of equal temperament is used in all modern music and is therefore the scale to which people are most accustomed. In table V it is seen that the intervals between some notes in a just tempered scale, and the same in a scale of equal temperament of the same average pitch, exceed the minimum audible pitch interval. Therefore an observer with a musically trained ear will be able to distinguish between both scales and in the case of a just tempered scale being used, will notice some discrepances from the musical scale he is accustomed to.

§ 21. Tolerances in the Musical Scales

Except for the manufacturing of new carillons there are other occasions where the consideration of the external tuning of a carillon is of importance, such as the filling up of empty places or renewing of bad bells in existing carillons. It will be shown in chapter IV that it is very difficult, or even impossible to tune any arbitrary rough bell so that the internal tuning attains the ideal values exactly at a predicted absolute pitch. Therefore the tuning of a bell will always be a compromise between internal- and external demands and certain deviations between the actual partial frequencies and the calculated values must be tolerated. The problem therefore arises what tolerances can be permitted within the general condition that the music produced on the instrument shall be concordant.

In § 12 it has been shown that under ideal circumstances frequency differences of 0,04 semitone can be detected by the ear. The average error of the electro-acoustical measurements is 0,03 semitones (§ 16). If a bell is to be judged by the results of the partial frequency measurements these tolerances have to be added; while the tuning of a bell must be done with the tolerances subtracted. In practice the accuracy with which bell casters claim the tuning of their bells is done is 1/16 semitone (0,06—0,07 semitone) which seems to be satisfactory. We will arbitrarily judge the measured carillons on a basis of the added tolerance of 0,07 semitone. In modern carillons where the results of the partial frequency measurements show with certainty a scale of equal temperament, deviations

of 0,07 semitone from the average scale of the carillon would therefore be tolerable.

Older carillons generally do not show a definite musical scale. If new bells are to be added to these carillons, the tolerance problem must be taken on a wider basis, as at the time these carillons were made the scale of equal temperament had not yet come into general use. If the bell-casters of that time had been able to tune their carillons with the same accuracy as that with which modern carillons can be intonated the following possibilities could be expected:

a) The carillon is tuned to a non-transposing scale of just temperament, with a choice made for the undefined semitones.

b) The just tempered scale of the carillon is transposing; the largest bell is labelled as C in the scale.

These possibilities can readily be recognised if the measurements of the partial frequencies are worked out according to the same method as has been done in § 13 for an exact just tempered scale. At first the musical scale is considered as equally tempered, with deviations. The average equally tempered scale is calculated and the deviations from this scale are considered. The deviations between an exact just tempered scale and its average scale of equal temperament are given in fig. 6. A hypothetical carillon according to the mentioned possibilities a) or b) will show the same pattern of deviations and the periodicity of the latter can decide wether a) or b) applies, and what the rate of transposition of the chromatic sequence is.

It can be remarked, that a pattern of deviations according to a scale of just temperament must be considered right from the musical aspect; at least for old carillons. Supposing that a carillon can be tuned to either scale with the same tolerances, a second series of tolerance values with respect to the average equally tempered scale are found that may be applied in two different ways, if the range of the carillon does not start with a C-note at the lower end.

For the notes E♭, F♯ and G♯ the tolerance ranges of the just- and the equal temperament scales do not overlap. In the general case where the carillon does not show the specific qualities of any of these scales, the tolerance ranges of the remaining notes may be joined. In this case the three mentioned semitones need a special consideration.

In § 13 it has been stated that if two tones are heard, having an interval that is gradually increased from a minor — to a major third the chord will be concordant for any value of the interval. There-

fore there are no musical objections against joining the tolerance ranges for just- and equally tempered D♯ (or E♭) notes.

The F♯ and G♭ notes in the just tempered scale show the largest discrepancy. It is entirely a matter of opinion which of the two should be taken for a carillon with just intonation. Probably the F♯ will be preferred in connection with the general major character of the music to be played on a carillon. In any case it is doubtful whether a bell should be disqualified if its note lies in between the two tolerance ranges.

The tolerance ranges of the G♯ note are only 0,02 semitone apart. In view of the roughness of the assumption of 0,07 semitone as tolerance value from the exact pitch, the interval between both tolerance ranges for G♯ can be neglected.

The final proposed tolerance limits for the general case of a carillon that does not show the specific pattern of any of the two musical scales of equal- or just temperament are given in table X.

TABLE X

Proposed tolerances in respect to the average equally - tempered scale of the carillon

Note	Tolerances (semitones)	
C	+0,11	—0,07
C♯	+0,07	—0,10
D	+0,15	—0,07
D♯	+0,27	—0,07
E	+0,07	—0,17
F	+0,09	—0,07
F♯ [1])	+0,07	—0,07
F♯ [2])	—0,20	—0,34
G	+0,12	—0,07
G♯	+0,26	—0,07
A	+0,07	—0,18
A♯	+0,07	—0,07
B	+0,07	—0,14

The results of the measured scales of the carillons will be compared hereafter with these suggested tolerances.

[1]) Range for equally-tempered scale.
[2]) Range for just-tempered scale.

§ 22. The Zierikzee Carillon

The carillon at Zierikzee (Town Hall) was cast in 1927 by Messrs. John Taylor and Co. at Loughborough, England. It is a typical example of a carillon that is apparently tuned in the equally tempered scale. The musical notation of the measured partial frequencies is given in table XI, while fig. 20 shows the Hum-Notes and the

TABLE XI

Carillon at Zierikzee Town Hall

(Numbering 10/8K. . .)

nr.	M (kg)	R_0 (m)	Hum-Note	Fundam.	Minor Th.	Fifth	Nominal
23	270	0,39	C 3 —0,27	C 4 —0,18	E♭4 —0,12	G 4 —0,17	C 5 —0,21
22	195	—	D 3 —0,22	D 4 —0,21	F 4 —0,09	A 4 —0,17	—
21	145	0,31	E 3 —0,16	E 4 —0,20	G 4 —0,07	B 4 —0,13	E 5 —0,19
20	123	0,28	F 3 —0,21	F 4 —0,19	A♭4 —0,06	C 5 —0,15	F 5 —0,24
19	121	0,28	F♯3 —0,16	F♯4 —0,20	A 4 +0,04	—	F♯5 —0,36
18	113	0,27	G 3 —0,18	G 4 —0,17	B♭4 —0,04	—	G 5 —0,02
17	95	—	G♯3 —0,26	G♯4 —0,22	B 4 —0,14	—	—
16	85	0,23	A 3 —0,14	A 4 —0,16	C 5 —0,07	—	A 5 —0,21
15	72	0,23	A♯3 —0,19	A♯4 —0,21	D♭5 —0,20	—	A♯5 —0,53
14	60	0,21	B 3 —0,19	B 4 —0,15	D 5 —0,20	—	B 5 —0,08
13	50	0,21	C 4 —0,17	C 5 —0,18	E♭5 —0,30	—	C 6 —0,17
12	39	0,20	C♯4 —0,15	C♯5 —0,10	E 5 —0,04	—	C♯6 —0,03
11	33	0,18	D 4 —0,17	D 5 —0,17	F 5 —0,55	—	D 6 —0,14
10	26	0,17	D♯4 —0,19	D♯5 —0,16	G♭5 —0,15	—	—
9	24	0,16	E 4 —0,17	E 5 —0,19	G 5 —0,04	—	E 6 —0,14
8	22	0,16	F 4 —0,19	F 5 —0,13	A♭5 —0,19	—	F 6 —0,15
7	19	0,15	F♯4 —0,18	F♯5 —0,13	A 5 —0,48	—	—
6	17	0,14	G 4 —0,18	G 5 —0,18	B♭5 —0,09	—	—
5	15	0,13	G♯4 —0,18	G♯5 —0,13	B 5 —0,13	—	—
4	13	0,13	A 4 —0,21	A 5 —0,12	C 6 +0,10	—	—
3	11	0,12	A♯4 —0,19	A♯5 —0,17	—	—	—
2	10	0,12	B 4 —0,19	B 5 —0,26	—	—	—
1	(lost)						

Fundamentals of the chromatic series in comparison with the suggested tolerances. The measurements were made according to the acoustical method mentioned in § 16.

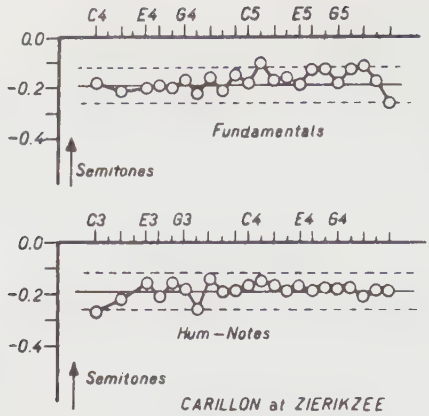

Fig. 20. The carillon at Zierikzee.
On the horizontal scale the notes are indicated; on the vertical scale the deviations from an equally tempered scale of international pitch A 3 = 440 c/s.

The average scale of equal temperament of the carillon is 0,19 semitone below that based on A 3 = 440 c/s. This corresponds to the older basis of international pitch, A'3 = 435 c/s that was adopted in 1891, and has been superseded in 1925 by the former.

According to the numbering of the bells by the Department for Preservation of Arts and Monuments, bell nr. 1, the top note C 5, has been lost; so the original range of the carillon has been two octaves C 3 to C 5 inclusive, with the lower two semitones missing, and not transposing.

For this equally tempered carillon the tolerances are as drawn in fig. 20. The only bell of which the Hum-Note frequency does not fulfill these tolerance conditions is the largest bass note nr. 23. It is further seen in fig. 20 that for the lower bass notes most attention is paid to the Fundamental frequencies to be as near as possible to the exact average scale of equal temperament, while for the treble notes this is done for the Hum-Note frequencies. This is well in accordance with the results to be mentioned in chapter VI, where it will be seen that during the normally occurring time intervals between the notes of a melody the pitch of the bass bells, having a longer reverberation time, is characterised primarily by the Fundamental frequency, and secondarily by the Hum-Note frequency, while for the top notes the Hum-Note frequencies are more important. It may also partly have been caused by the fact that the tuning fork method for measuring partial frequencies does not provide means for measuring frequencies above 1100 c/s, so that the internal tuning of small bells must be judged by the ear.

The internal tuning of the bells is mainly correct, apart from a notable discrepancy of the Nominal of bell nr. 15, and one of the Minor Third of bell nr. 11.

§ 23. The Carillon at Den Helder

The carillon of the Den Helder monument was cast in 1935 by Van Bergen at Heiligerlee. It is also a modern carillon with equal temperament scale. Table XII shows the measurement results, while fig. 21 gives the musical notation of the Hum-Notes and the

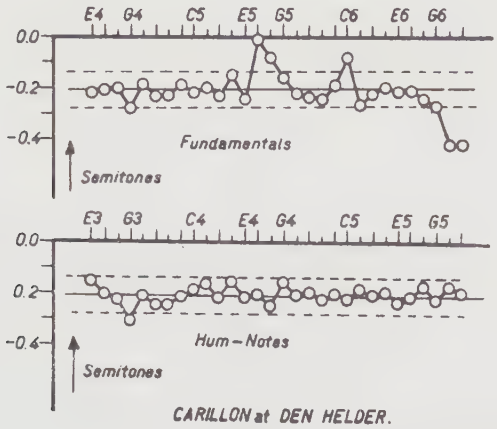

Fig. 21. The carillon at Den Helder.
Legend as for fig. 20.

Fundamentals of the chromatic series in comparison with the suggested tolerances. Like the carillon at Zierikzee the average scale is 0,21 semitone below the A 3 = 440 c/s basis, indicating the older basis A'3 = 435 c/s. The frequency measurements were made according to both methods indicated in chapter II. For the high partials of the small bells the excitation method proved to be more satisfactory.

The Hum-Note frequency of bell nr. 27 lies outside the tolerance range. Of the Fundamentals those of bells nrs. 27, 17, 15, 10, 1 and 9 show deviations larger than the suggested tolerances. In general the internal tuning of the bells of this carillon is more irregular than those of the Zierikzee instrument. The Nominals especially show the general tendency to be too sharp. It must be remarked that the carillon showed a serious degree of corrosion, so that the original tuning may have been different from that which has been found now.

TABLE XII

Carillon at Den Helder Monument

(Numbering 7/1K. . .)

nr.	M (kg)	R_o (m)	Hum-Note	Fundam.	Minor Th.	Fifth	Nominal
30	130	0,30	E 3 —0,16	E 4 —0,22	G 4 —0,26	B 4 —0,19	E 5 —0,22
29	109	0,28	F 3 —0,21	F 4 —0,21	A♭4 —0,21	C 5 —0,33	F 5 —0,21
28	87,5	0,265	F♯3 —0,23	F♯4 —0,20	A 4 —0,09	—	F♯5 —0,24
27	80	0,26	G 3 —0,31	G 4 —0,28	B♭4 +0,05	—	F♯5 +0,39
26	61	0,23	G♯3 —0,22	G♯4 —0,19	B 4 —0,12	—	G♯5 —0,12
25	58	0,225	A 3 —0,25	A 4 —0,23	C 5 +0,04	—	—
24	51	0,215	A♯3 —0,25	A♯4 —0,23	D♭5 +0,09	G 5 +0,07	—
22	39,5	0,20	B 3 —0,22	B 4 —0,19	D 5 +0,00	—	B 5 +0,16
23	41	0,19	C 4 —0,19	C 5 —0,22	E♭5 —0,40	A 5 —0,30	—
21	35	0,18	C♯4 —0,17	C♯5 —0,20	E 5 +0,22	—	C♯6 +0,43
20	29	0,175	D 4 —0,22	D 5 —0,23	F 5 —0,02	—	D♯6 +0,47
19	26	0,165	D♯4 —0,16	D♯5 —0,15	G♭5 +0,22	A♯5 —0,12	—
18	24	0,158	E 4 —0,22	E 5 —0,24	G 5 +0,25	B 5 —0,03	E 6 +0,57
17	16,5	0,145	F 4 —0,21	F 5 —0,01	A♭5 +0,47	B 5 +0,29	F♯6 —0,43
15	15,5	0,14	F♯4 —0,25	F♯5 —0,08	A 5 +0,35	—	—
16	16	0,14	G 4 —0,16	G 5 —0,16	B 5 —0,13	D 6 —0,27	A 6 —0,30
14	14,5	0,13	G♯4 —0,21	G♯5 —0,22	B 5 —0,03	D♯6 —0,23	G♯6 +0,01
11	10,5	0,123	A 4 —0,20	A 5 —0,23	D♭6 —0,26	E 6 —0,28	A 6 +1,17
13	12	0,12	A♯4 —0,23	A♯5 —0,24	D 6 —0,33	E 6 +0,41	D 7 +0,01
12	11	0,115	B 4 —0,21	B 5 —0,19	D 6 +0,43	F 6 +0,10	C 7 —0,40
10	9,5	0,11	C 5 —0,23	C 6 —0,08	E♭6 +0,09	G 6 —0,32	C 7 +0,3
8	8	0,108	C♯5 —0,19	C♯6 —0,26	E 6 +0,22	G 6 +0,40	C♯7 +0,50
4	7,5	0,10	D 5 —0,21	D 6 —0,22	G♭6 —0,05	—	E 7 —0,48
5	7,5	0,10	D♯5 —0,20	D♯6 —0,20	G♭6 —0,45	—	—
7	7,5	0,10	E 5 —0,24	E 6 —0,21	G 6 —0,43	C 7 —0,29	—
2	7	0,095	F 5 —0,22	F 6 —0,21	A♭6 —0,21	C♯7 +0,12	A♯7 —0,03
6	6,5	0,09	F♯5 —0,18	F♯6 —0,24	A♭6 +0,40	D♯7 +0,34	—
3	7	0,09	G 5 —0,23	G 6 —0,27	B♭6 —0,05	—	A 7 +0,25
1	6,5	0,09	G♯5 —0,18	G♯6 —0,42	B 5 +0,10	E 7 —0,35	G♯7 +0,13
9	8	0,108	A 5 —0,20	A 6 —0,42	—	—	G♯7 +0,20

§ 24. The Carillon at Gouda

The carillon at Gouda (Main Church) was originally made by P. Hemony in 1675—'76. Seven bells were replaced in 1930 by

John Taylor and Co. In the same tower four large swinging bells are hanging, made by H. Wegewart in 1605. The two largest swinging bells are already coupled to the carillon keyboard. The purpose of the measurements on this carillon was to decide whether the two smaller swinging bells, that had been removed during the war, could be coupled to the carillon at the time of the replacement in the tower.

The measurements on this carillon were made in the tower. As the bells were difficult of access the excitation method could not be used. A microphone was placed as near as possible to the bell that was tested and the acoustical method of measurement was used. As the background level of the disturbances by the wind and the sounds in the streets were rather high no attempts were made to measure the high partials of the smaller bells. The bells were struck by means of the existing clappers.

Table XIII gives the measurement results of the carillon and of the swinging bells. The average scale of the carillon bells is 0,37 semitone below the international pitch, corresponding to A'3 = 431 c/s. Fig. 22 shows the musical notation of the Hum-Notes in comparison with the suggested tolerances; this time those valid for the general case (table X).

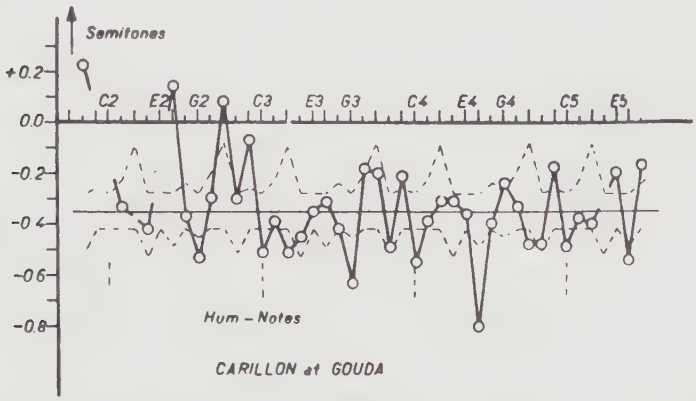

Fig. 22. The carillon at Gouda.
Legend as for fig. 20.

It is seen that the carillon does not show the specific pattern of just intonation. The scale of the carillon is transposing (see § 25) and so the tolerances of table X are drawn correspondingly in fig. 22.

Many deviations exceed the suggested tolerances. This carillon is not considered as one of the best Hemony carillons in the Netherlands.

TABLE XIII

Carillon and swinging bells at Gouda

A) Carillon bells by P. Hemony, 1675—1676

nr.	Hum - Note	Fundam.	Minor Th.	Fifth	Nominal
7	F♯2 —0,37	F♯3 —0,14	A 3 —0,16	C♯4 +0,08	F♯4 —0,26
4	G 2 —0,53	G 3 —0,49	B♭3 +0,47	D 4 —0,43	A♯4 +0,49
2	G♯2 —0,30	G♯3 —0,31	C 4 +0,03	D♯4 —0,3	—
1	A 2 +0,08	A 3 —0,16	C 4 +0,16	E 4 —0,19	A 4 —0,14
3	A♯2 —0,30	A♯3 —0,34	D♭4 —0,26	—	A♯4 —0,32
12	B 2 —0,07	B 3 —0,21	D 4 —0,01	—	B 4 —0,21
8	C 3 —0,51	C 4 —0,47	E♭4 —0,26	F 4 +0,48	C 5 —0,44
5	C♯3 —0,39	C♯4 —0,26	E 4 —0,19	G♯4 —0,31	C♯5 —0,28
6	D 3 —0,51	D 4 —0,52	F 4 —0,41	—	D 5 —0,46
10	D♯3 —0,45	D♯4 —0,26	G♭4 —0,21	—	D♯5 —0,37
19	E 3 —0,35	E 4 —0,19	G 4 —0,06	—	—
11	F 3 —0,31	F 4 —0,39	A♭4 —0,22	—	F 5 —0,40
9	F♯3 —0,42	F♯4 —0,16	A 4 —0,12	—	F♯5 —0,31
16	G 3 —0,63	G 4 —0,46	B♭4 —0,36	—	—
15	G♯3 —0,18	G♯4 —0,39	B 4 —0,22	—	—
18	A 3 —0,20	A 4 —0,22	C 5 —0,07	—	—
17	A♯3 —0,49	A♯4 —0,42	—	—	—
13	B 3 —0,21	B 4 —0,05	D 5 —0,13	—	—
14	C 4 —0,55	C 5 —0,36	E♭5 —0,30	—	—
27	C♯4 —0,39	C♯5 —0,29	E 5 —0,20	—	—

B) Swinging bells by H. Wegewart 1605

nr.	M (kg)	R_o (m)	Hum-Note	Fundam.	Minor Th.	Fifth	Nominal
—	—	0,86	A♯1 +0,22	B 2 —0,03	D 3 —0,28	F♯3 —0,48	B 3 —0,42
—	—	0,75	C♯2 —0,33	C♯3 +1,08	E 3 +0,38	G♯3 +0,07	C♯4 +0,29
11/23A	1117	0,62	D♯2 —0,40	D♯3 +0,91	G♭3 +0,32	A♯3 —0,23	D♯4 —0,01
11/22A	1092	0,59	F 2 +0,14	F♯3 +0,60	A 3 +0,04	C♯4 +0,17	F♯4 +0,04

TABLE XIII (Continued)

C) Smaller carillon bells

nr.	Founder	Hum - Note	nr.	Founder	Hum - Note
28	Taylor	D 4 —0,31	30	P. Hemony	A♯4 —0,48
31	Taylor	D♯4 —0,31	23	P. Hemony	B 4 —0,18
32	Taylor	E 4 —0,36	22	P. Hemony	C 5 —0,49
33	P. Hemony	F 4 —0,80	21	P. Hemony	C♯5 —0,38
26	Taylor	F♯4 —0,40	20	P. Hemony	D 5 —0,40
25	Taylor	G 4 —0,24	34	P. Hemony	E 5 —0,20
24	Taylor	G♯4 —0,33	35	P. Hemony	F 5 —0,54
29	Taylor	A 4 —0,48	36	P. Hemony	F♯5 —0,17

The internal tuning of the Hemony-bells is generally good. The largest deviation of the interval between Fundamental and Hum-Note is 0,26 semitone (bell nr. 9).

As to the coupling of the Wegewart bass bells to the carillon it can be remarked that, although the Hum-Notes of the two bells of which $R_0 = 0{,}75$ and 0,62 m respectively lie within the tolerances of the carillon scale, the interval between Hum-Note and Fundamental for the Wegewart bells is on the average 1,5 semitone more than an octave. Especially the bass bells should be arranged according to the Fundamental frequencies (comp. § 22). Thus false combinations will occur because of the long reverberation times of the Hum-Notes of the bass bells. It is therefore not advisable in this case to couple any of the Wegewart bells to the carillon.

The analogy between the partial frequencies of bell nr. 11A22 and the carillon bell nr. 7 leads to the replacement of the latter by the first mentioned bell for the half-hour stroke, which is now given on a Hemony carillon bell, thus causing unnecessary wear.

§ 25. The carillon at Den Briel.

The carillon at Den Briel (Catharina Church) is a small carillon made by F. Hemony in 1660. The results of the measurements are given in table XIV and in fig. 23. According to the numbering six bells out of this carillon have been lost during the war.

The two largest bells are by C. Crans Jzn. (nr. 25) and by P. Bakker, 1775 (nr. 24) respectively, whilst nr. 16 is by Van Bergen, 1898. These do not fit very well into the scale of the carillon bells and especially the internal tuning of nr. 16 is totally different from

TABLE XIV

Carillon at Den Briel (Catharina Church)

(Numbering 11/1K. . .)

A) Bells by F. Hemony, 1660

nr.	M (kg)	R_o (m)	Hum-Note	Fundam.	Minor Th.	Fifth	Nominal
2	8	0,11	C 5 —0,49	C 6 —0,55	—	—	—
4	11	0,12	A♯4 —0,61	A♯5 —0,50	D♭6 +0,28	—	—
6	15	0,135	G♯4 —0,37	G♯5 —0,39	—	—	—
7	17	0,145	G 4 —0,46	G 5 —0,48	B♭5 —0,33	—	—
8	20	0,15	F♯4 —0,28	F♯5 —0,27	—	—	—
10	26	0,17	E 4 —0,13	E 5 —0,11	—	—	—
11	31	0,18	D♯4 —0,46	D♯5 —0,34	G♭5 —0,24	—	—
12	38	0,19	D 4 —0,49	D 5 —0,41	—	—	—
14	48	0,21	C 4 —0,37	C 5 —0,39	E♭5 —0,27	—	—
17	75	0,245	A 3 —0,56	A 4 —0,56	C 5 —0,41	—	A 5 —0,50
18	102	0,27	G 3 —0,44	G 4 —0,4	B♭4 —0,30	—	—
20	119	0,28	F♯3 —0,23	F♯4 —0,19	A 4 —0,10	C♯5 —0,27	F♯5 —0,20
21	139	0,30	F 3 —0,36	F 4 —0,36	A♭4 —0,25	C 5 —0,36	F 5 —0,40
22	—	0,33	D♯3 —0,22	D♯4 —0,24	G♭4 —0,15	A♯4 —0,40	D♯5 —0,34
23	246	0,365	C♯3 —0,26	C♯4 —0,21	E 4 —0,09	G♯4 —0,24	C♯5 —0,27
			B) Bell by Van Bergen 1898				
16	59	0,23	G♯3 —0,05	A 4 +0,28	C 5 +0,11	—	—
			C. Bell by P. Bakker 1775				
24	363	0,415	B 2 +0,35	B 3 +0,21	E♭4 +0,05	F♯4 +0,21	—
			D) Bell by C. Crans Jzn 1744				
25	843	—	F 2 —0,56	F 3 —0,24	A♭3 —0,35	C♯4 +0,33	G 4 +0,09

that of the Hemony bells. It is therefore advisable to replace this bell if the carillon is to be restored.

The average equally tempered scale of the Hemony bells of this carillon is 0,38 semitone below the international pitch, which is in good agreement with the average scale of the carillon at Gouda. This could be expected as the Hemony's possessed a set of vibrating rods,

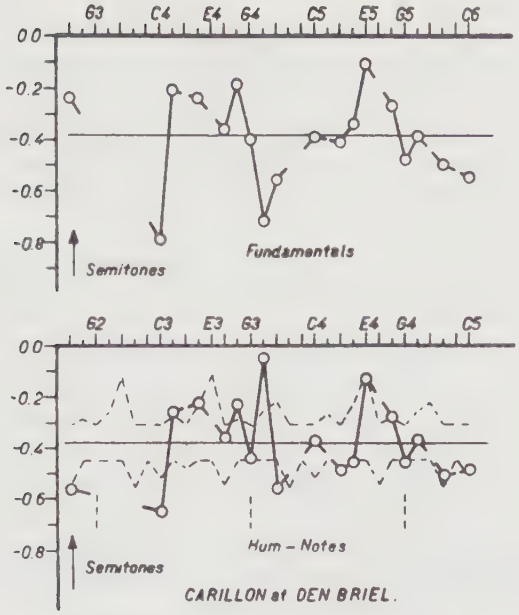

Fig. 23. The carillon at Den Briel.
Legend as for fig. 20.

which they used as a standard scale for tuning their bells[1]). The basis of this set must therefore have been A'3 = A 3 — 0,37 = 431 c/s.

Taking into account that the numbering of the bells has been done according to their fortuitous placing in the tower, it is seen that the original range of the carillon has been from C♯3 —0,26 up to C♯5 —0,49, while for the D 3 and E 3 notes no places can be found in the numbering. Therefore this carillon is also transposing and the tolerances of table X have been used correspondingly in fig. 23.

It is known that the Hemony's preferred to avoid the two lower semitones due to the high costs of these bells and the little use that is made of them. In this connection a correspondence has been preserved, concerning the carillon at Gouda, between P. Hemony and Mr. Quiryn Van Blankenburgh, organist at The Hague. Hemony protested against the making of "Cis" (C♯) and "Dis" (D♯) for the carillon at Gouda. This correspondence had lead to a paper by P. Hemony, about the general unnecessity of the two lower semitones

[1]) The life and work of the two brothers Hemony is described, amongst others, by: Bijtelaar, B., "De Zingende Torens van Amsterdam", 1947.

of a carillon[1]). His mentionning of "Cis" and "Dis" clearly indicates that at that time all carillons were considered transposing, with the largest bell as C-basis.

If, indeed, the Hemony's used a set of vibrating rods to tune their bells, they could not make pure just intonated scales on a transposing basis, as the frequencies of the notes depend upon the basis if a just intonated scale is used.

It is also remarkable that the carillons at Gouda and Den Briel both start with an augmented note. Therefore it could even be possible that the range of the carillon at Den Briel is meant to be C 3 to C 5, while that at Gouda is meant to be F 2 to F 5. In this case the pitch of the musical scale at the time of the Hemony's would have been a semitone different from that used nowadays. A tone of 431 c/s would have been known to the Hemony's as G♯3. This is in contradiction to the known tendency of later years to increase the standard pitch.

The deviations of the Hemony bells in the carillon at Den Briel beyond the suggested tolerance ranges are less than in the case of the carillon at Gouda (comp. fig. 23).

§ 26. The Carillon at Goes

The carillon at Goes (Maria Magdalena Church) is of a heterogeneous composition. Bells are present from the following founders:

Andr. van den Gheyn, 1765—1766	nrs. 2, 3, 5, 7, 8, 9, 10, 11, 12, 13, 14, 16 and 18.
St. van Aerschodt, 1886—1887 . .	nrs. 19, 22, 23, 25, 27 and 29.
Alex. Petit, 1776	nrs. 30, 35, 38, 39 and 40.
A. and P. Petit, 1769	nr. 34.
Petit and Fritsen	nr. 33.
Van Bergen (Heiligerlee), 1868 . .	nr. 37.
B. Eysbouts, 1930	nrs. 21, 24, 26, 28, 31, 32 and 36
No inscription	nrs. 6, 17 and 20.

The results of the measurements on this carillon are given in table XV. In fig. 24, where the Hum-Notes of the bells of this carillon are plotted in musical sequence, it is seen that the nrs. 39, 37, 36, 33, 32, 28, 27, 26, 20, 12, 9 and 8 are entirely different from the scale. Of the remaining bells the average equally tempered scale

[1]) Hemony, P.: "De On-Noodsakelijkheid en Ondienstigheid van Cis en Dis in de Bassen der Klokken". Cited by: Denyn, J., Handelingen van het tweede Beiaardcongres" (Proceedings of the Second Carillon Congress), 's Hertogenbosch, 1925. See also: B. Bijtelaar, l.c.

TABLE XV

Carillon at Goes (Maria Magdalena Church)

(Numbering 10/1K. . .)

nr.	M (kg)	R_o (m)	Hum-Note	Fundam.	Minor Th.	Fifth	Nominal
40	1040	—	F♯2 —0,38	F♯3 —0,38	A 3 —0,04	C♯4 —0,39	F♯4 —0,42
39	720	—	G♯2 —0,23	G♯3 —0,56	B 3 —0,13	D♯4 —0,49	G♯4 —0,43
38	610	0,47	A♯2 —0,38	A♯3 —0,34	D♭4 —0,10	E 4 +0,49	G 4 —0,16
37	360	0,42	B 2 —0,06	B 3 —0,89	D 4 +0,01	F♯4 —0,26	B 4 —0,51
36	242	0,375	B 2 —0,13	B 3 +0,63	E♭4 —0,30	F 4 +0,43	B 4 +0,62
35	277	0,37	C♯3 —0,55	C♯4 —0,33	E 4 —0,04	G♯4 —0,74	—
34	260	0,37	C♯3 +0,43	C♯4 +0,39	F 4 —0,46	A 4 —0,18	C♯5 +0,42
33	160	0,325	E 3 +0,18	D 4 +0,33	G♭4 —0,05	B 4 —0,46	—
31	99	0,275	F 3 —0,39	F 4 +0,21	A♭4 +0,15	—	—
30	104	—	F♯3 —0,33	F♯4 —0,38	A 4 +0,14	D 5 +0,10	G♯5 —0,50
32	124	0,30	G 3 —0,09	—	B♭4 +0,36	—	—
29	98	0,26	A 3 —0,56	A 4 —0,56	C 5 —0,34	—	—
28	68	0,24	A♯3 —0,03	A♯4 —0,04	E 5 —0,09	—	—
24	34	0,193	B 3 —0,49	B 4 —0,30	D 5 +0,10	G♯5 —0,03	B 5 +0,19
26	55	0,225	B 3 +0,21	B 4 +0,23	D 5 +0,53	—	—
21	26	0,18	C 4 —0,55	C 5 —0,34	E♭5 +0,09	F♯5 —0,04	C 6 —0,46
25	40	0,20	C♯4 —0,52	C♯5 —0,36	E 5 —0,52	—	—
27	60	0,23	C♯4 —0,26	C♯5 —0,31	—	G 5 —0,23	—
23	37	0,193	C♯4 +0,39	C♯5 +0,41	F 5 +0,01	—	E 6 —0,20
22	32	0,185	D 4 +0,43	D 5 +0,40	F 5 —0,05	B 5 +0,17	C 6 +0,41
19	22	0,15	E 4 —0,51	E 5 —0,50	—	—	C♯6 —0,37
20	29	0,165	F 4 —0,21	F 5 —0,10	A♭5 —0,21	C 6 —0,15	F 6 —0,10
18	18	0,15	F♯4 —0,33	F♯5 —0,33	—	—	—
17	23	0,15	G 4 —0,34	G 5 —0,44	B♭5 —0,20	—	G 6 —0,25
16	19	0,145	G♯4 —0,43	G♯5 —0,40	B 5 —0,02	—	—
14	15	0,13	A♯4 —0,65	A♯5 —0,42	D♭6 —0,11	—	—
13	10	0,12	B 4 —0,44	B 5 —0,18	D 6 +0,05	—	—
9	10	0,114	B 4 +0,23	B 5 +0,31	—	—	—
10	10	0,115	C♯5 —0,45	C♯6 —0,31	—	—	—
12	12	0,113	C♯5 +0,25	C♯6 +0,23	—	—	—
11	10	0,108	D 5 +0,41	D♯6 —0,01	G♭6 +0,5	—	—
8	9	0,105	E 5 —0,28	E 6 —0,28	—	—	—
7	9	0,10	E 5 +0,42	E 6 +0,42	—	—	—
6	7	0,095	F 5 +0,42	—	—	—	—
2	7	0,095	F♯5 +0,32	F♯6 +0,38	—	—	—
3	8	0,095	G♯5 —0,37	—	—	—	—
5	8	0,095	A 5 —0,27	—	—	—	—

is 0,50 semitone below the international pitch. The suggested tolerance ranges, plotted on a transposing scale above and below this average line (fig. 24) show that also the Hum-Notes of the nrs. 38, 31, 30, 23, 18, 17 and 2 lie outside the suggested tolerance ranges. In view of the general irregularity of the carillon these bells will, provisionally, not be rejected.

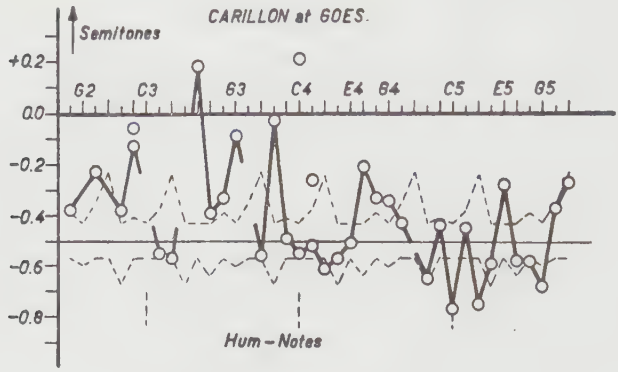

Fig. 24. The carillon at Goes.
Legend as for fig. 20.

Secondly, the internal tuning of the bells is taken into account. This leads to rejecting the nrs. 35, 31, 24, 13 and 11, because of too large deviations of the interval between Fundamental and Hum-Note from an octave. Of the remaining bells nr. 22 shows a Minor Third that is far too low, but this bell will not be rejected for this reason so as to keep as many bells as possible from the original set (the Van Aerschodt family of bell founders belong to the same generation as the Van Den Gheyn family).

Summing up, 19 bells out of the carillon at Goes can be kept as a skeleton that can be completed with a number of bells as given in table XVI to form an improved carillon. In fig. 25 the two lower partials of this scheme for renewing the Goes carillon are plotted in musical sequence. The new bells are expected to have definite deviations from the ideal internal tuning so as to fit into the scale of the corresponding partials of the carillon. Yet as far as possible the tolerance of 1/16 semitone in the intervals of the internal tuning of the new bells has been observed.

The scheme for renewing the Goes carillon ranges from F♯2 up to A 5, with the lower two semitones traditionally missing.

TABLE XVI

Carillon at Goes with improvements

nr. 10/1K. .	M (kg)	Hum - Note	Fundamental	Minor Third
40	1040	F♯2 —0,38	F♯3 —0,38	A 3 —0,04
a	700	G♯2 —0,42	G♯3 —0,40	B 3 —0,15
38	610	A♯2 —0,38	A♯3 —0,34	D♭4 —0,10
b	420	B 2 —0,44	B 3 —0,42	D 4 —0,20
c	360	C 3 —0,49	C 4 —0,43	E♭4 —0,25
d	277	C♯3 —0,55	C♯4 —0,53	E 4 —0,35
34	260	D 3 —0,57	D 4 —0,61	F 4 —0,46
e	215	D♯3 —0,51	D♯4 —0,54	G♭4 —0,40
f	175	E 3 —0,45	E 4 —0,48	G 4 —0,40
g	145	F 3 —0,43	F 4 —0,43	A♭4 —0,30
30	104	F♯3 —0,33	F♯4 —0,38	A 4 +0,14
h	105	G 3 —0,43	G 4 —0,43	B♭4 —0,20
i	90	G♯3 —0,48	G♯4 —0,49	B 4 —0,30
29	98	A 3 —0,56	A 4 —0,56	C 5 —0,34
j	63	A♯3 —0,56	A♯4 —0,52	D♭5 —0,32
k	53	B 3 —0,55	B 4 —0,48	D 5 —0,30
21	46	C 4 —0,55	C 5 —0,34	E♭5 +0,09
25	40	C♯4 —0,52	C♯5 —0,36	E 5 —0,52
23	37	D 4 —0,61	D 5 —0,59	F 5 +0,01
22	32	D♯4 —0,57	D♯5 —0,60	G♭5 —1,05
19	22	E 4 —0,51	E 5 —0,50	G 5 —0,30
l	22	F 4 —0,43	F 5 —0,43	A♭5 —0,25
18	18	F♯4 —0,33	F♯5 —0,33	A 5 —0,20
17	23	G 4 —0,34	G 5 —0,44	B♭5 —0,20
16	19	G♯4 —0,43	G♯5 —0,40	B 5 —0,02
m	14	A 4 —0,54	A 5 —0,50	C 6 —0,30
14	15	A♯4 —0,65	A♯5 —0,42	D♭6 —0,11
n	12	B 4 —0,50	B 5 —0,46	D 6 —0,30
o	10	C 5 —0,48	C 6 —0,43	E♭6 —0,25
10	10	C♯5 —0,45	C♯6 —0,31	—
p	10	D 5 —0,48	D 6 —0,45	F 6 —0,25
q	9	D♯5 —0,52	D♯6 —0,52	G♭6 —0,32
r	9	E 5 —0,55	E 6 —0,55	G 6 —0,35
7	9	F 5 —0,58	F 6 —0,58	—
6	7	F♯5 —0,58	—	—
2	7	G 5 —0,68	G 6 —0,62	—
3	8	G♯5 —0,37	—	—
5	8	A 5 —0,27	—	—

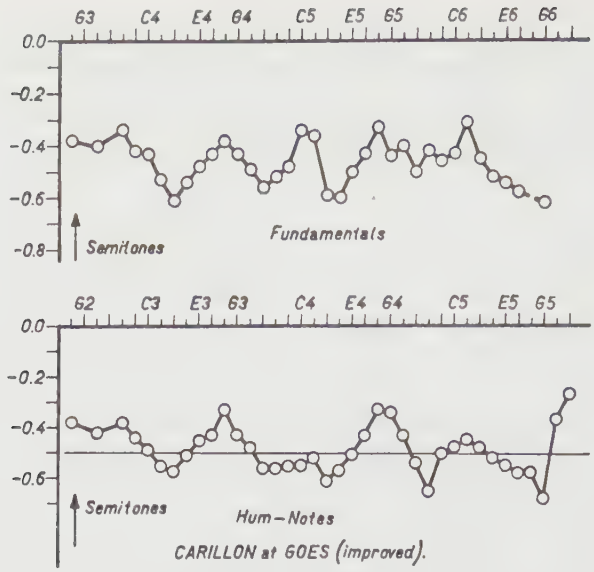

Fig. 25. The carillon at Goes with suggested improvements. Legend as for fig. 20.

§ 27. The Carillons at Tholen and St. Maartensdijk

Of the carillons at Tholen (Town Hall) and St. Maartensdijk (Dutch Reformed Church) only very few bells are remaining. The preserved bells, the measurements of which are given in the tables XVII and XVIII respectively, show that the original carillons had very irregular scales. Therefore it is not advisable to attempt to complete these carillons. The remaining bells might eventually give acceptable chimes.

In the carillon at Tholen the bells nrs 10/5K3, 10/5K6, 10/5K7 and 10/5K8 are probably broken. This can be concluded from the reverberation times of these bells (see § 44).

TABLE XVII

Carillon at Tholen (Town Hall)

(Numbering 10/5K. . .)

nr.	R_0 (m)	M (kg)	Hum-Note	Fundam.	Minor Th.	Fifth	Nominal
3	0,152	16,5	D♯4 —0,33	—	—	—	—
4	0,165	19,5	C♯4 —0,29	—	—	—	—
5	0,175	28	C 4 —0,11	—	E 5 +0,32	—	—
6	0,19	32	—	B 4 +0,28	D 5 —0,02	—	—
7	0,208	51	B 3 —0,24	A♯4 +0,09	D♭5 +0,46	—	A♯5 —0,01
8	0,24	48	A 3 +0,00	A♯4 —0,09	C 5 +0,28	—	—
9	0,24	66	G 3 —0,09	—	B♭4 +0,36	—	—
10	0,248	73	F♯3 —0,16	G 4 —0,24	A 4 +0,48	—	—
11	—	96	F 3 —0,11	F♯4 —0,31	A 4 +0,00	—	—
12	0,28	110	E 3 —0,05	F 4 +0,48	G 4 +0,31	B 4 +0,43	E 5 +0,10
13	0,298	128	D♯3 +0,38	E 4 +0,05	G 4 —0,15	A♯4 +0,31	—
14	0,32	161	D 3 +0,20	—	G♭4 —0,33	—	B 4 —0,49
15	0,372	260	C 3 +0,09	C♯4 —0,10	E 4 —0,11	G 4 +0,14	—
16	0,415	373	A♯2 +0,36	A♯3 +0,30	D 4 —0,12	F 4 +0,20	A♯4 +0,27
17	0,45	470	A 2 +0,20	A♯3 —0,38	D♭4 —0,20	E 4 +0,17	A♯4 —0,30
18	0,505	646	F♯2 +0,46	G 3 +0,33	B♭3 +0,32	D 4 —0,43	F♯4 +0,44
19	—	959	F 2 +0,33	F♯3 —0,4	A 3 —0,31	C♯4 —0,32	F♯4 —0,47

TABLE XVIII

Carillon at St. Maartensdijk (Dutch Reformed Church)

(Numbering 10/3K. . .)

nr.	R_0 (m)	M (kg)	Hum-Note	Fundam.	Minor Th.	Fifth	Nominal
2	0,155	26	G♯4 +0,48	G 5 +0,44	C 6 —0,08	—	—
3	0,155	16	F 4 +0,43	E 5 —0,14	—	—	—
4	0,18	33	E 4 —0,03	D♯5 +0,41	G 5 +0,05	—	—
5	0,205	47	C♯4 +0,36	D 5 —0,17	F 5 —0,02	—	—
10	0,283	125	G 3 +0,44	F♯4 —0,45	B♭4 —0,09	—	—
11	0,325	184	F♯3 —0,42	D♯4 —0,29	A♭4 +0,05	B 4 +0,42	F 5 +0,38
12	—	216	E 3 +0,12	D 4 +0,08	G♭4 +0,40	B 4 —0,05	—
13	0,37	259	D 3 —0,19	C♯4 —0,32	F 4 —0,41	G♯4 +0,01	—
14	0,375	270	D 3 —0,21	C♯4 —0,01	E 4 +0,43	A 4 —0,22	B 4 —0,46
15	0,44	402	A 2 +0,43	A♯3 +0,40	D♭4 +0,22	F 4 —0,06	A♯4 +0,21
16	0,50	638	G 2 —0,07	G 3 +0,31	A 3 +0,43	B 3 +0,49	—

9B

Reprinted from pages 74-96 of *Acoustical Measurements on Church Bells and Carillons,* DeGebroeders van Cleef, The Hague, Netherlands, 1949, 148p.

THE PROCESS OF TUNING

E. W. Van Heuven

§ 28. Literature on Tuning

It has been shown in the foregoing how, notwithstanding the relatively high degree of similarity in the profile shapes, the musical character of the sound of different bells may vary to a great extent. Therefore in the manufacture of a bell, of which the partials must have definite frequencies, such close tolerances cannot be kept in the profile shape by casting only. All the better founders finish their bells by turning them on a lathe whilst controlling the partial frequencies rather than the profile shape.

Most of the bell founders have experience about the qualitative alterations in the tuning due to turning the bell at given places. This experience is passed on in the founders families.

The usual frequency measuring method with tuning forks does not give exact enough values for quantitative information. Therefore the experience obtained is largely a statistical knowledge about the results of the tuning of many bells that have been made in the course of years. As the electro-acoustical measuring method does give more exact data, especially at high frequencies, it is of importance to apply this method to a systematical research on the relation between the partial frequencies and the profile. In this case data can be obtained on much smaller bells than would have been the case if tuning forks were used.

The only publications found about the effect of turning the bell, on the partial frequencies, are by Pfnor[1]) and by Simpson[2]).

Pfnor describes how the Fundamental may be flattened by decreasing the wall thickness near "A" (fig. 26), and also by taking off material between "C" and "G" and at the outside from "D" to "F". Not till after the wall thickness of the sound-bow has become equal to that of the waist of the bell is an increase of the Fundamental fre-

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[1]) Pfnor: Zur Akustik der Glocken. Verh. des Hessischen Gewertvereins, 1848.

[2]) Simpson: Pall Mall Magazine, October 1895.

quency possible by shortening the profile. Nothing is stated about the other partials.

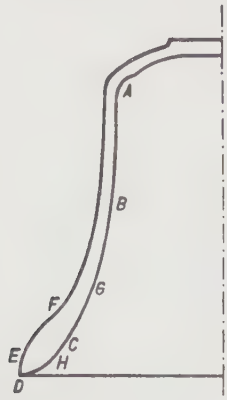

Fig. 26. Places for the tuning of partials.

Simpson gives the following rules (referring to fig. 26):

The Nominal frequency may be increased by taking off material at "HDE" (up to 1/4 semitone).

The Nominal frequency may be decreased by taking off material from "H" to "C" or from "G" to "B" (up to 1 semitone).

The Fundamental may be sharpened by taking off material along "C—D". This is possible until the line "C—E" is reached.

The Fundamental may be flattened by taking off material from "C" to "G", or even to "B" if necessary.

The Minor Third cannot be sharpened but may be flattened by thinning the wall between "G" and "B".

The Fifth in likewise manner cannot be sharpened but may be flattened by thinning the bell from "B" towards "G".

Simpson's conclusions are:

1) The Fundamental frequency cannot be decreased without flattening the Nominal.
2) The Nominal may be flattened without changing the Fundamental frequency by taking off material on both sides of "G".
3) The Nominal may be flattened to a small extent, at the same time increasing the Fundamental frequency by reducing the curved surface from "C" to "E".
4) The Fundamental frequency may be increased without much alteration of the Nominal.
5) An increase of the Nominal frequency causes an increase of the Fundamental frequency, though relatively less. Thus the interval between both partials will decrease.

It is not surprising that publications on this subject are rather scarce as most of the knowledge on tuning is considered the industrial property of the bell founders firms.

§ 29. The Profiles of a Number of Bells

In order to obtain an average profile shape of existing good bells the actual shapes of twelve bells were drawn. The bells involved were cast by F. and P. Hemony (4), St. Buytendyck, Burgerhuys (2), St. van Aerschodt, W. Moer and Jaspers', and Van Bergen (3).

To make the drawings the bell was placed with its sound-bow on a table, of which the leaf had been placed horizontally with the aid of a spirit-level. Beside the table a drawing-table was placed in a vertical position, by means of a plummet. On the drawing-table a ruler could slide with parallel leads. A second ruler, carrying a steel pin at one end, could be slid along the parallel ruler. The position of the drawing-table on the floor was adjusted till its plane went through the axis of the bell, so that the steel point of the sliding ruler touched the centre of the bell at the crown.

After these adjustments it is possible to draw the profile line of the outside of the bell by moving the sliding ruler with its pin against the bell surface for each position of the parallel ruler. Each time the rear side of the sliding ruler is marked on the drawing. Thus an actual size drawing of the outside profile of the bell is obtained.

The inside profile can likewise be drawn by placing the bell on its side in a position where the point of the sliding ruler can be brought both at the centre point of the bell near the fastening of the clapper, and at the geometrical centre of the sound-bow (which can be measured with a piece of string stretched across the rim), thus assuring that the plane of the drawing-desk goes through the rotation-axis of the bell.

The drawings of the bell-profiles were made on "Kodatrace" film to avoid errors in the measurements by elongation of the paper due to differences in humidity. The drawings of the inside- and the outside profiles could be combined by measuring the in- and outside diameters at a number of heights, indicated on the curve. In the combined figure co-ordinates were drawn with R_0 as unit of length thus giving the reduced profile.

Table XIX gives a survey of the results obtained. In this table the extreme values as found for the quantities R/R_0 (outside profile) and D/R_0 (the wall thickness measured in a direction perpendicular to the rotation axis) are indicated as a function of the reduced height

TABLE XIX

Range of reduced profiles

H/R_0	R/R_0 (max.)	R/R_0 (min.)	D/R_0 (max.)	D/R_0 (min.)	R/R_0 (Hemony)	D/R_0 (Hemony)
0,00	1,000	1,000	0,000	0,000	1,000	0,000
0,05	0,990	0,986	0,167	0,089	0,987	0,122
0,10	0,979	0,960	0,182	0,145	0,971	0,165
0,15	0,957	0,934	0,187	0,155	0,948	0,176
0,20	0,915	0,890	0,171	0,158	0,909	0,165
0,25	0,863	0,833	0,155	0,133	0,853	0,136
0,30	0,820	0,790	0,137	0,110	0,807	0,115
0,40	0,747	0,720	0,109	0,075	0,736	0,089
0,50	0,694	0,673	0,093	0,060	0,687	0,074
0,60	0,661	0,638	0,080	0,060	0,653	0,069
0,70	0,635	0,613	0,072	0,055	0,629	0,067
0,80	0,617	0,600	0,070	0,055	0,611	0,065
0,90	0,602	0,586	0,069	0,050	0,597	0,061
1,00	0,591	0,574	0,064	0,050	0,588	0,059
1,10	0,585	0,566	0,063	0,045	0,580	0,058
1,20	0,580	0,560	0,060	0,045	0,576	0,055
1,25	0,585	0,563	0,063	0,045	0,574	0,052
1,30	0,582	0,540	0,110	0,045	0,572	0,051
1,35	0,582	0,485	—	0,040	0,571	0,053
1,40	0,569	0,390	—	0,060	0,570	0,075
1,45	0,554	0,355	—	0,053	0,550	0,14
1,50	0,550	—	—	0,050	0,483	—
1,55	0,543	—	—	0,085	0,415	—
1,60	0,480	—	—	—	0,25	—

H/R_0. The last two columns give the average reduced profile of two Hemony bells that were included in the twelve of which the profiles were drawn.

Fig. 27 shows the range of differences in the reduced profile as found in the drawn bell profiles. It appears that the largest individual differences occur in the horizontal wall thicknesses near the sound-bow and in the total height of the bells. So probably these are important points in connection with the internal tuning.

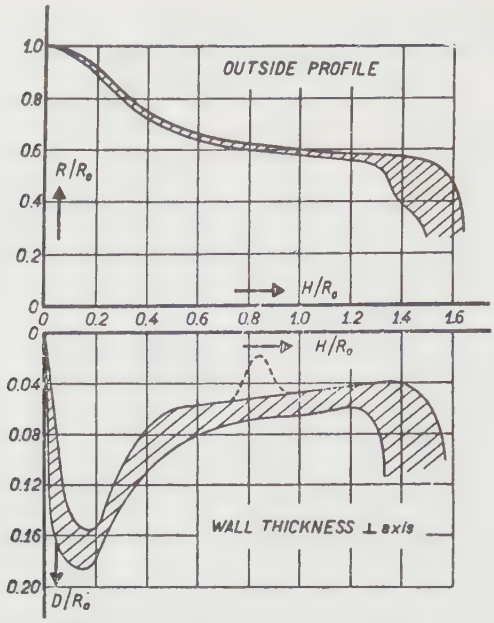

Fig. 27. Range of reduced profiles.
(The dotted line shows an extreme case of tuning).

§ 30. Theoretical Consideration of the Problem

The problem of the relation between the profile shape and the partial frequencies may be tackled with some assumptions.

We shall restrict ourselves to bell-shaped bodies with rotational symmetry, the reduced profiles of which lie within the range of existing profiles as indicated in fig. 27. It is further assumed that the alterations of the profile shape take place without disturbing the axial symmetry; in other words, that each time a ring-shaped amount of material is taken off. This is the case for instance, if the bell is turned on a lathe.

Every alteration of the profile shape will generally cause all partial frequencies to become different.

Let the original reduced profile of the bell be given by functions in cylindrical coordinates as:

$$R/R_0 = f(H/R_0) \text{ for the outside surface, resp.}$$
$$r/R_0 = f'(H/R_0) \text{ for the inside surface,}$$

where R_0 is the length unit in which all sizes are to be expressed.

The position of an infinitely small chip of height dH and depth dR that is taken off, is defined by its reduced height H/R_0, where

the radius is given by the above function, so that the volume of the chip is (see fig. 28):

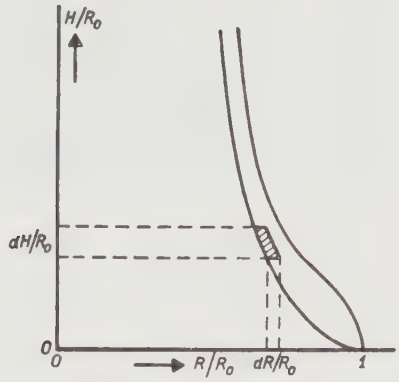

Fig. 28. Definition of the tuning functions.

$$\frac{dV}{R_0^3} \cong \frac{dR}{R_0} \cdot \frac{dH}{R_0} \cdot 2\pi f(H/R_0) \quad . \; . \; . \; . \; . \quad (17).$$

Let $\nu_1, \nu_2, \ldots \nu_i$ be the characteristic frequencies of the original bell and $(\nu_1 + d\nu_1), (\nu_2 + d\nu_2) \ldots (\nu_i + d\nu_i)$ the new partial frequencies after the chip has been taken off. Now for each partial a function $F_i(H/R_0)$ may be defined, giving at each position (H/R_0), either at the inside or at the outside surface, the ratio of the relative alteration of the partial frequency ν_1 and the surface, taken off the reduced profile:

$$F_i\ (H/R_0, \text{in- or outside}) = \frac{\dfrac{d\nu_i}{\nu_i}}{d\left(\dfrac{H}{R_0}\right) \cdot d\left(\dfrac{R}{R_0}\right)} \quad . \; . \; . \; . \quad (18).$$

The numerical values of these functions at a given position on the bell will generally depend upon the shape of the profile. It is finally assumed that (for reduced profiles within the range indicated in fig. 27) the functions F_i are independent of the profile shape $f(H/R_0)$, resp. $f'(H/R_0)$.

The consequences of this assumption are the following:

In the first place the total relative change in frequency of any partial, $\Delta\nu_i/\nu_i$, due to a finite amount of material, indicated by $\Delta R/R_0$, $\Delta H/R_0$, being taken off, will be proportional to the surface $\Delta R/R_0 . \Delta H/R_0$ of the reduced profile that is removed, provided

that the new profile shape after this finite alteration remains within the range of fig. 27.

The proportionality to $\Delta R/R_0$ is proved by dividing $\Delta R/R_0$ into a number of layers dR/R_0 where, according to the latest assumption each layer dR/R_0 will cause the same relative change in the partial frequencies. Likewise, the proportionality to $\Delta H/R_0$ is proved by adding the effects of taking off an amount of material $dR/R_0 . dH/R_0$ at $(H/R_0)_1$ and at $(H/R_0)_2$. If $(H/R_0)_2 \rightarrow (H/R_0)_1$, $F_i(H/R_0)_2 \rightarrow F_i(H/R_0)_1$ and the total value $\Delta\nu_i/\nu_i$ will be proportional to

$$dH_{(H/R_0)_2} + dH_{(H/R_0)_1} .$$

In the second place it follows that the total relative frequency alteration due to taking off material at a number of places will be the sum of the effects of each individual chip upon the partial frequencies.

If the assumption of the additivity of the effects of finite profile alterations within the prescribed range does not hold, this will show itself from the results of the tuning experiments, if consecutive layers of the test-bell are gradually taken off step by step.

If the assumption holds within reasonable tolerances, then the tuning problem of bells is solved if the functions F_i are known for each place on the bell surface. It must be possible in practice, however, to cast a rough bell with high enough accuracy, so that only limited amounts of material will have to be taken off to obtain the right partial frequencies.

§ 31. Determination of the Functions F_i

For the determination of the functions F_i a test-bell was cast which could be worked on a lathe. The reduced profile of this bell was based on the average Hemony profile as determined in § 29. On all sides of this profile about 5 mm extra wall thickness allowed for material to be taken off before the right profile shape was attained. Thus the experiments ranged from the tuning of a bell with a too large wall thickness to the same for a bell that was already too thin.

The size of the experimental bell was chosen as a compromise between two contradictory demands, namely:

a) The accuracy of the frequency measurements as carried out with a Wien-Robinson bridge is $\pm$ 1 c/s and therefore the Hum-Note frequency must not be lower than 500 c/s in order to obtain a relative accuracy of $\pm$ 0,2%. As the partial frequencies are expected to decrease during the experiments, the initial Hum-Note frequency must be at least 600 c/s.

Fig. 29. Experimental bell on the lathe.

Fig. 30. Apparatus for determining the tuning functions.

b) The bell must be as large as possible, since the profile must be measured after each alteration. The drawing of the profile of a bell can be made with a greater relative accuracy if the bell is large.

To meet these demands, a value $R_0 = 0{,}20$ m was chosen.

The bell must be fastened on the lathe by means of its top plate as in this case material can be turned off both at the inside and at the outside without alterations in the fastening. Also in this case the bell can vibrate freely on the lathe during the frequency measurements. The lathe to be used must therefore have a centre-height of at least 0,20 m. A lathe of this size was available at the mechanical technological workshop of the "Laboratorium voor Werktuig-, Scheeps- en Vliegtuigbouw" of the "Technische Hogeschool" at Delft, where, by kind permission of Prof. Ir. P. Landberg the experiments were carried out.

The experimental bell was cast by Messrs. Eysbouts at Asten, Holland, and kindly put at our disposal for the measurements. At the top plate a cylindrically shaped piece provided a firm and stable method of fastening on the lathe.

The excitation system used for the frequency measurements was mounted on the support in a position where it could be pressed against the rim of the bell (fig. 29). The method of measurement and the apparatus were described in § 16. Fig. 30 gives a picture of the apparatus near the lathe.

In the course of the experiments each time material is removed over a finite range ΔH and ΔR. The partial frequencies before and afterwards are measured. The values of the functions F_i, thus found, are average values over the range H to $H + \Delta H$. At places where F_i varies rapidly with H, small values of ΔH should be taken.

The parallel movements of the support of the lathe were used to determine the geometrical shape of the bell profile and also the place and the area of the material that was removed at each experiment. The heights H and ΔH were read on a dial in the movement of the lathe. Two methods were used to determine the area of the profile that was turned off:

a) By periodical drawings of the entire profile, at least each time before material was to be removed at any place for a second time. Thus the area can be determined graphically.

b) The bronze that came off was gathered and weighed. The average diameter of the place where the material was taken off was determined with a pair of callipers. The area of the profile change can then be calculated with the known density of the bronze.

The errors in the drawings of the bell profile are of the order of

magnitude of 0,5 mm. This leads to a relative error of $\pm$ 30% in the area that was usually turned off. The diameter of the turning place is of the order of magnitude of 200 mm and can be determined with an accuracy of $\pm$ 1 mm; the specific weight of the bronze can easily be determined with an average error of $\pm$ 1%. Therefore the precision of method b) is mainly determined by the possibility to gather as much as possible of the material spattering around during the turning. This is more difficult to do at the outside than at the inside of the bell.

After the bell had been turned over its whole surface to remove all asymmetry of the casting, three consecutive layers of material were taken off, both at the outside and at the inside surface. Except at the sound-bow, where smaller areas were removed, the standard value of ΔH at each experiment was taken as 10 mm = 0,05 R_0 approx. Between the turning processes the partial frequencies were measured each time and the relative frequency alterations of the partials were calculated in hundredths of a semitone. By expressing the area of the material with R_0^2 as surface unit, values for the functions F_i are found in "semitones per unit area R_0^2 removed".

§ 32. Measurement Results

The results of the measurements are given in graphical form in the figs. 31 to 41 inclusive. Each time F_i is expressed as a function of the position H/R_0, both for turning inside and outside the bell.

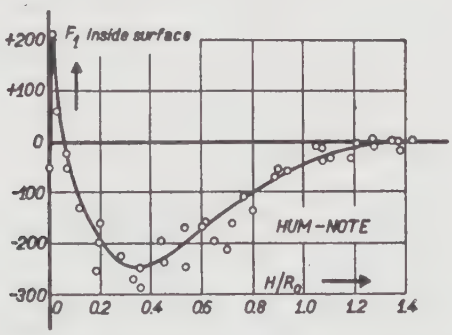

Fig. 31. Tuning function for the Hum-Note (inside surface).

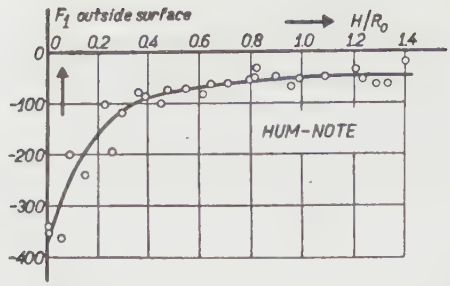

Fig. 32. Tuning function for the Hum-Note (outside surface).

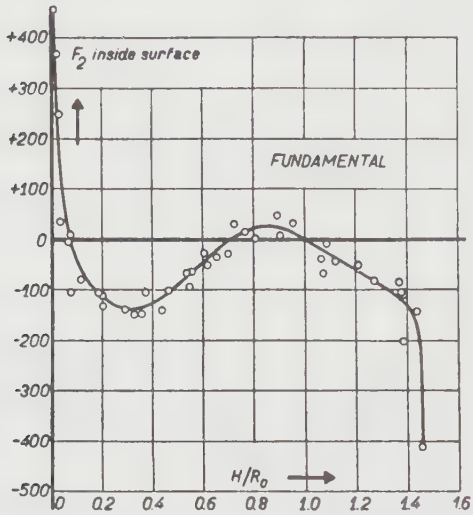

Fig. 33. Tuning function for the Fundamental (inside surface).

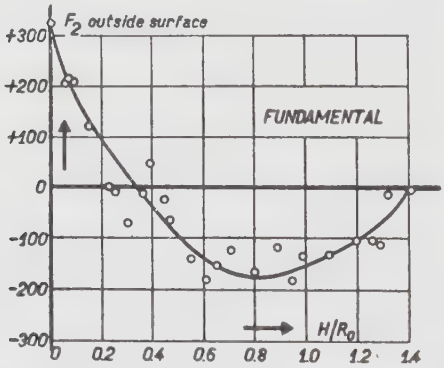

Fig. 34. Tuning function for the Fundamental (outside surface).

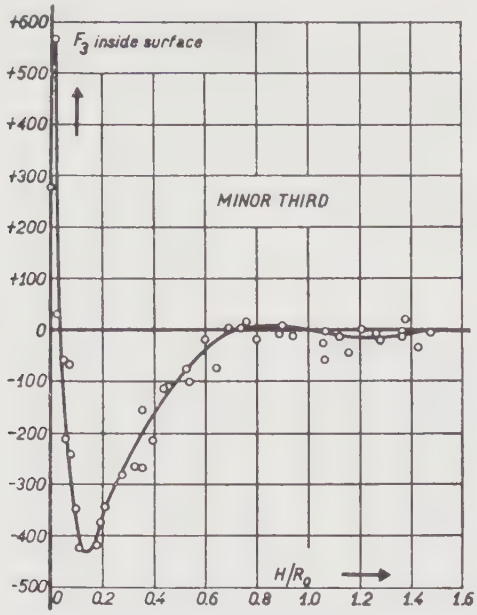

Fig. 35. Tuning function for the Minor Third (inside surface .

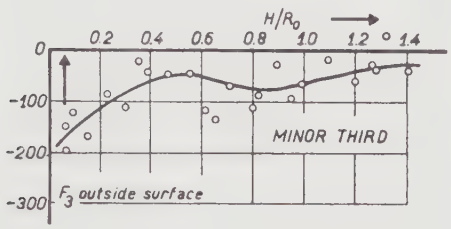

Fig. 36. Tuning function for the Minor Third (outside surface).

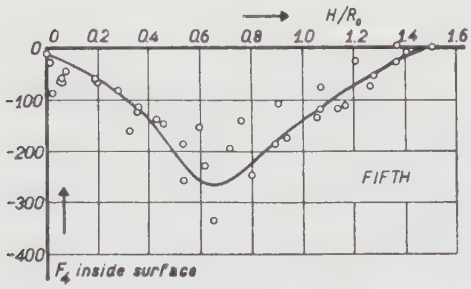

Fig. 37. Tuning function for the Fifth (inside surface).

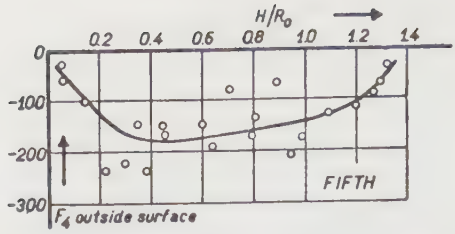

Fig. 38. Tuning function for the Fifth (outside surface).

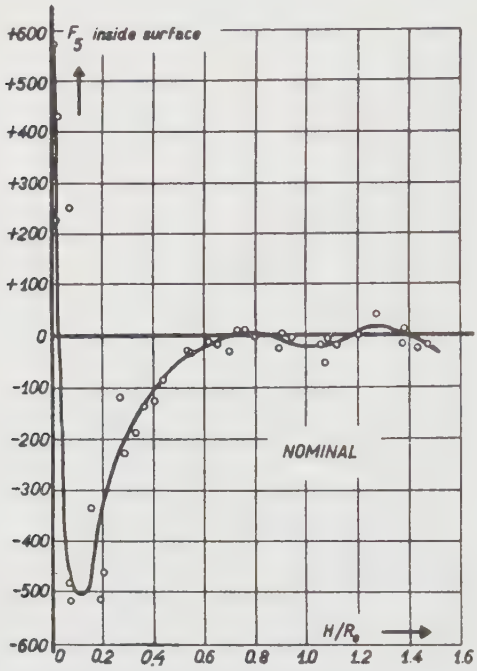

Fig. 39. Tuning function for the Nominal (inside surface).

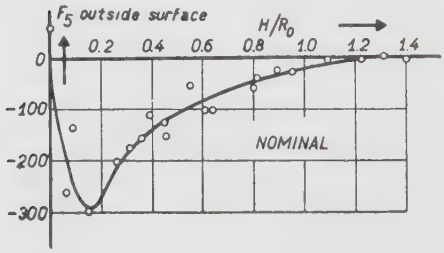

Fig. 40. Tuning function for the Nominal (outside surface).

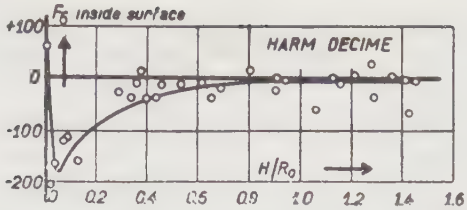

Fig. 41. Tuning function for the Harmonic Decime (inside surface).

There seems to be no systematic difference between the results obtained for the successive three layers. Thus the additivity assumption holds within the range of profile alterations considered and within the accuracy of the measurements.

It must be remarked that the sixth partial (Harmonic Decime) did not show any appreciable frequency alterations as a result of turning at the outside surface.

§ 33. Application of the Results

If a given roughly-cast bell is to be tuned the foregoing results can be applied in principle as follows:

It is supposed that a number of m partial frequencies are to be adjusted. If the actual frequency of the Hum-Note or first partial be ν_1', whilst the desired value for the same partial be ν_1, then Δ_1 is the interval in semitones between ν_1' and ν_1; in the same way Δ_2 equals the interval in semitones between the actual frequency ν_2' and the desired value ν_2 of the second partial (Fundamental), etc. These values of Δ_i are taken as positive if the actual partial is to be sharpened to obtain the desired pitch and negative if it is to be flattened.

Next a number of n positions on the bell are chosen, where material must be removed over the reduced profile areas K_1/R_0^2, $K_2/R_0^2, \ldots K_n/R_0^2$ respectively, in order to obtain the desired tuning of the bell. These n positions are defined by the reduced heights H_1/R_0, $H_2/R_0, \ldots H_n/R_0$ respectively, on the inside- or outside surface of the bell.

The areas $K_1/R_0^2 \ldots K_n/R_0^2$ can now be calculated from the following equations:

$$\left.\begin{aligned}
\Delta_1 &= K_1/R_0^2 \,.\, (F_1)_{H_1/R_0} + K_2/R_0^2 \,.\, (F_1)_{H_2/R_0} + \\
&\qquad \ldots + K_n/R_0^2 \,.\, (F_1)_{H_n/R_0} \\
\Delta_2 &= K_1/R_0^2 \,.\, (F_2)_{H_1/R_0} + K_2/R_0^2 \,.\, (F_2)_{H_2/R_0} + \\
&\qquad \ldots + K_n/R_0^2 \,.\, (F_2)_{H_n/R_0} \\
&\ldots\ldots\ldots\ldots \\
\Delta_m &= K_1/R_0^2 \,.\, (F_m)_{H_1/R_0} + K_2/R_0^2 \,.\, (F_m)_{H_2/R_0} + \\
&\qquad \ldots + K_n/R_0^2 \,.\, (F_m)_{H_n/R_0}
\end{aligned}\right\} \quad (19)$$

In case $n = m$ definite solutions can be found; in casa $n > m$ the solution is indefinite. There are some further conditions, however, that must be fulfilled:

a) Negative values for K are physically impossible, as material can only be removed.

b) The total area of a normal bell profile after Hemony (table XIX) varies for ranges $\Delta H = 0{,}1\ R_o$ between $0{,}016\ R_o^2$ at $H/R_o = 0{,}15$, and $0{,}005\ R_o^2$ at $H/R_o > 1$. The removal of material at any position during the tuning process must be restricted to approximately 20% of the total profile area at that place, unless the tuning is to be detrimental to the appearance and the mechanical strength of the bell (compare the extreme case of tuning indicated in fig. 27). This means that the total value of K for a range $\Delta H = 0{,}1\ R_o$ may not exceed $3.10^{-3}\ R_o^2$ at the sound-bow, or $1.10^{-3}\ R_o^2$ at the waist of the bell.

The solving of the equations (19) may be simplified, however, by the fact that in practice it is not necessary to find an exact solution. The partial frequencies that are to be tuned need only to be adjusted within certain tolerance limits.

Therefore a better method to solve the tuning problem is the cascade method. Here only one place of tuning is chosen at a time. This choice must be made in such a way that every partial frequency changes in the direction of the desired value. After each turning the partial frequencies are measured and a new choice is made after the results of these measurements. This process is to be repeated until all partial frequencies lie within the tolerance ranges of the desired values. The right choice of a place where material is to be removed is largely dependent on the rate at which the different partials must be altered. To fascilitate the choice of the place of turning a number of auxiliary graphs were deduced from the results in the figs. 31 to 41 inclusive. The idea was as follows:

Suppose only two partials are to be tuned ($m = 2$). Then the right place of turning will be that where F_1/F_2 is as closely as possible equal to Δ_1/Δ_2. Generally an angle $\Theta_{i,j}$ may be defined (fig. 42), according to:

$$(\Theta_{i,j})_{H/R_o} = \text{arc tan}\ \frac{(F_i)_{H/R_o}}{(F_j)_{H/R_o}} = \text{arc tan}\ \frac{\Delta_i}{\Delta_j} \quad \ldots\ldots\ (20).$$

From the graphs 31 to 41 inclusive $\Theta_{i,j}$ can be determined as a function of H/R_o, inside- or outside surface. As $\Theta_{i,j}$ is defined as a relation between two functions F, in semitones per R_o^2 area removed, at the same position on the bell, in the calculation of $\Theta_{i,j}$

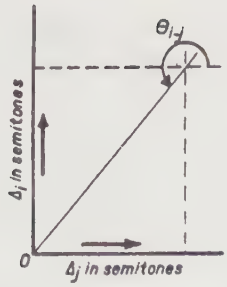

Fig. 42. Definition of $\Theta_{i,j}$

from the measurement results frequencies only are used, and the error due to the determination of the profile area of removed material will disappear.

In the figs. 43 to 50 inclusive, $\Theta_{i,1}$ is given for the four important higher partials in combination with the Hum-Note (j = 1) at every position on the bell surface, indicated by the reduced height H/R_0.

To use this method in practice, rectangular graphs are made in which Δ_i is plotted against Δ_1. In these graphs the angle $\Theta_{1,1}$ can be determined graphically (fig. 42) and also the approach of the righ tuning of the bell can be followed in detail.

A disadvantage of the use of the graphs for $\Theta_{1,1}$ is that they do not give the absolute values of the frequency alterations per unit of profile area removed, so that after a place of turning has been chosen

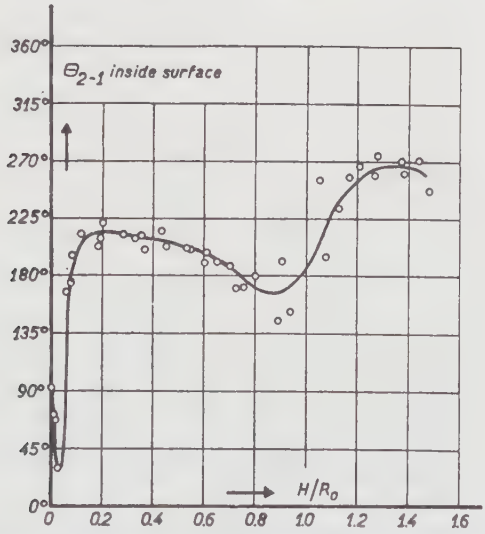

Fig. 43. Relative tuning function Θ of the Fundamental in combination with the Hum-Note at the inside surface.

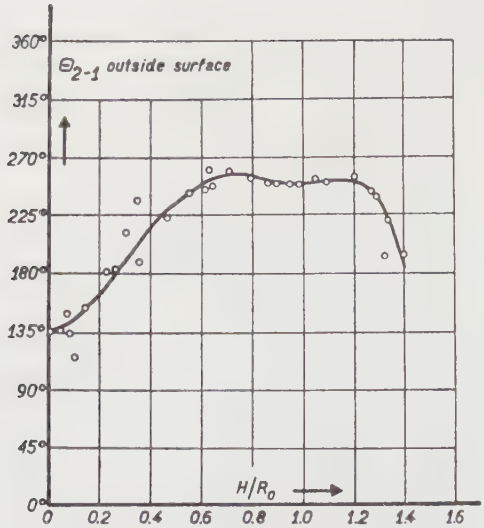

Fig. 44. Relative tuning function Θ of the Fundamental in combination with the Hum-Note at the outside surface.

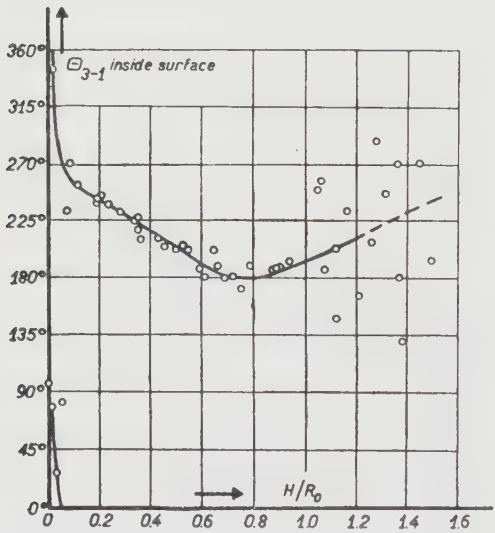

Fig. 45. Relative tuning function Θ of the Minor Third in combination with the Hum-Note at the inside surface.

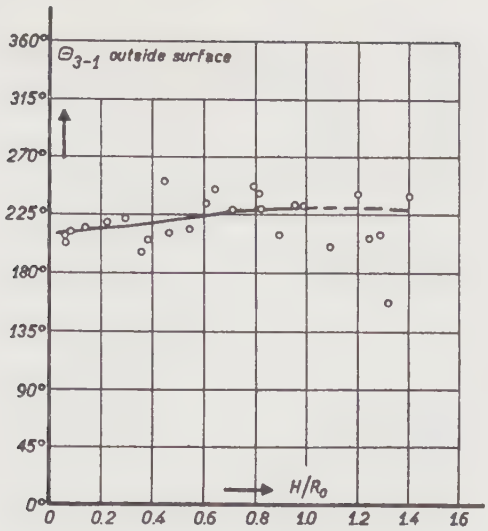

Fig. 46. Relative tuning function Θ of the Minor Third in combination with the Hum-Note, at the outside surface.

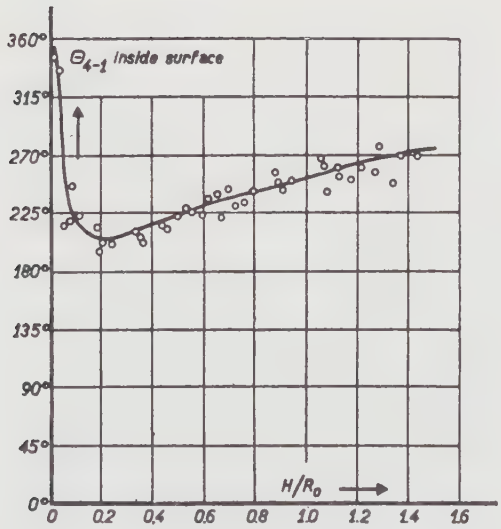

Fig. 47. Relative tuning function Θ of the Fifth in combination with the Hum-Note at the inside surface.

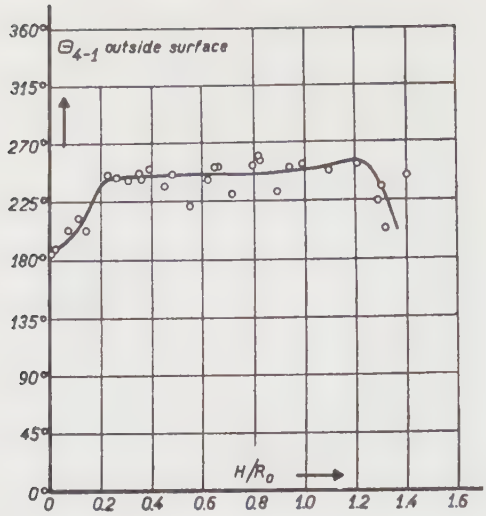

Fig. 48. Relative tuning function Θ of the Fifth in combination with the Hum-Note at the outside surface.

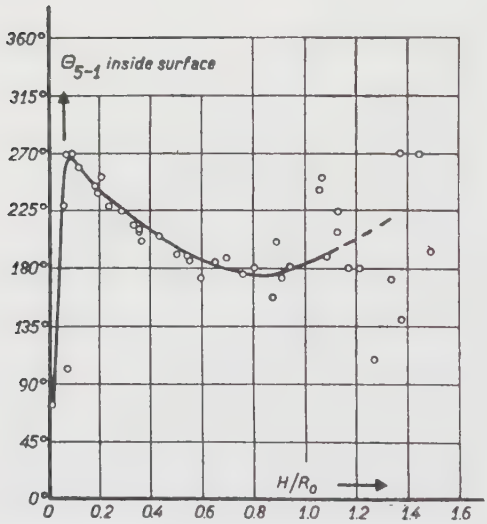

Fig. 49. Relative tuning function Θ of the Nominal in combination with the Hum-Note, at the inside surface.

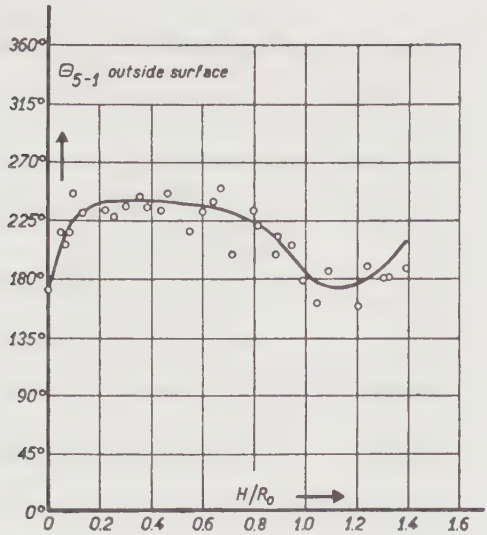

Fig. 50. Relative tuning function Θ of the Nominal in combination with the Hum-Note, at the outside surface.

the area of material to be removed must still be determined by using the figs. 31 to 41. There are also a number of regions where the values of some angles $\Theta_{1,1}$ are rather uncertain. During the measurements from which the figs. 31 to 41 were deduced, the frequencies before and after the turning were determined with a possible error of $\pm$ 1 c/s. So in cases where this frequency difference before and after the removal of material both for ν_i and for ν_1, was not large compared with 2 c/s, the values of $\Theta_{1,1}$ as calculated from the measurements will show large absolute errors. This is the case for instance in the figs. 45 and 49 for $H/R_0 > 1$. It is advisable to avoid these places of tuning, not only because of the uncertainty of Θ, but also because a finite frequency alteration, at least of one of the frequencies involved would mean the removal of a large area of the profile. The maximum values to be taken off at different heights were discussed earlier in this paragraph.

To avoid the use of both series of graphs, giving F_i and F_1 resp. as a function of H/R_0 in the control of the effectiveness of a place of turning, chosen with the appropriate value of $\Theta_{1,1}$, two additional graphs were constructed. In the Δ_i—Δ_1 diagram of fig. 42 the distance to the point of the desired tuning, measured in "semitone" distances in the figure, for $\Delta\nu/\nu$ in semitones, is given by:

$$d_{1,1} = \sqrt{(\Delta_i)^2 + (\Delta_1)^2} = \sqrt{\left(\frac{\Delta\nu_i}{\nu_i}\right)^2 + \left(\frac{\Delta\nu_1}{\nu_1}\right)^2} \quad . \; . \; . \; (21).$$

In the figs. 51 and 52 the values for $d_{1,1}$ per removed unit area R_0^2 are given as a function of H/R_0 for the inside- and the outside surface of the bell respectively.

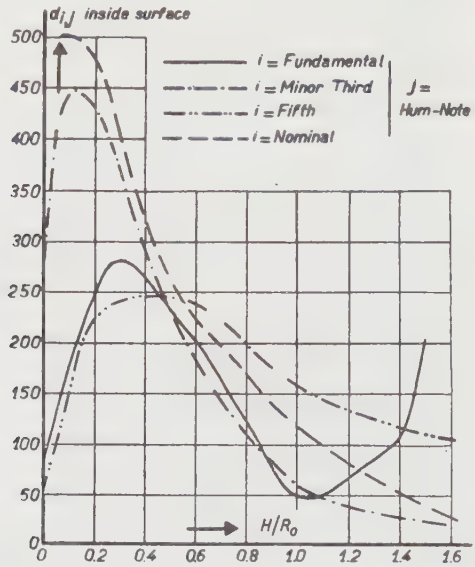

Fig. 51. Control of the effectiveness of a chosen place of turning at the inside surface.

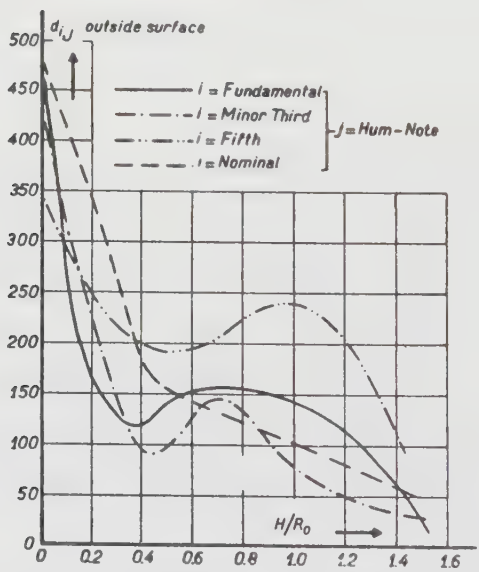

Fig. 52. Control of the effectiveness of a chosen place of turning at the outside surface.

§ 34. Example of Tuning

To illustrate the use of the foregoing results a simple example of the tuning of a swinging bell will be worked out. In this case the tuning of the octaves between Hum-Note, Fundamental and Nominal are considered as most important, while the Fifth is rather unimportant (to be discussed further in chapter VI). No requirements were laid down for the latter partial.

The bell did show the following partial frequencies after the casting:

Hum-Note: 248 c/s = B 2 + 0,08 semitones.
Fundamental: 504 c/s = B 3 + 0,35 semitones.
Minor Third: 601,5 c/s = D 4 + 0,41 semitones.
Nominal: 1004,5 c/s = B 4 + 0,29 semitones.

The desired tuning was that with a Hum-Note B 2 = 247 c/s, so the differences between the actual and the desired tuning of the partials are:

Hum-Note: $\Delta_1 = -0{,}08$ semitones.
Fundamental: $\Delta_2 = -0{,}35$ semitones, so $\Theta_{2,1} = 257°$.
Minor Third: $\Delta_3 = -0{,}41$ semitones, so $\Theta_{3,1} = 259°$.
Nominal: $\Delta_5 = -0{,}29$ semitones, so $\Theta_{5,1} = 255°$.

The shortest distance in the Δ_1—Δ_1 diagrams to the point of the desired tuning apparently occurs in the combination between the Nominal and the Hum-Note, where $d_{5,1} = \sqrt{(0{,}29)^2 + (0{,}08)^2} = 0{,}30$ "semitones". Therefore this combination was taken first to find a place of turning. From fig. 49 it can be seen that $\Theta_{5,1} = 255°$ corresponds to $H/R_0 = 0{,}10$ inside the bell. Fig. 39 shows that at this point $F_5 = -500$ semitones per R_0^2 area, whilst fig. 51 shows, that at $H/R_0 = 0{,}10$, $d_{5,1}$ per R_0^2 shows the largest value, so that none of the other partials will pass the desired tuning if at first the Nominal is brought to the right frequency. This can be done at $H/R_0 = 0{,}10$ by taking off:

$$K_1 = \frac{-0{,}29}{-500} \cdot R_0^2 = 0{,}58.10^{-3}\, R_0^2,$$

which does not surpass the limit of 20% of the total area for a range $\Delta H = 0{,}1\, R_0$. From figs. 31—41 the influence of an amount of material $0{,}58.10^{-3}\, R_0^2$ at $H/R_0 = 0{,}10$ is seen to be:

Hum-Note: $F_1 = -100$, so $F_1 . K_1 = -0{,}06$ semitones.
Fundamental: $F_2 = -60$, so $F_2 . K_1 = -0{,}035$ semitones.
Minor Third: $F_3 = -350$, so $F_3 . K_1 = -0{,}20$ semitones.
Nominal: $F_5 = -500$, so $F_5 . K_1 = -0{,}29$ semitones.

After this material has been taken off the tuning of the bell will therefore be:

Hum-Note:	B 2 + 0,02	semitones.
Fundamental:	B 3 + 0,315	semitones.
Minor Third:	D4 + 0,21	semitones.
Nominal:	B 4 + 0,00	semitones.

As the deviation between the actual Minor Third from the equally tempered ideal value of +0,21 semitones is tolerable (comp. § 13) only the Fundamental remains to be corrected. A decrease of the Fundamental frequency is always possible without altering the other partials by taking off material at $H/R_0 = 1{,}45$ on the inside surface. The value of K_2 at this place is determined by $F_2 . K_2 = -0{,}315$ semitones, where $F_2 = -400$ semitones per R_0^2 area according to fig. 33, so $K_2 = 0{,}79.10^{-3}\ R_0^2$, which is equally tolerable. Thus the tuning of this bell was completed.

It is seen that in practice the tolerable discrepancies from the right tuning will cause this cascade method to lead to a result more easily than the mathematical solution of the equations (19). Moreover, as the partial frequencies are controlled at each step any unexpected deviations from the calculated values can always be corrected in time.

§ 35. Comparison Between Own Results and Pfnor's and Simpson's Rules

When making a comparison between the results described in § 32, and the publications in the literature, mentioned in § 28, we find the following:

a) The flattening of the Fundamental can be done, according to Pfnor's results by turning at $H/R_0 = 0{,}2$ approximately at the inside surface, and at $H/R_0 = 1{,}45$, inside surface. According to Simpson it can be done by turning at $H/R_0 = 0{,}2$ and $H/R_0 = 1{,}0$ approximately, both at the inside surfaces. The tuning experiments indicate that the Fundamental may be flattened by turning at the inside surface both at $H/R_0 = 0{,}1$ to 0,7 and at $H/R_0 > 1{,}1$. Although Simpson does not mention the important point for tuning the Fundamental at $H/R_0 = 1{,}2$ to 1,45, inside surface, where none of the other important partials changes appreciably, the results are in agreement.

b) The sharpening of the Fundamental can be accomplished according to Pfnor by turning at $H/R_0 = 0{,}0$ after the sound-bow has been thinned considerably. The place mentioned by Simpson is at

$H/R_0 = 0{,}0$ to $0{,}2$ approximately, inside surface. According to fig. 33 the place is at $H/R_0 = 0{,}00$ to $0{,}05$ inside surface. The latter agrees with Pfnor's results, but not so well with Simpson's.

c) For the flattening of the Nominal Simpson mentions: at $H/R_0 = 0{,}05$ to $0{,}1$ approximately, inside surface, and at $H/R_0 = 0{,}5$ to $1{,}0$ approximately, also at the inside surface. According to fig. 39 it can be done by turning at $H/R_0 = 0{,}1$ to $0{,}6$ inside surface. This does not agree so well.

d) The sharpening of the Nominal can be done, according to Simpson, by turning at $H/R_0 = 0$ to $0{,}05$, inside- and outside surface. According to fig. 39 the Nominal may be sharpened at $H/R_0 = 0$ to $0{,}05$ inside surface, while according to fig. 40 the Nominal cannot be sharpened anywhere on the outside surface. This does not agree quite well.

e) The Minor Third may be flattened according to Simpson: at $H/R_0 = 0{,}5$ to $1{,}0$ inside surface; according to fig. 35 at $H/R_0 = 0{,}1$ to $0{,}6$ inside surface. This does not agree.

f) The Minor Third may be sharpened, according to fig. 35, at $H/R_0 = 0$ to $0{,}03$ approximately, whereas Simpson states that it is impossible. The tuning place mentioned cannot be used in practice however, because of the very narrow limits. It therefore may also very easily have been overlooked by Simpson.

g) The Fifth may be flattened, according to Simpson, at $H/R_0 = 0{,}5$ to $1{,}0$ inside; according to fig. 37 at $H/R_0 = 0{,}2$ to $1{,}3$ inside, with a maximum effect in the region mentioned by Simpson. The agreement between both results is therefore very good.

h) A good agreement is also reached as to the sharpening of the Fifth, which is impossible according to Simpson, as can also be seen in the figs. 37 and 38.

It can generally be concluded that a qualitative agreement exists between Pfnor's and Simpson's results on the one hand, based on experience in the tuning of bells, and the data of the laboratory measurements on the other hand. In practice, however, the quantitative data of the foregoing results will improve the accuracy with which the turning rules can be used in practice.

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Reprinted from pages 97–103 of *Acoustical Measurements on Church Bells and Carillons*, DeGebroeders van Cleef, The Hague, Netherlands, 1949, 148p.

THE DECAY OF SOUND

E. W. Van Heuven

§ 36. Causes of the Decay

After a bell has been struck the amplitudes of the vibrations show an exponential decay with time. The vibrational energy gradually passes into other forms of energy:

a) The material of the bell shows internal damping, the phenomenon for which Zener recently introduced the word "anelasticity" (comp. § 47). It means that in each cycle of the periodic deformations part of the elastical deformation energy passes into heat.

b) The bell sounds, i.e. it radiates acoustical energy into the surrounding air.

We will not consider any damping resulting from the suspension of the bell at the crown, as this place is a nodal point for all modes of vibration that participate as partials in the sound of the bell.

In the following the character of these causes of the decay will be analysed in further detail, with the aid of similarity considerations.

§ 37. The Internal Damping of the Bell Material

We will consider two similarly shaped bells of different sizes made from the same material. In the following all quantities of these bells will be denoted with indices 1 and 2. Let all dimensions of the second bell be a factor γ times those of the first bell:

$$(R_0)_2 = \gamma . (R_0)_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (22).$$

Let the material of both bells be homogeneous and isotropic. If points on the bells are indicated by means of cylindrical coordinates (comp. § 2), then points for which the three coordinates ψ, R/R_0 and H/R_0 have the same values on both bells will be named corresponding points.

Both bells are vibrating sinusoidally with the same mode of vibration. It is further assumed that the vibrational amplitudes are

such as to give sinusoidal elastical tensions with the same amplitude at corresponding points of both bells. This means that the sinusoidal relative dilatations at corresponding points will have the same amplitude as well, so that the amplitude of the vibration at every point of bell nr. 2 will be γ times that at the corresponding point of bell nr. 1.

The masses of corresponding particles of both bells will be as $1 : \gamma^3$. The total elastic forces on both particles have the same ratio as the cross sections perpendicular to the direction of the — identical — elastic tensions, i.e. as $1 : \gamma^2$. As the acceleration equals the force divided by the mass, it follows that the acceleration amplitude of particle 2 is γ^{-1} times that of particle 1.

From the foregoing it is seen that the frequencies of the sinusoidal vibrations of the bells, that have the same mode of vibration, ν_1 resp. ν_2 are related by: $\nu_2 = \gamma^{-1}.\nu_1$, and the times of whole cycles of vibration, T_1 resp. T_2 by:

$$T_2 = \gamma . T_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (23).$$

Further the velocity amplitudes of the movement of corresponding points are equal:

$$v_2 = v_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (24).$$

The energy taken up per second by each mass element dM at any moment can be written as the product of force and velocity at that moment. For corresponding elements of bell nr. 1 and 2 resp. these products are related as $1 : \gamma^2$. Therefore the energies, taken up per cycle (during a time T) are related as $1 : \gamma^3$; these are therefore proportional to the bell volume under similarity conditions.

The internal damping of the bell material means that in each volume element of the material during each cycle of vibration a certain fraction of the total vibrational energy in that volume element is dissipated into heat. If $A_{mech.}$ be the vibrational energy of the bell particle and $\delta A_{mech.}$ the amount of energy, dissipated per cycle in this particle due to the damping of the bell material then the damping angle $\varphi_{mech.}$ of the bell material is defined by:

$$\varphi_{mech.} = \frac{1}{2\pi} \cdot \frac{\delta A_{mech.}}{A_{mech.}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (25).$$

As it is supposed that this fraction is independent of the amplitude of the mechanical tension it will be identical for each volume element of the same bell and the same definition applies, if $\delta A_{mech.}$ and $A_{mech.}$ are taken over the total bell volume.

For different materials $\varphi_{mech.}$ will generally be different. It is

possible that $\varphi_{mech.}$ may be dependent upon the frequency of the vibrations and therefore different numerical values of $\varphi_{mech.}$ may apply to corresponding partials in bells of different sizes, made from the same material, and also to different partials of the same bell.

§ 38. Acoustical Damping

If in the foregoing considerations the vibrating system is taken as consisting of the bell surrounded by an infinitely thick layer of air, and that the second bell in the same way is surrounded by an infinitely ($\gamma . \infty$) thick layer of air, the vibrating systems are again similarly shaped and the results of the similarity principle as to the partial frequencies are the same as without the air.

In the infinitely thick layer of air there will nowhere occur any reflections of vibrational energy. Thus there will be a continuous stream of vibrational energy in the air in an outward direction from the bell. This energy can be considered as a loss of vibrational energy of the bell itself.

At each point on the surface of the bell the velocity amplitude of the bell material and of the air will be the same. Analogous to the considerations of § 37 the ratio between the energies transmitted per cycle into the air at corresponding surface elements of both bells will be as $1 : \gamma^3$. As the corresponding surface elements have the same velocity amplitudes under similarity conditions, the relative decrease per cycle of the total vibrational energy in the bell volume, through radiation of vibrational energy of the total bell surface, will be independent of the size of the bell.

Analogous to the damping angle $\varphi_{mech.}$ of the internal damping of the bell material, an acoustical damping angle $\varphi_{acoust.}$ can be defined as:

$$\varphi_{acoust.} = \frac{1}{2\pi} \cdot \frac{\delta A_{acoust.}}{A_{mech.}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (26),$$

where $\delta A_{acoust.}$ equals the total vibrational energy passed on per cycle through the surface of the bell material into the air.

Generally $\varphi_{acoust.}$ will be different for each mode of vibration of a bell. The foregoing considerations show that for bells of different sizes of the same material $\varphi_{acoust.}$ will be identical for corresponding modes of vibration. This latter statement can also be proved by the following considerations:

The amplitude of the sound at any point in the radiated sound field is the vectorial sum of the sound radiated by all surface elements of the bell. The complex ratio between the sound pressure amplitude

at a point on the surface of the bell, and the velocity amplitude $\hat{v}$ at the same point, is called the acoustic unit-area impedance $\bar{z}$. If the surface element were part of an infinite surface, vibrating homogeneously as a piston, $\bar{z}$ is real and equals the characteristic acoustical impedance $\rho'.c'$ of the air (ρ' = the air density and c' is the velocity of sound in the air). For a surface element of a bell $\bar{z}$ will be P times $\rho'.c'$. The — generally complex — factor P comprises the baffle-effect of the remaining parts of the bell which partly screen the omnidirectional sound radiation, and the addition to the sound pressure $\hat{p}$ at the considered point due to the acoustical radiation of the remaining bell surface; therefore P will generally be different at different points on a bell. As the wavelength in the air of tones of corresponding modes of vibration of similar bells as well as the distances to other corresponding points on the bell are all proportional to the ratio γ between the dimensions of the bells, P, and thus the acoustical unit-area impedance will be equal at corresponding points on bells of different sizes, vibrating in corresponding modes. This applies only to cases where the material from which the bell is made, remains the same. For similar bells with equal velocity amplitude at corresponding points (thus the amplitude of vibration proportional to γ) the total radiated acoustical energy per second will therefore be proportional to γ^2, and the radiated energy per cycle, $\delta A_{acoust.}$, to γ^3, so to the volume of the bell, and thus to the total amount of vibrational energy $A_{mech.}$ of the bell. Therefore $\varphi_{acoust.} = \delta A_{acoust.}/A_{mech.}$ is independent of γ for corresponding partials.

For similar bells of different materials there will exist a different ratio between the dimensions of the bell and the wavelength in the air of the sound of corresponding partials. Therefore P will not be identical at corresponding points, and different values of $\varphi_{acoust.}$ will apply to corresponding partials if the materials of the bells are different.

It is also possible to alter the mechanical behaviour of the surrounding air, for instance by varying the atmospheric pressure p_o. In this case the similarity of the vibrating system consisting of the bell together with the infinite layer of surrounding air is disturbed, and the partial frequencies of the bell will be altered. The acoustical unit-area impedance at points on the bell surface will be modified, both through changes of the factors P and through differences of the characteristic impedance $\rho'.c'$ of the air. As to the latter it is seen that ρ' is proportional to p_o, whilst c', defined according to Laplace's formula by:

$$(c')^2 = \frac{dp_o}{d\rho'} = \varkappa \cdot \left(\frac{p_o}{\rho'}\right)_{\text{at atmospheric pressure}} \quad . \; . \; . \; (27),$$

is independent of the atmospheric pressure p_0. ($\varkappa$ = the adiabatic gas constant).

The influence of p_0 on P is related with the change in the partial frequencies. As c′ is independent of p_0, the wavelength of the sound in the air changes relatively to the same amount as do the partial frequencies. As ρ' is very small in comparison to ρ, the changes in the partial frequencies and also the changes of the factor P due to variation of the atmospheric pressure will be negligible in comparison with the linear dependence on p_0 of the characteristic impedance $\rho'.c'$ of the air. Therefore the radiated acoustical energy per cycle, and thus $\varphi_{acoust.}$ will be in good approximation proportional to p_0:

$$\varphi_{acoust.} = \text{const. } p_0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (28).$$

§ 39. The Total Damping

If both $\varphi_{mech.}$ and $\varphi_{acoust.}$ are small in comparison with unity, both dampings are additional and the total decay of the bell is given by:

$$\varphi_{total} = \varphi_{mech.} + \varphi_{acoust.}.$$

The total vibrational energy of the bell, $A_{mech.}$ diminishes during each cycle with δA, where

$$\frac{\delta A}{A_{mech.}} = \frac{\delta A_{mech.} + \delta A_{acoust.}}{A_{mech.}} = 2\pi\,(\varphi_{mech.} + \varphi_{acoust.}) = \\ = 2\pi.\varphi_{total} \quad . \; . \; (29).$$

This corresponds to an exponential decay according to

$$A_{mech.} = A_0 \exp\left(-2\pi.\varphi_{total} \cdot \frac{t}{T}\right) \quad . \; . \; . \; . \; (30),$$

where $T = 1/\nu$ = the period of the vibration.

In the Dutch bell foundries the practice of determining the decay has hitherto been rather simple: the bell is struck, at the best with a standardised hammer, and the time is determined during which the sound of the bell is audible after the stroke. It is clearly seen that the initial sound intensity depends upon the impulse of the stroke and on the place of the observer and in addition the threshold value where the bell becomes inaudible is rather dependent upon the masking effect of the random noise wich is always present, even at "quiet" moments in a factory, or in a tower. It is therefore not surprising that the values of the "reverberation time" of a bell, measured in this way can differ by as much as 100%. Even in perfectly quiet surroundings and for a standardised impulse of

striking and place of observation, a correction should be made for the frequency dependence of the threshold of hearing[1]).

For the decay measurements, to be described hereafter, a reverberation time τ of the bell was defined as the time for the sound intensity at any point to decrease to one millionth of the initial value after the stroke. This definition was chosen for two reasons:

a) A difference between the initial and the final sound levels of 60 dB (one millionth) will not be very different from that occurring under practical circumstances. So the given values for τ will be plausible to bell founders, used to the simple method of measuring reverberation times.

b) The same definition is used in architectural acoustics.

From equation (30), with $A_{mech.} = 10^{-6} A_o$ at $t = \tau$ it follows that:

$$\frac{\tau}{T} = \nu.\tau = \frac{2{,}20}{\varphi_{total}} \quad . \; . \; . \; . \; . \; . \; . \; . \; (31).$$

§ 40. Other Quantities Used for Denoting Numerical Damping

In the literature on damping of materials, other quantities are used. One of these for instance is t_H, the time for the vibrational amplitude to be halved.

In this case in equation (30) $A_{mech.} = 0{,}25\ A_o$ at $t = t_H$, so:

$$t_H = \frac{{}^{10}\log 0{,}25}{{}^{10}\log 10^{-6}} \; . \; \tau = 0{,}10 . \tau \quad . \; . \; . \; . \; . \; . \; (32).$$

The ratio between two successive amplitudes with the same sign is given according to equation (30), with $t = T$, by:

$$\frac{\hat{a}_1}{\hat{a}_2} = \sqrt{\frac{A_1}{A_2}} = \exp\left(-\pi . \varphi_{total}\right) = \exp\left(-\delta\right) \quad . \; . \; . \; . \; . \; (33).$$

This quantity δ is called the logarithmic decrement. The relation between τ and δ is as:

$$\delta = \frac{6{,}9}{\tau . \nu} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (34).$$

Sometimes, especially where the movement of a vibrating mechanical system under the action of a sinusoidal force is described, the damping is denoted in the form of the "width of the resonance curve" (compare § 11), or of the "resonance rise of the amplitude",

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[1]) Fletcher and Munson: Jl. Acoust. Soc. Am. 5, 1933, 82.

Q. These quantities are also used in radio technics in connection to tuned circuits. Computation of the amplitude $\hat{a}$ of the system[1]) if the amplitude of the sinusoidal working force is kept at a constant value, and the frequency of it is varied near the resonance frequency, gives in case the damping angle φ_{total} is small compared to unity:

$$\hat{a} = a_{stat.} \cdot \frac{1}{\left(1 - \frac{f^2}{\nu^2}\right) + j \cdot \varphi_{total}} \quad \ldots\ldots\ldots \quad (35),$$

where $a_{stat.}$ = the amplitude or displacement, if a static force is applied equal to the peak value of the sinusoidal force; f = the frequency of the acting force; ν = the resonance frequency of the system and $j = \sqrt{(-1)}$.

At resonance ($f = \nu$), $\hat{a}$ is a maximum. In this case, in analogy to the electrical tuned circuit, the resonance rise of the amplitude, Q, is given by:

$$Q = \frac{\hat{a}_{res.}}{a_{stat.}} = \frac{1}{\varphi_{total}} = \frac{\nu . \tau .}{2{,}20} \quad \ldots\ldots\ldots \quad (36).$$

The width of the resonance curve is given by the ratio

$$\frac{f_2 - f_1}{\nu}$$

where f_2 and f_1 are the two frequencies above and below the resonance frequency ν, where the amplitude $\hat{a}$ for a constant amplitude of the exciting force is:

$$\hat{a} = \frac{1}{\sqrt{2}} \cdot \hat{a}_{res.} \cdot$$

This is the case if

$$\left(1 - \frac{f_1^2}{\nu^2}\right) = -\left(1 - \frac{f_2^2}{\nu^2}\right) = \varphi_{total},$$

so that, if $f_1 + f_2 = 2 . \nu$ approximately,

$$\frac{f_2 - f_1}{\nu} = \varphi_{total} \quad \ldots\ldots\ldots\ldots \quad (37).$$

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[1]) Terman: Radio Engineering, l.c.

[*Editor's Note:* Material has been omitted at this point.]

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THE SYSTEM OF THE HEMONY-CARILLONS TUNING

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Summary

A tone-recording of three HEMONY carillons (namely: Delft, Kampen and Middelstum) has been made with electronic measuring instruments. The object was to find out which sort of tuning the HEMONYS used. The results show that the HEMONYS used meantone tuning.

Sommaire

On a enregistré électroniquement la tonalité de trois carillons HEMONY (à savoir: ceux de Delft, Kampen et Middelstum). Il s'agissait de déterminer quel tempérament a été utilisé. Les résultats montrent que c'est celui du son moyen.

Zusammenfassung

Mit elektrischen Meßinstrumenten wurde eine Klangaufnahme von drei HEMONY-Glockenspielen (nämlich Delft, Kampen und Middelstum) gemacht, mit dem Ziel herauszufinden, welche Stimmung die HEMONYS benutzten. Die Ergebnisse zeigten, daß die HEMONYS die Mitteltonstimmung benutzten.

1. Introduction

The carillons of François and Pierre HEMONY which are so often found in the Netherlands have always been a point of interest to historians, bell-experts and acoustic-scientists. In spite of this the problem has never been considered: what type of tuning the bros. HEMONY used, just-tempered, meantone or another sort of tuning? It is generally assumed that their carillons were mostly under just-tempered tuning. Strange, for during the 17th century meantone tuning was often applied. For correct investigation of this matter it was necessary to make an accurate tone recording of HEMONY carillons.

This has lately been done by the Nederlandse Klokkengieterij Eijsbouts-Lips, Asten, Holland to three carillons, namely Delft, Kampen and Middelstum, using electronic measuring instruments. The note diagrams Figs. 1 and 2 give the HEMONY-bell-recordings of Kampen and Delft. Bells which have been cast later are not quoted here. On the lower side of the horizontal axis the semitones are given corresponding with the equitempered tuning taking the base as $a' = 435$. On the vertical axis the deviation for the hum note (large circles), prime (small circles) and octave (dots) (the three important partials of the bell) is stated in "cents". An accurately tuned bell is thus one for which the large circle, the small circle and the dot coincide. This is for example the case for the f′ an a′ of the Kampen carillon. If the mutual deviatio is not larger than 10 cents i. e. the circles in th graph are still in contact, we can still talk abou a well-tuned bell. This is usually the case. On th upper side of the horizontal axis (top of Fig.), th notes are stated as they appear on the keyboard In the Delft carillon cis-28 cents is labelled c (Fig. 1 This is normal for the HEMONY's; they tune higher than $a' = 435$. In the Kampen carillo gis-14 cents is called c (Fig. 2). Gis-14 cents is th normal g of HEMONY. Since it is called c the car illon is said to be transposed from g to c. Trans position is common practice for economical reason since in that way the total mass, and therefore th cost of the carillon, is reduced. Considering th graphs carefully, the first thing we notice is tha the semitones are not the same, for, then, all th bells would lie on a horizontal line. In what systen of tuning are these bells made then? Firstly w must state the exact pitch of the sound. It is know now that the pitch of the bell is defined by th striking note.

It gives us the first impression of the tone quality but can only be heard for a very short time The octave of a well-tuned bell is always a clea octave higher than the striking note, and, since th octave can be heard much longer than the striking

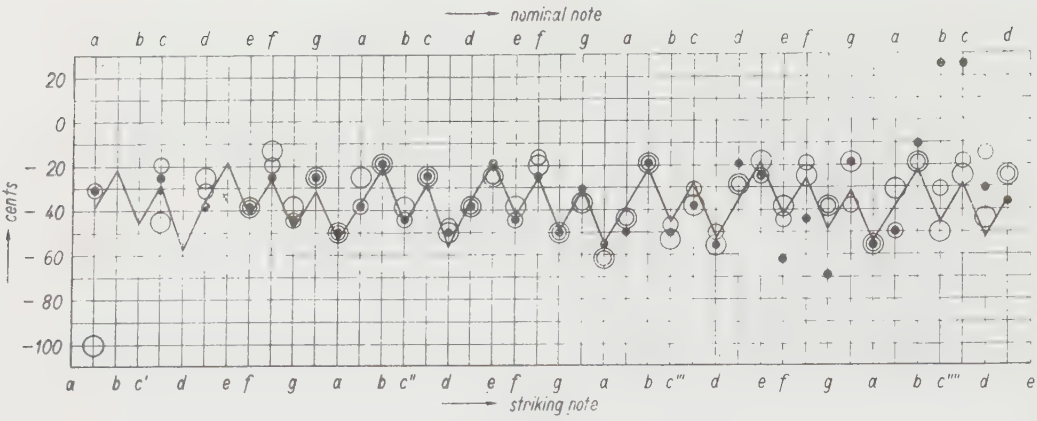

Fig. 1. Carillon Delft.

note, the octave is a very suitable partial for measuring the pitch of a bell. The pitch will be defined, therefore, as one octave below the partial called "octave". The prime is less suitable to serve our purpose since only in bells with an extreme preciseness of tuning is the striking note of the same pitch as the prime.

2. The Delft carillon

As Fig. 1 shows us, this carillon belongs to one of the superior categories. François Hemony founded and tuned it in 1661. Practically for all the bells, the hum note, prime and octave lie within the limited tolerance.

Only the smallest bells show clear deviations in the octaves originating in the difficulties, which Hemony had while tuning these small carillon-bells. It is advisable to omit the highest octave from our investigations, noting only the bells from cis′ till cis‴. The octaves which are proportional to the striking note are given in Table I, second and third column. Errors that have inevitably been made at the tuning and the tone-recording are

Table I

Carillon Delft

Interval	cis′-cis″	cis″-c‴s	average	meantone tuning base cis-25	meantone tuning base cis-28
prime	- 25	25	25	25	28
semitone		— 50	— 50	— 49	— 52
second	— 38	— 38	— 38	— 32	— 35
minor third		— 19	- 19	— 15	— 18
third	— 38	— 44	— 41	— 39	— 42
fourth	— 25	— 25	— 25	— 22	— 25
augm. fourth	— 44	— 50	— 47	— 45	— 48
fifth	— 25	— 31	— 28	— 28	— 31
augm. fifth	— 50	— 56	— 53	— 52	— 55
sixth	— 38	— 50	— 44	— 35	— 38
dimin. seventh	— 19	— 19	— 19	— 18	— 21
seventh	— 44	— 50	— 47	— 42	— 45

reduced by averaging these two octaves (fourth column). Applying the just tempered tuning system, it is clearly shown that the carillon is indeed not just tempered, for the second should be 204, the minor third 316 cents higher than the prime. From the table we see that this is definitely not the case. Let us now try meantone tuning. This means that the fifths from F till a′ are reduced to such a state, that f′—a′ form a clear third; so the fifths only become 5 cents too small. For the greater part the third remains just tempered. In Table II the intervals of the meantone tuning are indicated as deviations (in cents) in relation to the equi-tempered tuning. Comparing the average recording of the Delft carillon (Table I, 4) with the meantone tuning, begin-

Table II

The meantone tuning compared with the equi-tempered tuning

Interval	Deviations in cents
prime	0
semitone	— 24
second	— 7
minor third	+ 10
third	— 14
fourth	+ 3
augm. fourth	— 20
fifth	— 3
augm. fifth	— 27
sixth	— 10
dimin. seventh	+ 7
seventh	— 17

ning at cis-25 cents (Table I, 5) it is obvious that we can surely regard this carillon as made in this systém of tuning. It is not absolutely positive however that the base is cis-25 cents, for we have to consider an average, i. e. the number 25 should be adopted so as to make the sum of the deviations between column 4 ("average") and column 5 ("mean-

tone tuning") zero. Starting from this, we can calculate the correct base, only of course if we can assume that the above conclusion is correct. This base then must be cis-28 cents. From this we can again make a table (Table I, 6) that shows the arrangement of the bells after an accurate tuning by HEMONY, still assuming that our theory is right. Comparing with the reality, the mean deviation seems to be ± 4 cents, including the errors made by HEMONY, those made by us in recording the tone and later alterations in precision due to weather (erosion etc.). Bearing this in mind, ± 4 cents is practically negligible, so giving us reason to think that François HEMONY has made the Delft carillon in meantone tuning. The drawn line in Fig. 1 gives this system of tuning as Table I, 6 does. Applying the above to the Kampen carillon, we get practically the same result.

3. The Kampen carillon

It would be useful to go through some of this carillon's history. Kampen, the town where the most well-known bellfounder prior to HEMONY GERRYT VAN WOU, lived and worked during the end of the 15th century, possessed various heavy bells made by the latter founder. François HEMONY cast and tuned the carillon in 1659 and in 1662; VAN WOU's bells (gis, bes, c′ and g′) have been incorporated in the carillon. Naturally the pitch was not right, this is why HEMONY tuned the bells. He did the same with the es′, cast by WEGEWAERT in 1627. The tuning did not turn out as he had expected it to, which can also be seen in Fig. 2. In the carillon however they are satisfactory, for the octaves (proportional to the striking note) are correct. The bell g‴ was recast later. Like the Delft carillon only the bells from gis till gis‴ are considered. From this, a corresponding table, III, can be formed. It is quite clear now that this carillon, which has an average base gis-14 cents, is made in the meantone tuning. The greatest deviation appears at the prime of the third octave, which is 11 cents too low. The average deviation is ± 5 cents, a factor which we can take to be correct. The line drawn in Fig. 2 indicates the system of the meantone tuning.

Table III
Carillon Kampen

Interval	gis-gis′	gis′-gis″	gis″-gis‴	average	meantone tuning base gis-14
prime	— 12	— 6	— 25	— 14	— 14
semitone		— 37	— 40	— 38	— 38
second	— 25	— 25	— 28	— 26	— 21
minor third		— 12	0	— 6	— 4
third	— 31	— 31	— 31	— 31	— 28
fourth	0	— 12	— 19	— 10	— 11
augm. fourth		— 37	— 38	— 38	— 34
fifth	— 16	— 19	— 19	— 18	— 17
augm. fifth		— 34	— 34	— 34	— 41
sixth	— 25	— 25	— 31	— 27	— 24
dimin. seventh	0	— 9	— 9	— 6	— 7
seventh	— 28	— 34	— 31	— 31	— 31

4. The Middelstum carillon

Like Kampen and Delft, this carillon is also made in the meantone tuning. Calculations are given in Table IV. Searching for the average base, it was proved to be cis-3 cents. The greatest deviation is 9 cents and the average ± 4 cents. This justifies the conclusion that HEMONY made these bells

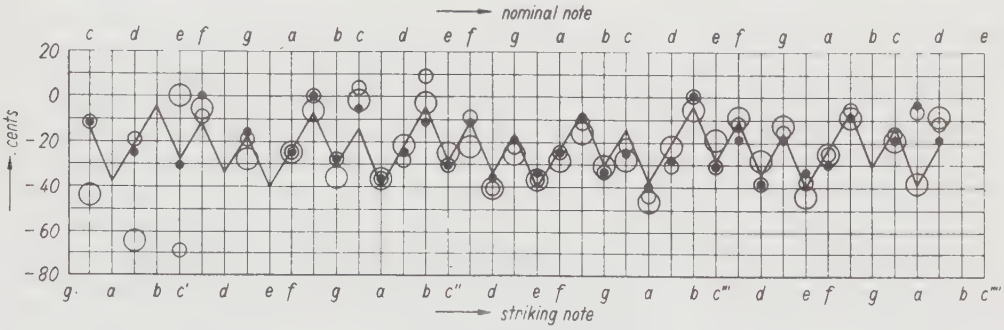

Fig. 2. Carillon Kampen

Table IV
Carillon Middelstum

Interval	cis″-cis‴	cis‴-cis⁗	average	meantone tuning base cis-3
prime	0	— 6	— 3	— 3
semitone		— 22	— 22	— 27
second	— 3	— 6	— 5	— 10
minor third		+ 6	+ 6	+ 7
third	— 15	— 19	— 17	— 17
fourth	— 6	— 9	— 7	0
augm. fourth	— 31	— 28	— 29	— 23
fifth	— 3	— 12	— 7	— 6
augm. fifth	— 31	— 25	— 28	— 30
sixth	— 12	— 9	— 11	— 13
dimin. seventh	0	+ 6	+ 3	+ 4
seventh	— 22	— 22	— 22	— 20

in the meantone tuning too. What does the above conclusion mean to our ears? From the early beginning, when the modern carillons were cast, they were made in equi-tempered tuning, which is now generally used. The popular opinion is that these carillons, even though excellent, have a different character and lack the charm of the HEMONY carillons. This is natural, as the tempered tuning enforced a levelling influence.

Could this be the key of the HEMONY secret? How about the practice? This has spoken for itself, for HEMONY carillons are still used very often and surely in more than one key. The meantone tuning permits many keys indeed; the carillon can be played in six keys in major as well as in minor, a range that is generally not surpassed, at least not in carillon music.

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11

CONTEMPORARY DUTCH BELL-FOUNDING ART

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HISTORICAL ORIENTATION[1]

The bell, a very primitive, strongly symbolistic, percussion instrument, originated probably in the second millenium before Christ in the steppe region of central Asia. Excavations as well as drawings have shown that the dimensions of such bells were very small (10 cm diameter); moreover, their sound was highly nonmusical

Naturally, our concept of a bell of that time, if it really merits the name bell, is not very clear. On the other hand, using our present-day values, we can show very clearly what a stark contrast there is between this primitive beginning and a contemporary west European bell. The latter is melodious and can well serve a musical function. However, before the bell reached this stage, which happened for the first time in seventeenth century Netherlands, there had been a long and laborious development.

Without any clear proof, most publications on bells put the origin of the west European bell in ancient China.[2] This is done presumably because the bronze culture, the first condition for founding a bell, was developed in China long before Christ. Moreover, a series of bells have been found from that time that were based on the Chinese scale and must have served as a tuning standard for other musical instruments.

Nevertheless, there are several reasons to doubt that the art of bell founding was introduced into Europe from China. Not only is the Chinese bell with its cylindrical shape of a totally different profile than the oldest European bell, but also in Europe a different and better bronze alloy had been used from the beginning. Above all, the development of the European founding art started from a very primitive stage, which makes any comparison with the highly refined bell art of China at the same time seem rather imaginary.

More plausible seems to be the hypothesis that bell art in the second millenium B.C. spread from the central Asiatic steppes both east to China and to the south. Equestrian tribes from the north also played a decisive role in penetrating into the near East. Even later, the Celts, during their migrations west, did the same thing when they

provided their horses and chariots with small bells, which had the function of frightening unfavorable gods during battle. This seems to be the origin of the peaceful cowbells of the Swiss Alps.

The question is, were the conditions in ancient China before Christ really much more favorable for a rich bell art? Probably they were. We cannot rely on the fact that the Chinese possessed the undoubtedly highly developed bronze founding technique that found its expression in innumerable ritual vases. If the bell also found its place in this rich bronze culture, there had to be other factors present as well. We find these present in the curious fact that, in the entire Orient, musical instruments were much more varied than in the countries around the Mediterranean Sea.

The Far East was, and still is, that part of the world where idiophonic instruments are of prime importance. We only have to think of the gamelan, which is also known in The Netherlands, for support of this hypothesis.[3] It is therefore not surprising that at an early stage in China, bells were jointed together in series for the purpose of sounding melodies. We can continue this line further by observing that all idiophonic instruments, whether the iron or brass rod of a metallophone, or a brass gong; or even a bell, without special precautions will contain non-harmonic overtones and will thus be unsuitable for music that has a definite structure, that is, polyphony. Is this the reason why in the Far East music has remained primarily melodic? Do we have to assume that the earliest musicians there unconsciously understood the impossibility of sounding instruments together harmonically which themselves had no harmonic structure? This all seems more plausible if we also consider the contrast with West European music, which came to be as we understand from the Near East. For there, even if one deviated from the human voice with its harmonic structure, one chose predominantly stringed instruments that also possessed harmonic overtones.

Making a big jump in time, to the end of the Middle Ages, when polyphony started to clearly form and instrumental music obtained an independent value, primarily harmonic instruments still determined the musical culture. On a theoretical level, the discussion of the division of the octave into 12 tones demonstrates this situation very clearly.[4] The great problem was the question of how one could make this division without creating too many disturbing deviations from the harmonic series. If strings and organ pipes were not mutually tuned harmonically, it would be necessary to minimize the expected beats for polyphonic consonance. The tuning principle that remained valid for the longest time, until the eighteenth century, was the so-called mean-tone tuning, in which one minimized the beats in the

most usual chords. The consequence was an imaginary fifth $g^{\sharp}$–$e^{\flat}$ in which the beats were intolerable and resembled the howling of wolves, thus the wolf-fifth.

From this contrast between the polyphonic music of west Europe with harmonic overtones and the homophonic Chinese music with nonharmonic overtones, one can expect that if anyone were to develop a harmonic bell or carillon, it should have been done in Europe. For it is clear that a carillon of bells without harmonic overtones does not properly belong in Europe, at least if one were to try playing polyphonic music on bells, or with other instruments.

In contrast with the single pealing bell, in which one could accept the nonharmonic overtone structure, is the carillon where one is faced with the disagreeable consequences of a nonharmonic overtone structure. It is not incidental that this problem became more and more urgent as polyphony developed.

From all of this one can get the impression that the problem of tuning bells was especially one of internal tone structure. There were additional complications, however, in that one also had to tune the strike note of one bell to another, much more complicated than the mutual tuning of the strings of a piano or the pipes of an organ, although the problems are similar. This problem of tuning the internal harmonic structure for each bell (the ratios of the eigenfrequencies) as well as external tuning from bell to bell (the correct ratio between the principal tones) was already felt in the sixteenth century but only resolved in the seventeenth century. As a chronicle writer of the sixteenth century has indicated, the carillon is typically a development of the Low Countries, where one is impressed by the beauty and wealth of the churches and by the size and beautiful sound of the bells, on which one can play songs as on guitars.

Can one discern a clear evolution starting with the steppe region of Central Asia a thousand years before Christ to the end of the Middle Ages in the Low Countries? Not at all. For in spite of the fact that all civilizations around the Mediterranean have had small bells since antiquity, the bells never emerged beyond the beginning stage, and their function in daily life was limited to simple signal instruments or the clearly sacred function of frightening demons. Any significance of the bell as a musical instrument was absent. Illustrating this is, for example, the Old Testament story about the high priest who enters the holy of holies. The prescripts required that little bells should hang from the border of his robe, not to enhance the spectacle of his entrance, but to frighten evil so that he could enter in a state of purification.

If we really want to look for the true origin of present-day bell art,

we only have to go back a thousand years in history to the time when missionaries from Anglo-Saxon countries brought in bells of Celtic origin. Not long after, ca. 1000 A.D., cymbal playing originated in several Benedictine monasteries; it is the most ancient precursor of the present-day carillon. Eight or more little bells tuned to the diatonic scale were joined together. These were used for musical instruction, clockworks of later dates, and as an additional organ register.[5]

The method used by the Medieval monks to design this tone sequence was mathematically beautiful but fundamentally incorrect. They reasoned, for example, that an octave interval between two bells (frequency ratio of 2:1) could be realized by having the weights of the bells also in the ratio of 2:1; this meant that the octave bell was to be roughly twice as thick.[6] One can take the frequency to be linearly proportional to wall thickness. This rule, by Rayleigh, is only valid for hypothetically thin-walled shells.[7] In practice one really cannot obtain a frequency doubling, but less than that, so that the octave bell must have been considerably low. But perhaps we should be thankful that the Medieval people applied this simple theory because, completely in accord with the spirit of that time, they did not dare doubt their arithmetic rule, but suspected that the deviations were caused by mistakes in the founding. This was a reason for them to start tuning the little bells. Thus the origin of bell tuning. One may have noticed the cymbals were considered only as far as their external tuning and that one only paid attention to the mutual tuning of the little bells. There was no objection to this, because one can assume that these cymbals only produced simple homophonic music. Nevertheless, even if one had some idea of the frequency ratios of overtones, one would have been in an even more hopeless situation. For the discovery that internal tone structure is highly dependent on the profile came much later. The profile, however strange this may seem, was not controlled until very late in the Middle Ages.

During the entire Middle Ages, until the beginning of the fourteenth century, the bell was not formed with the help of a template, but was almost entirely made free hand.[8] Only a few main dimensions had been fixed a priori. Particularly uncertain were the various curves in the bell model that are so important for internal tuning. A systematic profile investigation had never been made, because two bell models were never the same. In fact, the bell founder of that time was mainly impressed with relating the main dimensions of various bells by simple ratios. It is clear that this cannot have any significance for the sound of any particular bell.

However, from the beginning of the fourteenth century, templates were used in bell founding, and we see some great changes in bell

science. This happened in particular at the end of the fourteenth century when the principle of dynamic similarity was discovered experimentally. This principle states that corresponding eigenfrequencies with corresponding nodes, cast in the same bronze with the same structure, are inversely proportional to the dimensions. An octave bell could therefore be realized by having *all* dimensions a factor of two smaller. Nevertheless, this does not mean that from that time the strike tones of bells were correct relative to each other. For in spite of the fact that the profile could be designed very accurately, and one could scale the profiles using pythagorean numbers, even to this day uncertain factors during forming and casting make the cast bell something different than what was designed on paper. The consequence was that correction of the bell profile after forming remained necessary, and one therefore had to tune the bell. Since the middle of the sixteenth century this has been done with most carillons.

Adoption of the template meant that the fundamental conditions for determining the relationship between bell profile and overtone structure had been found. One could now cast various profiles and try to relate these to the variation of the relationship between the overtones of a bell. Without a template this would have been impossible.

The introduction of the template was not the final development because one also had to measure the frequencies. This was only done a couple of centuries later. The development of the measuring technique is due to the famous seventeenth century bell founders, brothers Francois and Pieter Hemony.[9] Our conclusion is that, since the end of the Middle Ages until the beginning of the seventeenth century, attempts to give bells internal harmonic structure were more directed than earlier times but were never successful because they lacked a good measurement technique for the overtones. This will be discussed in the next section.

FREQUENCY MEASUREMENTS IN BELL FOUNDING

Once the bell was accepted as a musical instrument in western Europe at the end of the Middle Ages, an appropriate method for measuring the sound of a bell was sought. Of course the oldest method was to compare the pitch with that of another musical instrument such as the organ. In the sixteenth century one finds that occasionally the bell founder was sent an organ pipe with the request to make a bell that had the same pitch.[10] Clearly this method could be successful only for external tuning; one could only compare the general impression of the bell tone with that of the organ pipe. Adequate bell tuning at that time meant that one tried thinning the wall to make the

impression of the bell sound the same as the organ pipe. Really fruitful bell-sound analysis could begin only by analyzing the sound of the bell. This only would make it possible to draw conclusions about the internal structure of the bell sound and relate this to the bell profile.

The impulse to do this was given by the impossibility of playing polyphonic harmonized music on bells. Jacob van Eijck, famous musician and carilloneur in the beginning of the seventeenth century, pointed this out repeatedly, particularly to Bacon, Descartes, and Mersenne. Certainly Van Eijck's greatest accomplishment is that he developed a method for measuring the bell partials individually. He would demonstrate the method, by whistling at the pitch of one of the eigenfrequencies of wine glasses, so that the wine glass would start resonating. This instructive experiment was repeated with bells. It is known that he was able to bring some of the eigenfrequencies of the bells of the cathedral at Utrecht into resonance by whistling. One should therefore not be surprised that, as a result of this discovery, Van Eijck was able to accurately formulate objections against the nonharmonic bell of his time. For he could sound each overtone of a bell separately, one by one, for the purpose of comparison of the overtones to acceptable musical intervals.[11]

Van Eijck's ingenious method made heavy demands on one's musical hearing, therefore his discovery was not applied very much in bell founding. A more appropriate variation had to appear. This variation, first used by the Hemony brothers in the first half of the seventeenth century, was also based on the resonance phenomenon but in the reverse sequence. The Hemonys had a number of sounding rods of iron or brass, which by means of an organ or a harpsichord had been tuned accurately to the desired frequencies of a bell's partials. A correctly tuned bell, when struck, would then bring the rods into resonance. Because of the great intensity of the sounding bell, it was not always easy to tell if the rod did come into resonance. This was later refined by the use of sand; the resonating rod would leave the sand around it lying at the nodes and jumping away from the anti-nodes. This formed the simplest vibration patterns of Chladni, long before he described them.[12]

Working with these fascinating resonating rods remained a custom until the nineteenth century. Though this method was correct, the difficulty was that the rods could not be tuned continuously. Thus one had only a series of fixed frequency references. This meant that one had to compare an untuned bell overtone with the rod by listening, a method completely fine for good musical ears, but with the potential for error due to fatigue.

In the nineteenth century an instrument was developed with continuously variable frequency. This was done by fastening weights on the prongs of a tuning fork. By shifting these weights the frequency of the tuning fork could be changed. By calibrating the tuning fork frequency with weight position, one could set any desirable frequency. Also, bell partials could be brought into resonance by striking the tuning fork and placing its stem on the surface of the bell.

Though this technique was used in many bell foundries, since 1945 an electronic device has come into use that uses an electronic oscillator and transducer. The oscillator frequency is settable to an accuracy of 0.2% or about 3 cents. The coil and driving amplitude can be set to put the bell into continuous resonance at various intensities.

TUNING THE BELL

When Pieter Hemony died in 1680, his niece Anna Margaretha Hemony wrote to a friend that she doubted if her Uncle Pieter had really taught the "real secret" to his successor Mammes Fremy. Almost ten years later the Antwerp bell founder Melchior de Haze sighed unhappily that Pieter probably took the secret with him to the grave. In both cases they were talking about the tuning of bells.

In spite of all the romantic theories, in the present time the tuning technique cannot be called a secret. In the nineteenth century A. B. Simpson, an English cleric, published some interesting data on this. The latest revelation, based on his own investigations, was prepared in 1949 by E. W. Van Heuven of Delft, in his dissertation on "Acoustical Measurements on Church Bells and Carillons." Without any romanticizing elements, it describes the first elements of tuning.

One could define bell tuning as changing a bell's profile by thinning its wall so that the lowest five overtones attain harmonic proportions. It might seem much simpler to correct the bell-wall thickness beforehand in casting, but this is not possible. During forming and pouring there are so many uncalculable factors such as mold shrinkage, that it is practically impossible to give the bell exactly the shape that theory prescribes. It is more practical to cast the bell thicker than necessary, as was done in the beginning of the seventeenth century. The process of tuning would then turn the bell down to the correct thickness.

Although in theory it should be possible to give the cast bell its correct form by measuring the wall thickness in various places and then calculating how much should be turned down, this also is impossible. It has always been the practice to measure the overtone

structure during thinning, and use this as a guide to determining which places have to be thinned.

Because of the different vibration pattern for each bell overtone, it is clear that thinning the bell wall at any particular place will not give the same frequency change for all overtones. This means that for each bell overtone one has to know the frequency change as a function of wall thinning along the profile of the bell.

Such functions for tuning are only used for the inside surface of the bell, because the outside of the bell, according to tradition, is provided with ornamentations and inscriptions. One wants to tune a bell by turning down the inside, not the outside, which is reserved for ornamentation.

What is the ideal tone structure? Can this be imitated by other musical instruments or does the bell have its own ideal?

As noted before, no percussion instrument, including the bell, has a harmonic overtone structure "by nature." This means that the eigenfrequencies cannot be expressed as arithmetically simple ratios. This is, as one knows, very different from a strong or an organ pipe whose eigenfrequencies can be expressed as 1:2:3:4:5, and so on. However, if we take any bell from the time when little attention was given to the overtone structure, one finds for example lowest eigenfrequencies in the ratios of 1.00: 2.07: 2.24: 3.09: 3.76 or 1.00: 1.78: 2.61: 2.76: 4.41. In contrast with the completely elastic string or vibrating air column, then, is the bell or other percussion instrument where certain provisions have to be made to make this arithmetically complicated series simpler and more acceptable for musical hearing.

A simple comparison between the example above and the harmonic tone series shows that differences appear not only in their anharmonic and harmonic character, but also that the partials for the bell are much more condensed. In the case of the stringed instrument, the fifth partial is five times the fundamental, while for the bell the fifth partial is only about four times the lowest partial. Therefore, it is possible to see that it is almost impossible to transform the overtone series of a bell to that of a string. One should therefore be content with obtaining a harmonic relation of a higher order where only certain ratios are lacking.

This latter idea turns out to be possible. After the introduction of the template in the beginning of the fourteenth century, bell science gradually developed and made it possible in the beginning of the seventeenth century to give the lowest eigenfrequencies of the bell the ratios 5:10:12:15:20. It is interesting that these can also be expressed as $1/12$: $1/6$: $1/5$: $1/4$: $1/3$—a remarkable fact that indicates that the minor third is dominant in the bell's sound.

It is natural to understand that the bell founder, and even more the musician, do not work with frequencies or ratios of frequencies. They prefer to express the frequencies as notes to show directly the musical acceptability of certain bell sounds. In this case the tone series can be expressed as $c\text{-}c^1\text{-}e^{b1}\text{-}g^1\text{-}c^2$, ($C_3\text{-}C_4\text{-}E^b_4\text{-}G_4\text{-}C_5$) in which the hum tone is c in this example. In bell founding these partials have acquired their own names of hum tone, prime, tierce, quint, and nominal.

One could remark that this harmonic series for the bell is realized only for the lowest five eigenfrequencies, while for the string or organ pipe the harmonic series is maintained over a considerable greater number of overtones. This is correct, but in connection with other bell-sound factors, it is not as important as it seems. In the first place, higher overtones have a lesser volume and are of shorter duration. Furthermore, a large number of higher partials have a node at the place where the ball is struck; these overtones will be considerably weakened. One can remark that in the development of the bell, the aim has been to localize the striking point, or conversely, to pay attention to those partials that are the loudest. Finally, one can say that those higher partials are heard best which have fairly exact harmonic values and, as will be shown, are strongly anchored in the structure of the lowest five partials. Indeed, with very fine tuning, the higher strong partials join themselves to some degree into the harmonic scheme.

The bell founder will first aim at realizing the first five partials. Let us assume that the profile is approximately correct and that the wall thickness is a little greater than necessary (about ½ mm too thick). Then he tries to reach his bell ideal by tuning.

Take for example the tone structure of a freshly cast bell given in Table 1. If we assume that the bell in this example must be tuned to a standard pitch, then each of the partials must be tuned down. If local wall thinnings were to affect each partial equally, then it would be impossible to tune the bell. To tune, one has to have accurate

	bes	bes^1	des^2	f^2	bes^2
ongestemd	44	48	34	56	26
c - t	24	40	20	32	14
d - t	6	34	10	13	6
d - f	4	29	8	10	2
e - g	0	27	6	8	0
q - s	0	11	5	4	0
s - t	0	0	4	1	0

Table 1. The tuning process for a bell bes^1 (B^b_4), diameter 860 mm.

Note: The first line gives the lowest five partials, the second their deviation in cents from the correct tuning. In the next lines the first column shows in which sectors bronze has been removed, while the next columns give the result. Before tuning, this bell weighed 427 kg, after tuning 418 kg.

information relating how metal removal at every level affects the pitch of each partial to be tuned. The production of such tuning graphs is experimentally not very difficult. The inside of the bell is divided into a number of ring-shaped sectors of equal width. As a unit of width one can take 1/25 of the diameter of the bell (Fig. 1). Next, in every sector starting with "a", a measured amount of brass is turned down and the pitch shift for each partial is recorded. One moves on to sector "b." In this way Figure 2 has been obtained. The sectors are graphed horizontally while the pitch changes are noted vertically.

With the help of these graphs the bell founder will easily tune an untuned bell, see for example, Table 1. In the first column one finds the sectors in which bronze has been turned off. The following columns show the results.

One can see from this table that tuning takes place step by step. The reason is that the smallest amount of excess removal of bronze would irreparably change the bell. Moreover, elaborate calculations would be necessary to determine a priori with the tuning curves the

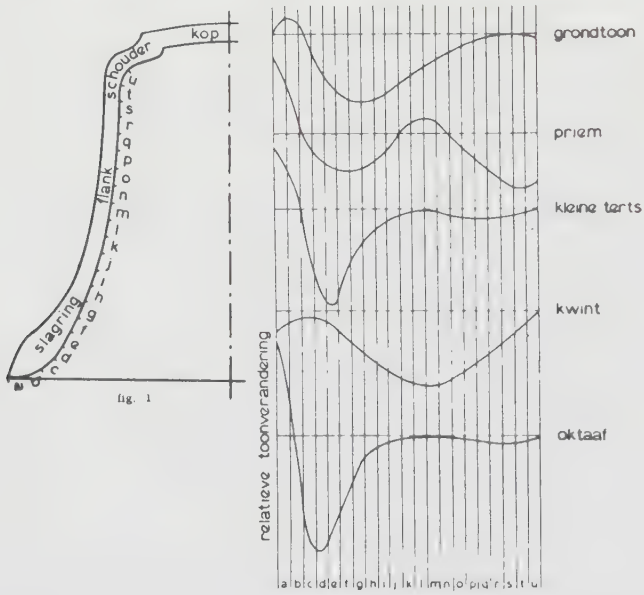

fig. 1

Figure 1. To determine the tuning curves the inside of the bell is divided into horizontal sectors of width 1/25 of the bell diameter. Then for each partial the tone change is measured for removing metal from each sector. The results of the measurements are graphed in Figure 2.

Figure 2. The relative pitch deviation of the lowest five partials as a function of the sector when a certain amount of bronze is removed.

amount of bronze needed to be removed. Therefore, the bell tuner always works with relative values as used in Figure 2.

It should be noted that, although these tuning graphs are very useful, there are limitations. For example, if the fourth partial were too low, this could not be corrected by means of wall thinning. The tuning curve shows that in each sector the pitch of the fourth partial will fall if metal is removed. Does this mean that the other partials can be tuned up? Theoretically this seems possible. In practice there are considerable limitations. The tuning curves show that the sectors for pitch raising are very few. If one wanted to raise the pitch of an overtone a large amount, a large amount of metal would have to be removed. In principle this is possible but the profile would be changed by excessive local removal. One must consider that a smooth profile strongly affects the tone color. Local wall thinning can be used to change overtone tuning over a wide range, but there are other reasons why this is not advisable. One can generalize that extreme local metal removal introduces discontinuities that are disasterous. For example, if one would like to tune the fifth partial by removing much metal in sector d, one would get a groove that would hurt the sound of the bell.

Tuning a bell is not simply a question of setting a number of characteristic frequencies. One always has to rely on hearing for the tone color of the bell, or apply certain rules of experience. All of this shows that designing a bell profile is certainly not a random matter, and that all imperfections cannot be corrected during tuning.

The designing of a bell is in fact a rough pretuning. For this the tuning curves for the inside of the bell are not sufficient. Besides having tuning curves for the outside of the bell, one also has to have other rules which relate tone structure to bell profile—the relationship between the eigenfrequencies and the height of the bell, for example. One can ask how appropriate the tuning curves are for various profiles. Of course this question should have been asked much earlier. If one starts in sector "a" and shaves off metal to sector "u", he ends up with a different bell. Experiments show, however, that even if one removed metal in the other direction the same results are obtained. This independent character of tuning curves is also valid for not very different bell profiles. In these cases, only minor changes in the tuning curves occur. Therefore, the curves are a very reliable tool in the design of a bell.

A last and very important question in connection with bell tuning is what degree of tone purity should be aimed for? Of course the criterion here can only be musical, although simultaneously a very complicated problem is introduced.

Considering a single bell, not heard in consonance with others,

one has the requirement that the intervals to be realized between the lowest five partials should be perfect, and heard as such. Experience has shown that this requirement is only satisfied when the eigenfrequencies of the bell are tuned to an accuracy of 0.2% or 3 cents. Nevertheless, it is dangerous to apply such a rule without further thought. In low bells this can lead to disturbing beats between the partials themselves. For example, if the hum tone of a bell forms a subjective harmonic an octave higher and the hum tone is 3 cents low, then the subjective harmonic will give beats with the prime. This shows that in the final analysis musical hearing is the ultimate criterion.

When several bells sound together, the same rules are valid; however, the danger of beats is much larger and therefore more attention has to be paid to this aspect. For example, if a bell with hum tone c^1 (C_4) whose tierce is es^2 ($E_5^\flat$) were to sound with a bell whose prime was es^2 ($E_5^\flat$), then very disagreeable beats will occur if both partials do not have exactly the same frequency. One needs to tune a set of bells externally as well as each individual bell internally. This makes for a complicated tuning process. Each amount of bronze turned down from a bell should be extremely well considered. A bell thinned by too much, however little, cannot be restored to its original condition.

Curiously enough, in this process of beatless tuning, the technique of using accurate measurement apparatus is insufficient. In the final fine tuning one is entirely dependent on the musical ear—the same ear that in carillon playing hears beats between bells immediately and experiences them as disturbing the harmonic consonance.

I should mention that the picture given so far of tuning carillon bells reflects the developments of the last few years. Before this, one was content to tune bells within 3 cents and not to care too much about beats that would occur. Even the famous Hemony bell founders of the seventeenth century, who left numerous excellent carillons in The Netherlands, limited themselves to this accuracy of 3 cents,[13] an amazing accomplishment for the bell foundry art of that time. In spite of this, one must state that considerable improvement has now been made in tuning techniques so that beat-free carillon tuning has now become possible. This is especially important for the place of bells in the development of contemporary music.

PARTIAL GROUPS IN THE BELL SOUND[14]

In the foregoing we concerned ourselves exclusively with the lowest five partials of a bell and spoke only secondarily about higher

partials. This does not mean, however, that higher partials are musically of less value or that their spectral position is arbitrary. On the other hand, not all high partials have the same significance. The criterion here is, without any doubt, their strength during the bell strike. This would mean that the overtone strength for every bell would have to be measured to determine which higher partials are really important. Unfortunately, determining such a spectrum during normal bell founding is nearly impossible. Of course, there are analyzers that will isolate certain bell frequencies, making it possible to measure partials separately. However, these analyzers usually have too low a selectivity to measure strength of partials one by one unless time-consuming measurements are made. If one considers that many higher partials have frequencies less than 10% of each other, then this difficulty will be very clear. Fixing the frequency spectrum of a bell generally must be done in a laboratory, and this is a considerable limitation. One has to look for other methods for finding the strengths of bell overtones.

It is clear that one has to look for the characteristic function of the overtone in question, that is the function in which the radial wall amplitudes for each point of the bell surface are given. It is the radial wall amplitudes that determine the total volume of an overtone. Determining such functions is done in the following way.

After the bell has been brought into steady-state vibration for a certain overtone by means of an electrodynamic excitator, one uses a recorder employing an exciter used inversely to convert the mechanical bell vibrations into an A.C. voltage. Because the voltage is proportional to velocity, one has to use an electronic integrator to convert the velocity signal to a displacement signal. If, then, the recorder has scanned the whole bell surface one would have enough information to put together the necessary characteristic function. Because of the particular symmetry of the bell one can best split these measurements into circular and meridional sections. Of course the meridian measurements are the most interesting. In Figure 4 the characteristic functions for a large number of overtones of an arbitrary bell are given for an outside meridian. The bell had a diameter of 97 cm and weighed 567 kg. Its strike tone was $g15^1$ ($G_4^\sharp$). The measured characteristic frequencies for this bell are given in Table 2 with the traditional partial names.

We should give an elaborate discussion of errors. However, only some summary observations will be made. When one examines other bells for comparison there are small deviations in the characteristic functions, but the form of the graph is nevertheless the same. This means the above graphs can be applied to any bell provided it has normal proportions. One could aim at measuring absolute amplitudes.

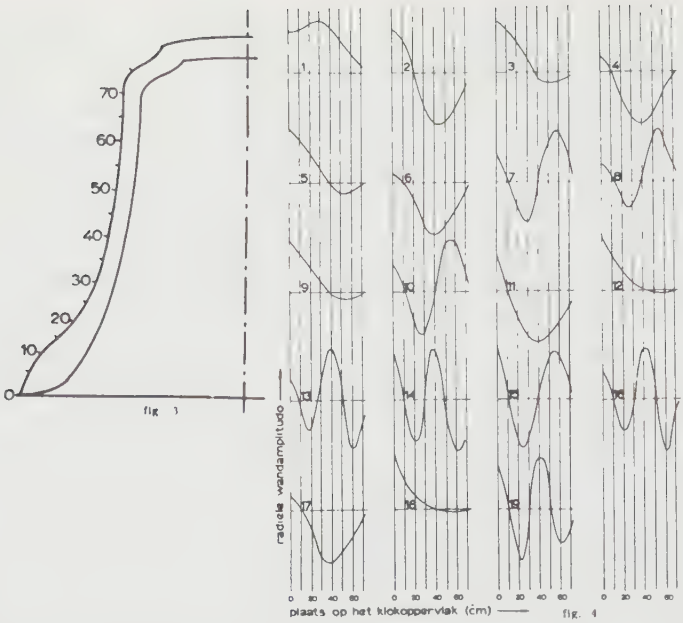

Figure 3. A bell, tone gis^1($G_4^{\#}$), diameter 97 cm, weight 567 kg, on which the radial wall amplitudes for Figure 4 have been determined. The strike point for this bell lies on the outside of the bell 10 cm above the lip measured along the bell surface.

Figure 4. The radial wall amplitudes in a relative measurement, measured on the outside surface of the bell along a meridian of the bell in Figure 3. Measurements for each partial of Table 2 are given. The strike point is about 10 cm from the lip.

This does not make sense, however, for the graphs above were created by electrodynamic excitation, not by a clapper. Even if one could consider tone formation by the clapper one would have to also consider the hardness of the clapper. A soft clapper would diminish upper partial strength, as would a relatively heavy clapper. The mean amplitude of each function is variable, dependent upon the manner in which the bell is struck. In view of the way in which these characteristic functions have been obtained, it does not make sense to measure absolute amplitude. Therefore, all the graphs are normalized to have the same peak amplitude.

In spite of these limitations one can say something in a relative sense about the volume to be expected from each partial. If a bell is struck on one of the nodal lines for a certain mode, one can certainly expect a low amplitude for the corresponding partial. Striking at an antinode would emphasize the partial.

no.	muzikale naam	toon	frekw.	s	groep	fysische naam				relat. toon.
1	grondtoon	gis-21	205,1	2	I	I-2				c_1
2	priem	gis^1-21	410,3	2	II		II-2			c^1
3	kleine terts	b^1-9	491 ,3	3	I	I-3				es^1
4	kwint	dis^2-22	614,4	3	II		II-3			g^1
5	oktaaf	gis^2-21	820,6	4	I	I-4				c^2
6	grote deciem	c^3+20	1058	4	II		II-4			e^2
7	2^e undeciem	cis^3-51	1076	3	III			III-3		f^2
8	1^e undeciem	cis^3-31	1089	2	III			III-2		f^2
9	duodeciem	dis^3-17	1232	5	I	I-5				g^2
10	grote tredeciem	f^3-32	1371	4	III			III-4		a^2
11	grote kwartdeciem	g^3+15	1582	5	II		II-5			b^2
12	dubbeloktaaf	gis^3+46	1706	6	I	I-6				c^3
13		a^3-25	1735	2	IV				IV-2	
14		a^3+12	1772	3	IV				IV-3	
15		ais^3-10	1854	5	III			III-5		
16		b^3+4	1980	4	IV				IV-4	
17		c^4+56	2162	6	II		II-6			
18	dubbelundeciem	cis^4+18	2241	7	I	I-7				f^3
19		d^4+21	2378	5	IV				IV-5	

Table 2. Frequency analysis of a bell, $G_1^{\#}$, diameter 970 mm, weight 567 kg.

Note: The second column gives the names customary in bell founding for the most important partials. The fifth column gives the number of nodal meridians; the sixth column, the eigenfunction group derived in Figure 5 from Figure 4. The seventh column gives the physical name of the partials based on their eigenfunction group and number of nodal meridians. The last column gives the relative musical position of most of the partials when the hum tone is defined as C.

Considering that the striking point for a bell is nearly fixed, then one can quite reliably say that partials with nodal circles near the strike point would be heard weakly. Partials with nodal circles far from the striking point will be heard loudly. In this way one can observe that the hum tone, prime, tierce, nominal, twelveth, double-octave, etc. will be dominant in the spectrum. Interesting is that this sequence gives a relative tone scale $c - c^1 - es^1 - c^2 - g^2 - c^3$ ($C_3 - C_4 - E_4^b - C_5 - G_5 - C_6$), which is very acceptable. Conversely the weaker partials give a much less harmonic series $g^1 - e^2 - f^2$ (double) $- a^2 - b^2$ ($G_4 - E_5 - F_5 - A_5 - B_5$), and so forth.

All of the amplitude functions given are measured over meridians of the bell surface. To complete the picture of the vibrating bell one should also investigate wall movements along circular lines around the bell. Because of the circular symmetry of the bell, these will be much simpler. One can expect a strong analogy with a vibrating ring.[15] Each overtone can be characterized by a number of nodes equally spaced around the circumference of the bell in which the wall movement between nodes can be approximated by a sine function. In Table 2, column 4, the number of nodal lines vertically down the surface of the bell are given for each overtone. We have deviated from the usual notation in bell literature. Usually one counts the number of

nodes found when going completely around the bell once. However, if one correctly considers the bell as a bent plate, we can follow the notation that is customary in scientific literature; this is half of the bell founder's count.

The nodes that one counts going around an arbitrary parallel will be seen as radial lines. Observing the bell from above one sees the radial lines as half the number of lines simply crossing at the crown of the bell. This halved count is indicated in column "s" of Table 2 for every partial; for example, the hum tone having four radial nodes is seen as having only two intersecting nodal lines when viewed from above.

Since we are dealing with a circularly symmetric body, the positions of the meridional nodes are determined by the strike point of the bell. For the hum tone and the prime the meridional nodes will be 45° and 135° both to the left and right of the strike point. In this consideration one assumes an antinode for all partials is formed at the strike point.

By means of the meridional and circular nodal lines the modes of vibration of a bell can be completely described. The next step is to order and interpret these eigenfunctions. The number of circular nodes, as described before, serve as one sort of classification. Certain patterns return time and again. Figure 5 collects the averages of the characteristic amplitude functions divided into the classes of modes. Every basic pattern, or class, is indicated by a Roman numeral.

Aside from the fundamental and the prime, it is possible to classify partials into groups which have the same circular node pattern. The averages of the first four classes of partials are taken from Figure 4 and given above in Figure 5. The Roman numeral groupings in Table 2 correspond with the Roman numerals above.

In view of the fact that one particular circular node can be contained in several different partials, a basis for grouping the partials is formed. Partials with a particular nodal pattern can be grouped together. From Table 2 and Figures 4 and 5 one can distinguish the following groups:

Group I. Partials from this group are characterized by a large amplitude at the sound bow and a nodal circle in the waist of the bell. The lowest partial with $s=2$ is lacking since neither the hum tone or the prime have a nodal pattern corresponding to this group.

Group II. Partials in this group contain a nodal circle in the sound bow and an antinode in the waist of the bell. Here again, for reasons mentioned earlier, the nodal pattern with $s=2$ is lacking, although the prime is a close approximation to the $s=2$ member.

Group III. The partials in this group contain one more nodal circle than the foregoing groups. (2 nodal circles)

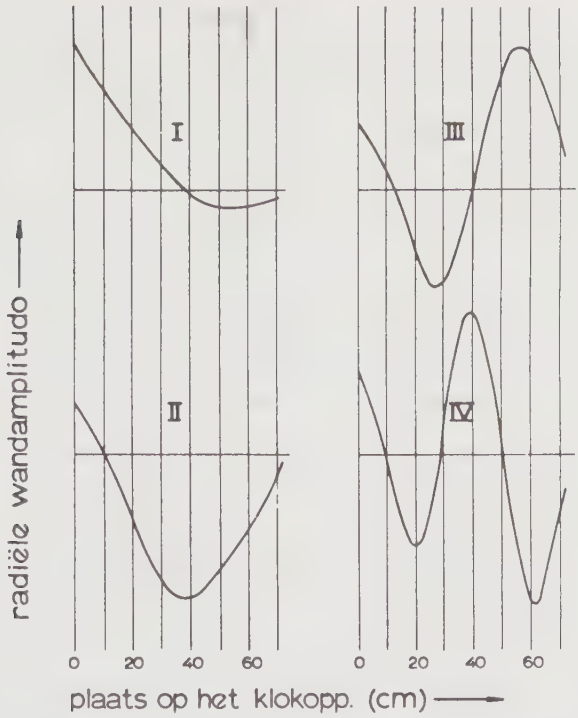

Figure 5. Aside from the fundamental and the prime, it is possible to classify partials into groups which have the same circular mode pattern. The averages of the first four classes of partials are taken from Figure 4 and given above in Figure 5. The Roman numeral groupings in Table 2 correspond with the Roman numerals above.

Group IV. Three nodal circles characterize members of this group. The amplitude function over the surface of the bell is described approximately by a sinusoidal curve.

Group V, and higher contains each time one more nodal circle than the preceding group. The amplitude functions tend more and more towards a pure sinusoidal curve.

The lowest two partials of the bell, the hum tone and the prime, curiously seem to stand outside of the group classifications given. Nevertheless, one can show that the hum tone is the lowest member of Group I and the prime the lowest of Group II. This can be shown by constructing an experimental bell profile which is purely conical. If one investigates the modes of vibration on such a conical bell, one notices the following interesting changes. It appears that the nodal circle for Group I is shifted to the crown of the bell. This means that the modes of vibration for Group I are practically identical with the mode for the hum tone, so it seems justified to consider the hum tone

as the lowest member of Group I. Something similar is valid for the partials of the second group. The nodal circle for Group II in a conical bell lies near the position of the nodal circle for the prime in the normal profile bell. For this reason the prime can be considered the lowest member of Group II.

Starting with the foregoing considerations, it should not be difficult to represent the partial groups graphically. This has been done in Figure 6 for the bell of Table 2. The number of nodal meridians is given horizontally, the partial frequencies, vertically. The connecting curves represent the progression of members of one group.

Any partial can now be unambiguously identified by its group and the number of vertical meridional nodes. Is it easy to indicate this in a partial's name? This seems advantageous, since in the past partials have been named by the relative musical position, such as the prime.[16] Such labeling cannot be unambiguous since the musical interval position of a partial is highly dependent on the bell profile. It is much better to consider the physical aspects of each overtone in giving a name, aspects which are less dependent on bell shape. In this way partials may be named by two members. The first is a Roman numeral indicating the group and the second number indicates the number of nodal meridians, s. Thus the hum tone becomes I-2, the prime II-2, and so forth (see Table 2).

The starting point of our considerations was to understand simply which partials could be considered musically important. After measuring the eigenfunctions for one bell, one asks if it is necessary to

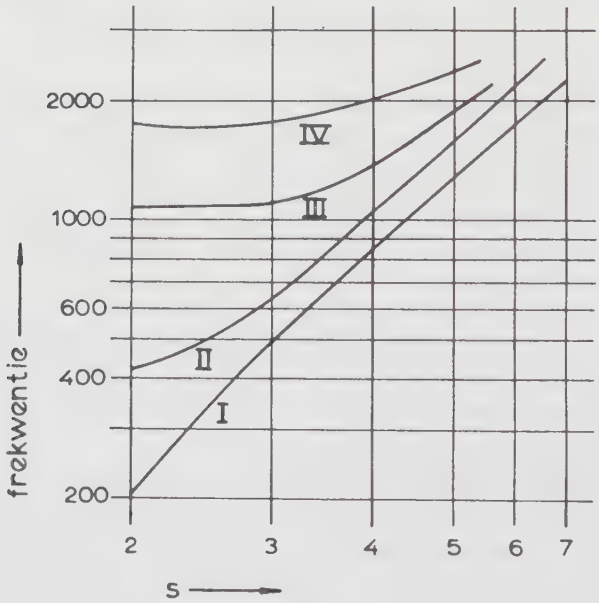

Figure 6. Graphic representation of the group classifications for the bell in Table 2.

repeat the measurements for each bell. Fortunately, it is not necessary; every bell has the aforementioned modes of vibration. Each mode is characterized by its physical name which gives the circular node group and the number of nodal meridians. One only needs to look for these characteristic modes; the following procedure is used. By means of the electrodynamic excitator described earlier, the bell is brought into steady-state vibration for the partial to be investigated. The bell surface is scanned with a pickup transducer; the voltage induced on the pickup will show where the nodes occur on the bell. Another variant consists of carefully going over the bell surface with a stethoscope. For tones not too high in pitch the minima can be observed by hearing. The final method, which is only suitable for finding nodal circles, makes use of the fact that a partial cannot be excited by striking on one of its nodal circles. In Table 3 results are given for a bell with strike tone $g^{\#1}$, diameter 974 mm, and weight

no.	muzikale naam	toon	frekw.	fysische naam							
1	grondtoon	gis+31	211,4	I-2							
2	priem	gis1+31	422,6		II-2						
3	kleine terts	b1+40	505,4	I-3							
4	kwint	dis2+23	630,6		II-3						
5	oktaaf	gis2+31	845,6	I-4							
6	grote deciem	o3+58	1082		II-4						
7	1eundeciem	cis3-34	1087			III-2					
8	2e undeciem	cis3-14	1100			III-3					
9	duodeciem	dis3+9	1251	I-5							
10	grote tredeciem	f3+2	1398			III-4					
11	grote kwartdeciem	g3+32	1597		II-5						
12	dubbel oktaaf	gis3+91	1751	I-6							
13		a3+13	1773				IV-2				
14		a3+22	1782				IV-3				
15		ais3+45	1914			III-5					
16		b3+29	2009				IV-4				
17		c4+60	2167		II-6						
18	dubbelundeciem	cis4+56	2290	I-7							
19		d4+57	2428					V-2			
20		d4+64	2438				IV-5				
21		dis4+11	2505					V-3			
22		dis4+31	2534			III-6					
23		e4+65	2738					V-4			
24		f4+18	2823		II-7						
25	dubbelkwartdeciem	f4+46	2869	I-8							
26		fis4-7	2948						VI-2		
27		fis4+16	2987						VI-3		
28		fis4+36	3022				IV-6				
29		g4-6	3125					V-5			
30		g4+53	3233			III-7					
31		gis4-11	3301							VII-2	
32		gis4+17	3355							VII-3	
33	tripeloktaaf	gis4+82	3484	I-9							
34		a4+12	3544		II-8						
35		a4+37	3596							VII-4	
36		a4+58	3640								VIII-2
37		a4+60	3644					V-6			
38		ais4-4	3721				IV-7				
39		ais4+29	3792						VI-5		

Frekwentie-analyse van een gis^1-klok, diameter 924 mm, gewicht 428 kg. De partiaal VI-4 werd niet gevonden. Voor een verdere toelichting zij verwezen naar die bij tabel 2.

Table 3. Frequency analysis of a bell gis^1 ($G_4^{\#}$), diameter 924 mm, weight 428 kg.

Note: The partial VI-4 was not found. For further explanation, see Table 2.

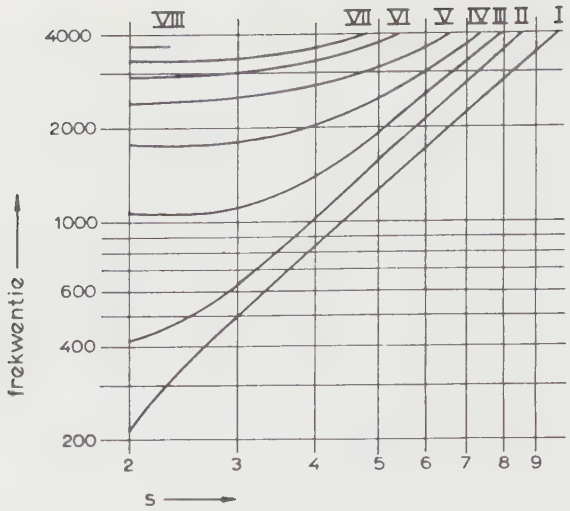

Figure 7. Graphic representation of the group classifications for the bell in Table 3.

428 kg. In Figure 7 the group classifications are represented. One can easily see that this graph has great similarities to that of Figure 6. This indicates that the group classification graphs for all approximately tuned bells will give the same results.

Can one assume from the examples that a bell is completely described by its frequency structure? This is practically true, but on a theoretical level the description needs to be expanded further.

The eigenfunctions considered so far are all produced by vibrations of the flexural type. This can be made very plausible if one keeps in mind the analogy between a horizontal bell section and a vibrating ring. The theory of flexural vibrations is used when realizing that in the execution of wall movements, one can locate a surface that neither expands nor contracts. That is, a surface which only makes inflections along which no normal or tangential forces appear. Usually this surface is considered to be the midplane of the wall. The radial wall amplitudes would then be perpendicular to this cylindrical surface. In order to imagine such a non-expanding or contracting cylinder there would have to be at least four motionless points on the circumference, that is, at least four radial nodes. The number "s" should therefore be at least two. For vibration nodes where $s=0$ or 1, there cannot be such a neutral surface. In this case one has to speak of stretch vibrations. Do these stretch vibrations occur in a bell?

From the nature of these modes one would expect that they would naturally have a high frequency. Indeed, this is confirmed experimentally. The first stretch type partial in a normal bell occurs at

a frequency between that of partials I-4 and II-4. All others have a higher position yet in the frequency spectrum.

In spite of these circumstances that make the stretch vibrations seem less important, one can still ask why they have not been included in the foregoing considerations. It turns out that all the stretch vibrations have an antinode in the crown of the bell, from which the bell is normally hung fixed. Thus, these stretch modes will barely be heard, if at all. Even if the antinode in the crown of the bell is allowed to develop completely, the amplitude of vibrations in the side and sound bow of the bell remain small. For this reason it makes no sense to completely describe the stretch vibrations. Only in small bells with a relatively thick wall can they be of any significance at all.

Apart from the musical consequences, which we will deal with later, the system of group classification offers a means of profile investigation. In the past, a bell could not be physically related to other vibrating bodies such as the gong. Now, however, on the basis of the eigenfunctions, this is possible. For if the bell is gradually changed through several intermediate shapes, one can always identify in the new shape those partials that correspond to partials of the original bell shape, regardless of how much the partials have shifted or changed position in the sound spectrum. It is even possible to bypass these intermediate shapes and by means of eigenfunctions alone indicate partials that relate to bell partials. In the gong, for example, a study by Grützmacher and Wesselhoft[17] shows that partial groups in a bell compare to partial groups in a gong, where similarly the first group dominates, while other groups have a secondary function. Such a connection with the bell would be impossible to make on the basis of the sound spectrum alone.

MUSICAL CONSEQUENCES

By means of group classification it should not be difficult to predict the relative spectral distribution of the various eigenfrequencies of a bell.

We observed earlier that those partials having no nodal circles at the sound bow especially dominate the sound spectrum of a bell. This includes the partials of group I and partial II-2, the prime, as well. In musical terms, this is approximately the series $c - c^1 - es^1 - c^2 - g^2 - c^3 - f^3 - a^3 - c^4$ ($C_3 - C_4 - E_4^b - C_5 - G_5 - C_6 - F_6 - A_6 - C_7$). In Figure 8 the average sound spectrum of ten bells is given, and it appears that these tones clearly are stronger than others. The essentially nonharmonic spectrum of the bell has been made quite acceptable in this way, for it is the dominant overtones that reinforce the tuned pitch of the bell. Nevertheless, some questions remain that merit consideration. One

can ask why only the lowest five partials are tuned when our consideration above involves higher partials as well. One assumes then that tuning higher partials might be possible. In practice however this is not feasible. It has been found that partials with nearly identical eigenfunctions have very similar tuning curves as well. One can observe this for the partials I-3 and I-4, for example (see Fig. 2). This implies that one can rarely tune separate partials within the same group. We illustrate this with an example. It is clear from Tables 2 and 3 that the pitch of partial I-6 is rather sharp, a common occurrence in most bells. Naturally it would be attractive to make this partial I-6 be a perfect octave above partial I-4. The tuning process for this uses practically the same tuning curves for the partials I-4 and I-6. This means that if the pitch of one of the partials is changed by some amount the other partial will change practically to the same degree. To tune these partials separately, then, one would have to make use of detailed differences between the tuning curves of the two partials, and the bell profile would have to undergo much localized thinning. One would introduce in the profile a discontinuity that would have undesirable consequences for the timbre of the bell. Other partials would take on quite arbitrary positions in the sound spectrum of the bell, so that what one gains with one partial, one loses with the others.

In this way we have indicated the possibilities for tuning with greater accuracy. If one assumes that partials with the same eigenfunctions have practically the same tuning functions, then large tuning changes are possible only for the hum tone I-2, the prime II-2, and among different groups. Partials within one group allow only small independent corrections. One should realize that this last statement is quite significant for the musical aspects of the bell!

Finally, one has to consider that tuning bell partials upwards is generally not advisable. All of these factors limit tuning possibilities considerably, and one should weigh these factors when designing a bell. Does this mean that one is totally bound to the tone structure of an individual group and that relations within a group are practically independent of bell profile? Yes, up to a certain point. The design of a bell allows great possibilities, but one has to avoid discontinuities in designing a bell profile. Consideration must be given to more than just wall thickness. One can vary the slope of the bell profile, the height or diameter of the bell, and so forth. This means that the designer can do something about internal relationships within a group. But one cannot expect too much, for experience shows us that to conserve the harmonic structure of the lowest partials, a variation of 4% in frequency is maximum. As a consequence, the major third bell, with partial I-3 a major third above the prime, has not yet been realized, in spite of the claims of certain bell founders. Also it is not possible to make the interval between I-3 and I-4 a perfect octave. Even in the Chinese

gong with a design much different than our normal bell, partial I-3 is 10% from the octave frequency of I-4.[18]

Although the strong structure within a group can have some bad consequences musically, this is an advantage in other ways during tuning. To take a very crude attitude towards the tuning process, one might consider it sufficient to tune only the important lower harmonic partials that have different eigenfunctions and therefore different tuning curves. This would be the sequence: hum tone I-2, prime II-2, quint II-3 and octave I-4. If then, by the design of the bell, sufficiently pure relationships within a group were chosen nothing essential will be changed if tuning is done within reasonable bounds. This means that the tierce I-3 would have a reasonable purity, as would the twelfth I-5 and double octave I-6.

The bell founder however, cannot be satisfied with such a crude approach to tuning. He will at least try to use the detailed differences in the tuning curves of I-3 and I-4 to fix the partial I-3 as harmonically pure as possible, for the partial I-3 occupies a very dominant position in the spectrum of a bell (Fig. 8). But with this the possibilities for

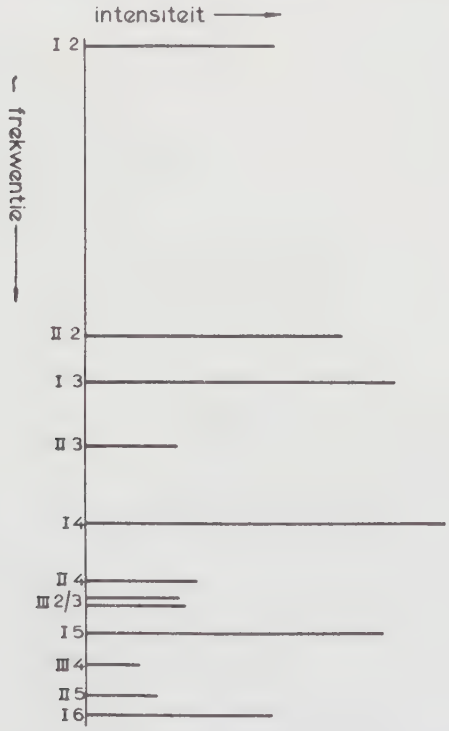

Figure 8. Average intensity distribution of the partials of 10 normal bells struck in the customary way. It is clear that the first group of partials occupies a dominant position.

fine tuning are practically exhausted. The bell founder will have to be content with fixing only reasonably well the 12th I-5 and the double octave I-6, without being able to change them much during tuning. This is also true for the partial II-4. The ability to correctly place partials is finally exhausted when considering group III with its curious lowest partials of flatted tenths.

Summarizing, one can state that the final result of designing and tuning a bell can at most assure that the partials I-2, II-2, I-3, II-3 and I-4 are highly pure, that is, within 0.2% of the correct frequency. The higher partials will relate to the lowest five partials, depending on their particular eigenfunctions, with a purity of 0.5 to 5% of the desired frequency. So bell tuning is a compromise, particularly involving the tone purity of the higher strong partials.

We have seen that the group classification of partials is of great significance to the tuning and designing of a bell, as well as many other aspects of a bell's sound. Repeatedly in scientific bell literature we find a listing of the eigenfrequencies as if it would give us complete insight into the sound of a bell. One even goes so far as to draw extreme conclusions from such listings of eigenfrequencies.[19] Such a procedure however is not at all possible. For how can one locate the double octave in such a list?

Tables 2 and 3 show that near the partial I-6 there occur several partials, for example those from group IV. Based on the frequency analysis of several bells, it is not at all impossible for a tone of group IV to form an exact octave with I-4, while I-6 remains too high. Can one consider this tone of group IV as a double octave? Certainly not. In general, the impression of a musician relates to the strongly audible partials. It therefore makes no sense to let a weak but pure partial prevail over a strong but less pure partial because of its more precise harmonic position. The conclusion is then that even if the partial I-6 has deviations in tonal purity, the ear will still consider this tone as the double octave. This means that when one is given the eigenfrequencies, one also has to be informed of the corresponding eigenfunctions. This information is easily stated with our nomenclature, in which the number of nodal circles and nodal meridians is given. This difficulty not only occurs with the double octave, but also with other partials. Among others this is true for the flatted double tenth I-7, a tone that can be very disturbing, especially in heavier bells. Another phenomenon that can be illuminated by group classification is one many musicians have correctly observed: that the bell, with its conspicuous minor third, gives rise to confusion in carillon playing when a melody is in a major key. Correctly then, the musicians state that the minor third should be as weak as possible. Physically this

means that by suitable bell model choice the nodal circle for the third has to be lowered as much as possible toward the sound bow.

Apart from the question of whether this is possible or not, one has to consider seriously that the nodal circles of the other partials of group I would be shifted down by approximately the same degree. However, it is exactly in this group that the bell sound finds a strong basis for unambiguous tonal height. Thus, a weaker third opposes the production of a strong tonal height sensation for the bell; it is clear in such a choice what we would prefer. All this and more demonstrates that group classification is an important aspect of bell science. One can return to group classification time and again if one wants to understand the observed phenomena. This systematic ordering on the basis of the most fundamental properties of partials is also important in the decay rates of the partials in the production of difference tones, and in the problem of the strike tone about which Prof. Schouten reports more extensively [*Editor's Note:* See Paper 15.]

The same group classification will also form a foundation for the investigation of timbre, a subject for which a coherent physical picture is totally lacking. Certainly several have been given, but the problem is so complicated that research on this very important aspect of bells is only beginning. Many factors are involved, because apart from the profile of the bell, the upper structure and the decay times play an essential role. Also important are the bronze used to cast the bell, the quality of the cast work, the manner in which the bell is struck, the place where the bell is hung, the audition location, and especially the interpretation that the listener gives to the physically measurable facts. Here we return to our point of departure, for however fascinating a bell is as a physical object, finally it is only the listener that can give value to the measurable aspects.

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Part IV

SEARCH FOR THE STRIKE NOTE

Editor's Comments on Papers 12 Through 16

12 JONES
The Strike Note of Bells

13 MEYER and KLAES
On the Strike Note of Bells

14A ARTS
The Sound of Bells. Jottings from My Experiences in the Domain of the Sound of Bells

14B ARTS
The Sound of Bells. Jottings from My Experience in the Domain of the Sound of Bells. The Secondary Strike Note

15 SCHOUTEN and 'T HART
The Strike Note of Bells

16 PFUNDER
On the Strike Note of Bells

Early investigators were aware of a strike note or strike tone distinct in character from the various partials of the bell (see, for example, Blessing, 1911). So far as I can tell, A. T. Jones was the first to notice that the strike note could not be amplified by a Helmholtz resonator, and thus is a subjective tone (Jones, 1920). Paper 12 describes his experiments on the strike tone and considers several alternative explanations for its origin.

Other early investigations of the strike note were carried out by Meyer and Klaes (Paper 13) and by Arts (Paper 14). Meyer and Klaes conclude that the strike note is a difference tone between the fifth and seventh partials. Arts, on the other hand, favors the hypothesis that the fifth partial alone determines the strike note. In Paper 14B he describes a secondary strike note observed in bells of about 800 kg and larger, which usually occurs a musical fourth above the fifth partial (or an eleventh above the primary strike tone). Other investigators place it in the same octave as the primary strike note, however.

The work of J. F. Schouten and others, beginning about 1938, on the apparent pitch of complex tones led to an enhanced understanding of the origin of the strike tone. The ear has a remarkable ability to pick out partials, which are almost harmonic, and assign to them a single pitch that is the fundamental of the harmonic series.

In Paper 15 Schouten and 't Hart conclude that the strike note is determined by the nearly harmonic partials I-4, I-5, and I-6 (partials 5, 9, and 12), while the secondary strike note is determined by partials I-6, I-7, I-8, and I-9. Pfunder (Paper 16) agrees with this hypothesis and makes a connection with the hearing sensitivity curve. In small bells, partials I-6 to I-9 have such a high frequency that the ear does not use them to construct a secondary strike tone.

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THE STRIKE NOTE OF BELLS

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Introduction

A bell, like most sounding bodies, usually gives out several different tones at the same time. For the production of each of these tones there is a definite "normal mode" of vibration. The different normal modes are characterized by different numbers of vibrating segments separated by nodal lines. Some nodal lines run up and down the bell, and some are circles parallel to the plane of the mouth.

When a bell is struck its vibration consists of a number of normal modes all occurring at the same time, and in the sound from the bell there are the various corresponding tones. The one of these tones which has the lowest pitch will be called the *first partial* or *first partial tone*, the one of next higher pitch the *second partial*, and so on.[1]

The partial tones of a bell do not in general follow the harmonic series. For instance, on the largest bell of the Dorothea Carlile Chime at Smith College the first seven partial tones have frequencies which are approximately in the ratios 1.00:1.65:2.10:3.00:3.54:4.97:5.33. Thus the partial tones are quite different from those of strings or pipes.

There are a number of ways in which the separate partial tones may be picked up. If a bell is struck and a Helmholtz resonator is held to the observer's ear, it is often possible to pick out some of the partial tones. The resonator may be tuned by rolling a finger over its mouth, and when the natural pitch of the resonator comes near to one of the partial tones of the bell the loudness of that partial increases. A second

[1] It frequently happens that there are two or more normal modes with the same number of vibrating segments. For instance, each of two modes may have four nodal lines running up and down the bell, the nodal lines for one mode lying between those for the other. The stiffness and inertia coefficients for the two modes are likely to differ in such a way as to give rise to two tones of nearly the same pitch. This is the cause of the beats that are often heard from a bell. When two or more modes have the same number of vibrating segments, similarly distributed, and give rise to frequencies so close together as to beat, I shall count them as a single partial tone.

If we were to use optical terms the different modes in a single partial might be regarded as giving rise to "multiple lines." On the bells of the Dorothea Carlile Chime the lower partials are often double, the sixth is usually a multiplet of higher order, and the seventh is sometimes a doublet. In the experimental work described in this article the frequencies of the component "lines" have usually been determined, but the values used are averages.

method makes use of the beats between one of the partial tones and some other source of sound. Suppose, for instance, that a tuning fork has a pitch which differs slightly from that of one of the partial tones of a bell. If bell and fork are struck the beats may be very distinct. A third method makes use of resonance from the bell. Suppose as before that the pitch of a tuning fork is close to that of one of the partial tones of a bell. This time the bell is not struck. The fork is struck and then its stem is pressed against the bell. The bell picks up the vibration and sings out that one tone.

Some of the partial tones of a bell are easily heard without any resonator or tuning fork. They may, in fact, be heard so easily that when a single bell is ringing it is not always easy to decide what the pitch of the bell is. But when several bells follow each other in fairly rapid succession there is usually no difficulty in deciding on the pitches. The note which is most prominent when several bells sound in succession is called the *strike note* of the bell.

More than forty years ago Lord Rayleigh[2] was surprised and disturbed at finding that the strike notes of a number of bells did not agree with the pitches of any of the first few partial tones. For instance, the second bell (reckoned from the highest) of the Terling peal was supposed to be b_3, i.e. the first b above middle c. The first five partials were: d_3-6, $a_3\sharp-5$, d_4+8, $g_4\sharp\sharp+10$, and b_4+2, where the numbers added and subtracted indicate the numbers of beats per second made with notes on a harmonium. It will be seen that the only partial which is a b is the fifth, and that this fifth partial is an octave higher than the strike note. The strike note is not one of the partial tones of this bell.

Rayleigh examined the first five partials of seven bells. On all of them the fifth partial was an octave higher than the strike note. On two of the bells the second partial had nearly the pitch of the strike note, but on the others no partial was very close to the strike note.

Ten years ago I published[3] a study of the twelve bells of the Dorothea Carlile Chime at Smith College, and more recently[4] a study of the ten bells of the Harkness Memorial Chime at Yale University. On all of these bells the fifth partial is just about an octave above the strike note. On the bells of the Harkness Chime the second partial is close to the strike note, but on most of the bells of the Dorothea Carlile Chime

[2] Rayleigh, Phil. Mag., **29**, 1 (1890).
[3] A. T. Jones, Phys. Rev., **16**, 247 (1920).
[4] A. T. Jones, Phys. Rev., **31**, 1092 (1928).

the second partial is lower than the strike note and the third partial is higher.

During the study of the Dorothea Carlile Chime it seemed to me that the fifth partial came out very promptly when a bell was struck, and that the second and third partials grew to their maxima more slowly. I suggested that the pitch of the strike note may be determined, except for octave, by the fifth partial, and that the octave in which we hear it may be determined by the more sluggishly responding second and third partials. The work on the Harkness Chime was not decisive, but it seemed to indicate that the fifth partial really does come out promptly, and the third partial not so promptly.

The work last summer was again on the Dorothea Carlile Chime, and was done in the hope of getting further evidence as to the cause of the strike note.

Determinations of Pitch

Frequencies of Partials.—For the work last summer a set of nine tuning forks with adjustable loads was made and calibrated by the Riverbank Laboratories. These forks run from 168 cycles/sec to 1400 cycles/sec. They are graduated so that from 168 to 960 the successive settings differ by four cycles/sec, and from 960 to 1400 they differ by eight cycles/sec. These forks cover the range of the first seven partials on the six largest bells of the Dorothea Carlile Chime, and the work now reported is confined to those six bells.

With these tuning forks the pitches of the first seven partial tones were determined by the frequencies of the beats. On the average the frequencies thus determined differ from those found ten years ago by about one per cent. The present values are much the more reliable. The average deviation of several determinations is now almost always less than a tenth of one per cent.

Frequencies of Strike Notes.—Since a strike note does not beat with a tuning fork, its frequency must be found in some other way. A fork tuned to the octave below the fifth partial of a bell sounds to me a little sharper than the strike note, and it seemed desirable to have the judgments of a number of musicians as to the exact pitch of the strike note. Seven persons with some musical training—several of them professional musicians—were good enough to assist me. Each of them gave me from half an hour to an hour of his time. I selected three bells which are readily accessible, and each of the seven musicians gave me his judgment on each of the three bells. I would adjust a fork to a

pitch near the strike note and then ask whether the fork was sharper or flatter than the bell. This was repeated until it seemed that sufficient data had been obtained.

The results are given in Table I. It will be seen that the average frequency is in each case close to an octave below the fifth partial of the bell. It is also true however that the range from the lowest value on any

TABLE I.
FREQUENCIES OF STRIKE NOTES*

Observer	*f*	*g*	*a*
1	329	380	429
2	340	384	423
3	346	383	434
4	343	387	442
5	346	390	437
6	348	389	439
7	349	392	448
Average	343	386	436
Octave below fifth partial	345	387	438

* The observers are not listed in the order in which they gave their judgments, but beginning with the lowest values and going up to the highest.

one bell to the highest value on that same bell is in each case just about a half step.[5] Several of the musicians said that when fork and bell had nearly the same pitch some difference in the quality of the sounds made it difficult to decide which was the sharper. The average deviation of the values for one bell is in each case a little over one per cent, so that the difference between the average and the octave below the fifth partial is probably without significance. Although the range covered by the separate values is considerable, it may be reasonable to conclude that on these three bells the strike note is fairly close to an octave below the fifth partial.

Griesbacher's Method.—Griesbacher[6] has found that the strike note of a bell can be elicited by a tuning fork of the same pitch. The fork is struck and then pressed against the bell in such a way as to have a

[5] I find that some persons are not familiar with the terms *step* and *half step*. The same intervals are also called *tone* and *semitone*. Since the terms "step" and "half step" are in use I prefer not to use the word "tone" to indicate an interval between two tones.

[6] P. Griesbacher, Glockenmusik, pp. 53–55, Alfred Coppenrath (1927).

very small area of contact. Griesbacher finds it desirable to bring the stem of the fork, or even better a sharp edge near the bottom of one of the prongs, against the lip of the bell—where the inner and outer surfaces meet. I find it satisfactory to hold the axis of the fork at an angle of about 45° with the surface of the bell and then touch the stem lightly to the soundbow—on which the clapper strikes. The particular azimuth at which the fork is held seems not to be important. The tone elicited from the bell in this way is rather faint and of short duration, but the effect is clearly a real one.

Although a fork tuned to the octave below the fifth partial sounds to me a little sharper than the strike note, I find that a fork thus tuned brings out the strike note, and that a fork tuned so that it sounds to me to have more nearly the pitch of the strike note does not bring it out so well. This result is further evidence that the strike note is an octave below the fifth partial.

Beats from Octave of Fork.—It is possible to get beats from a bell and a fork when the fork is struck and the bell is not. Let a fork be tuned so that its pitch is near to that of one of the partial tones of a bell. Let the fork be struck, its stem pressed against the bell, and the fork then removed. Both bell and fork will now vibrate freely, and the tones from the two will beat. Even if the fork remains in contact with the bell beats may often be heard.

When a fork is tuned so as to elicit the strike note there are often similar beats. They are evanescent, but a few may be heard. If different settings of the fork are made and the beats in each case are timed, the frequencies calculated from the beats differ. But if we assume that the vibration of the fork contains the octave,[7] and that this octave beats with the fifth partial, I find that the frequencies calculated from the different settings of the fork are in agreement.[8]

I have used this same method to determine the frequencies of the seventh partials on five bells. This was possible because in the bells of the Dorothea Carlile Chime there is no partial close to an octave below the seventh partial. When I first determined these frequencies I had

[7] Although the ordinary normal modes of vibration of a tuning fork are inharmonic, it is nevertheless known that the octave may occur (See Helmholtz, Sensations of Tone, tr. Ellis, p. 54; Rayleigh, Theory of Sound, ed. 2, vol. 2, p. 463; Barton, Text-Book on Sound, arts. 309–314; Richardson, Sound p. 114). I had not supposed that the octave could be obtained with sufficient intensity to produce the above effect. The suggestion that it might be possible was made in a letter written to me by Professor A. Wilmer Duff.

[8] I have actually carried out this determination in connection with the fifth partial in only one case. But see the next paragraph with regard to the seventh partial.

no fork that went as high as the seventh partial for any bell in the Dorothea Carlile Chime except the largest. Later another fork was made for me, and the frequencies of the seventh partials found with this new fork agreed excellently with those already found by the beats of forks an octave lower.

When using this method the tone heard has the pitch of the fundamental of the fork, and it is that tone, rather than the one an octave higher, that sounds as if it were beating. This fact, and the possibility of making satisfactory determinations of the seventh partial in this way, seem to be good evidence that when a fork brings out the strike note of a bell it is really because the fifth partial is being elicited. It would seem then that this fifth partial heard along with the fundamental of the fork gives the impression of the strike note.

If this last statement is correct it seems to be additional evidence that the strike note of a bell is determined, except for octave, by the fifth partial.

Other Tests

Listening at a Distance.—If the fifth partial is sufficiently prominent to determine the note heard from a bell it seemed that this partial might still be prominent when the bell is heard from a distance. I made a few attempts to find out whether the pitch from a bell was the same when I was so far away that the sound was very faint. These attempts were few and not conclusive. The pitch seemed, however, not to be affected by distance.

I spent some time listening at a distance of about three hundred meters. The bells were struck on the soundbow, and in most cases by the regular clapper. I would adjust a fork to beat with one of the partial tones of a bell, give a signal to have that bell struck, and listen for the beats. The results seemed to have most significance when the bells were struck so gently that I could just detect the beats. The strength of blow that was necessary for the hearing of beats differed for the different partials.

The results are not as definite as I had hoped, but seem to be as follows: The fourth and sixth partials are not detected when other partials can be. This is to be expected from the fact[9] that the fourth and sixth partials are not brought out strongly when the bell is struck on the soundbow. The second partial can be detected from a distance

[9] I have pointed this out on p. 258 of reference 3.

when the first cannot. The third, fifth, and seventh are the most easily detected, and the fifth perhaps more easily than the third or the seventh. A number of times I heard the fifth partial of a bell begin to beat with the fork when the bell was struck so gently that the beats were the only evidence I had that the bell had been struck—I did not hear the stroke of the bell at all.

Time of Contact of Clapper.—An attempt was made to determine the time of contact between clapper and bell. This was to be accomplished by sending an electric current through an oscillograph, the circuit being closed during the time that the clapper was in contact with the bell. On most of the bells the electric connection between clapper and bell proved to be already so good that the blow of the clapper did not produce any marked change in resistance. An attempt was made to put over the clapper a thin insulating coating and then a thin layer of metal. But the compressibility of the material added is probably too great for the results obtained to be of any value.

The largest bell of the chime is provided with three clappers. Two of these are electrically insulated from the rest of the bell, and so needed no additional insulation. Making use of one of these clappers four satisfactory records were obtained. The times of contact were found to be: 700, 820, 760, and 830 microseconds. The average of these, 780 μ sec, is about 0.28, 0.48, and 0.72 of the period of the third, fifth, and seventh partials of this bell. That is, on this bell the clapper was in contact for about half the period of the fifth partial. The data are too meager to permit of drawing any general conclusions, but if the clapper is usually in contact for about half the period of the fifth partial, it would seem likely that the development of the fifth partial would be favored. The question is to be examined further during the coming summer.

Combination Tones

It was early suggested[10] that the strike note of a bell might be a combination tone. In my 1920 paper I listed those first order difference tones which might arise from the partials I had observed and which were within a half step from the strike note. With the more accurate determinations of frequency that I now have it seems worth while to examine this question again. Among the first seven partials of the six largest bells of the Dorothea Carlile Chime I now find that the only

[10] P. J. Blessing [Physikal. Zeitschr. 12, 597 (1911)] states that Rudolph König made this suggestion.

first order difference tones which lie within a half step from the octave below the fifth partial are these: The difference tone from the fourth and sixth partials on the *f* and *a* bells, and the difference tones from the fifth and seventh partials on all six bells. Since the fourth and sixth partials are neither of them brought out strongly when a bell is struck on the soundbow, and since the difference tones from the fourth and sixth partials are within the given range on only two of the bells, it seems that the difference tone from the fifth and seventh partials is the only one that need be considered. The difference tones from these partials are shown in Table II. On the average the difference tone from the fifth and seventh partials differs from the octave below the fifth partial by about a quarter of a half step.

Table II.

First Order Difference Tones from Fifth and Seventh Partials

Bell	Frequency of Octave below Fifth Partial = A	Cycles/sec to add to A to get Dif. Tone	Cycles/sec to add to A to raise Pitch one Half Step
eb	305.6	4.8	18.1
f	345.5	6.5	20.5
g	387.2	2.6	23.0
ab	411.0	12.0	24.4
a	438.4	6.3	26.1
bb	462.0	− 1.0	27.4

For frequencies in the neighborhood of 300 or 400 cycles/sec Knudsen[11] finds that the ear can detect changes in pitch of something like a twentieth of a half step. On the other hand Guttmann[12] finds that singers and violinists do not attain this accuracy. He finds that the pitch of a note may be regarded as satisfactory if it is not in error by more than one per cent, which is about a sixth of a half step. If Guttmann is right the difference tone from the fifth and seventh partials on the six largest bells of the Dorothea Carlile Chime may be regarded as agreeing moderately well with the strike note.

In my work on the Harkness Chime[4] I found that the frequencies of the fifth, seventh, and tenth partials have ratios not far from 2:3:4, so that a difference tone from the seventh and tenth partials would also lie not far from the strike note. On the Dorothea Carlile Chime I have

[11] Vern O. Knudsen, Phys. Rev., 21, 84 (1923). The value given in the text is calculated from the curve in Knudsen's Fig. 10.

[12] Alfred Guttmann, Zeitschr. f. Physchol., 58 [2d abt.], 209, 247 (1927).

not been able to determine the frequency of the tenth partial except on the two largest bells. On these two the relative frequencies of the fifth, seventh, and tenth partials are respectively: 2:3.01:4.17 and 2:3.01: 4.13. On these two bells the tenth partial seems to be too far from the double octave to contribute much to the strike note.

It may be that a difference tone from the fifth and seventh partials is of some importance in determining the strike note. On the other hand, difference tones are not usually heard unless the combining tones are fairly loud. The strike note of a bell can be heard when the bell is tapped very gently.

Conclusion

The work described in this article seems to indicate that the strike note of a bell is an octave below the fifth partial, and that the fifth partial is probably important in producing the strike note. It does not settle the question as to whether a difference tone from the fifth and seventh partials may be of importance, although that seems rather doubtful, and it gives no clear answer to the question why the pitch of the bell is judged to be a octave lower than the fifth partial.

I wish to express my gratitude to the seven musicians who gave me their judgments as to the pitches of the strike notes. They were: Dr. Frank E. Dow, Professor Rebecca W. Holmes, Professor Cary F. Jacob, Professor Wilson T. Moog, Mrs. William Allan Neilson, Mr. George C. Vieh, and Professor Roy D. Welch. I wish also to record my hearty thanks to Professor George W. Alderman, of the Department of Physics at the Massachusetts Agricultural College, who worked with me for a part of the summer. We have begun some further studies, but the results are not sufficiently definite for any report as yet. We intend to to continue the work next summer.

December, 23, 1929.

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ON THE STRIKE NOTE OF BELLS

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This article was translated by H. Leighton and T. Rossing from "Uber den Schlagton von Glocken," in Naturwissenschaften ***39:****697–701 (1933)*

INTRODUCTION

The problem of the strike note, the most important sound of a bell, has been treated in several investigations and publications without any comprehensive explanation having been reached. The sound spectrum of a bell differs substantially from those of other sound sources. For one thing, it is not the lowest partial that determines the pitch in a bell; in fact, the pitch does not even have to coincide with a physically measured partial of a bell. Furthermore, the partials of the bell spectrum do not in general arrange themselves in a harmonic series throughout, but rather they deviate from it in varying degrees. Therefore, in order to simply recognize them, we are using a numbering of the partials in order of increasing frequency. For purposes of comparison, in Table 1 we have added the customary identifications used by bell founders and bell inspectors.

All remaining partials, those lying between the fifth and seventh partials and between the seventh and eighth partials as well as those of higher frequency, are neglected in the present research. Figure 1 is a cross section of a bell showing the customary shape in Germany. The position of the stroke ring (sound bow), struck by the clapper on the inside or a hammer on the outside, is indicated by arrows.

2. STATUS OF STRIKE NOTE RESEARCH.

We designate as the strike note (strike tone) of a bell the pitch which is received by the ear at the moment of the clapper stroke. The strike

Table 1

Partial tone	*Customary names*
1	Underton; Unteroktave (Hum tone)
2	Hauptton, Grundton (Prime, fundamental)
3	Terz (Third)
4	Quinte (Fifth)
5	Oberoktave (Octave, nominal)
6	Dezime (Upper third)
7	Duodezime (Twelfth)
8	Oktave der Oberoktave (Upper octave)

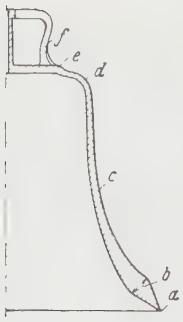

Figure 1. Bell cross section. a. Lip; b. Sound bow; c. Waist; d. Shoulder; e. Plate; f. Crown.

note is not identical with any of the partials presented on Table 1; however, it always lies in the region of partial note 2, and in certain bells, it may coincide with it in pitch. It is widely known that the strike note cannot be excited through resonance, and that it cannot be strengthened through resonators. Lord Rayleigh[1] found in his research on seven bells that the strike note lay one octave below the fifth partial, but that it coincided approximately with the tuning of the second partial only in two bells. Biehle,[2] in a thorough analysis of bell sound, deals with the vibration classification of the bell, starting with the surface vibrations. He labels the strike tone "an imaginary tone" that is the result of the totality of the real notes. Griesbacher,[3] in an extensive treatise on the strike tone, comes to the conclusion as a musician that "the octave and the strike tone always become entirely incorporated into each other. However, both the strike note and the octave are each a very definite sound entity, in no way concealed, but rather physically observable." According to him, the strike note is given by half the vibration frequency of the octave. Tyzzer[4] examined the vibration modes of bells in relation to number and location of the nodal lines, and expresses himself as follows: "In the bells examined here, the tuning (of the strike tone) seemed to be the same as the second partial, which lay approximately an octave lower than the fifth partial." In the last few years, extensive bell investigations have been undertaken by A. T. Jones. In a work from the year 1928,[5] he writes that "the best hypothesis that I see at the moment is that of the falsely-interpreted octave." In a work of 1930,[6] Jones comes to the conclusion that "The . . . research seems to show that the strike note of a bell lies one octave below the fifth partial, and that the fifth partial is probably important in the origin of the strike note. The research does not raise the question whether a difference tone between the fifth and seventh partial can be of significance, although this seems somehow dubious, and it does not give a clear answer as to why the pitch of a bell as an octave is determined below the fifth partial." In an investigation from the year 1931,[7] Jones and Alderman came to the following conclusion: "The present work gives us further proof that the strike tone of a bell is not present objectively in the air and that most likely it is not a combination tone . . . Up to now we see no sufficient reason why the ear values the pitch of a bell one octave

lower than the fifth partial." In a more recent investigation by these two authors,[8] which deals with the mechanical vibration forms of bells and the Fourier analysis of bell oscillograms, there is the following: "The spectra show further, in agreement with previous research, that when a bell is struck in the usual way, the most pronounced tone which appears first is the fifth partial."

3. NEW RESEARCH CONCERNING THE STRIKE NOTE.

On the basis of the strike note research outlined thus far, our goal was to undertake research on the strike note using the new procedures of electroacoustics, automatic sound analysis, and electrical filters. According to what has been presented thus far, there are three possibilities for the origin of the strike note: it is either physically present; or it originates as a physiological difference tone; or it is psychologically created, that is, it represents a kind of octave-deception. For each one of these possibilities, investigations were made using a special bell whose strike tone and principal tone were separated greatly in frequency.[9] The mass of the bell is 230 kg and its height is 70 cm. The strike note was given by the foundry as c^2 (C_5) + ⅜ semitone = approx. 530 Hz, and the second partial as h^1 (B_4) + ¼ semitone = 495 Hz. The bell was set outside. A striking apparatus, driven by an electric motor, released a hammer strike on the sound bow of the bell at intervals of 2.5 seconds and 1.2 seconds, respectively.

In order to first determine the position and strength of the individual partials, a sound spectrum was recorded by an automatic sound analysis with a search note according to Grützmacher.[10] The bell was struck constantly at the intervals indicated. The reception took place with a distortion-free condenser microphone whose frequency response was flat from approximately 50–10,000 Hz and whose nonlinear distortion was negligible. The microphone was 7 m from the bell. Figure 2 shows the total spectrum; the most important partials have the same numbers as in Table 1. The frequencies of the partials accurately measured are also given. We see that the hum tone and the fifth are equally small, a fact that corresponds to the actual circumstances. It should be noted that while receiving the spectra, the position of the microphone in relation to the bell has an important influence on the strengths of the partials. The reason is the interference field around the bell which has strong maxima and minima. The main result of many surveys, such as Figure 2, is that the strike note, with its frequency of 530 Hz, is missing. That is, it is not physically present, as indicated by all the above-mentioned investigations. Even with greater precision of analysis, there resulted no indication of the strike tone. The ordinate in Figure 2, that is the strength of the individual partials, is the time-averages over 0.2 s. In it there are included both the strike and the decay. It is also to be noted in the spectrum of Figure 2 that the lower partials have a different form than the higher ones. The lower tones sound sufficiently long after the strike that there is

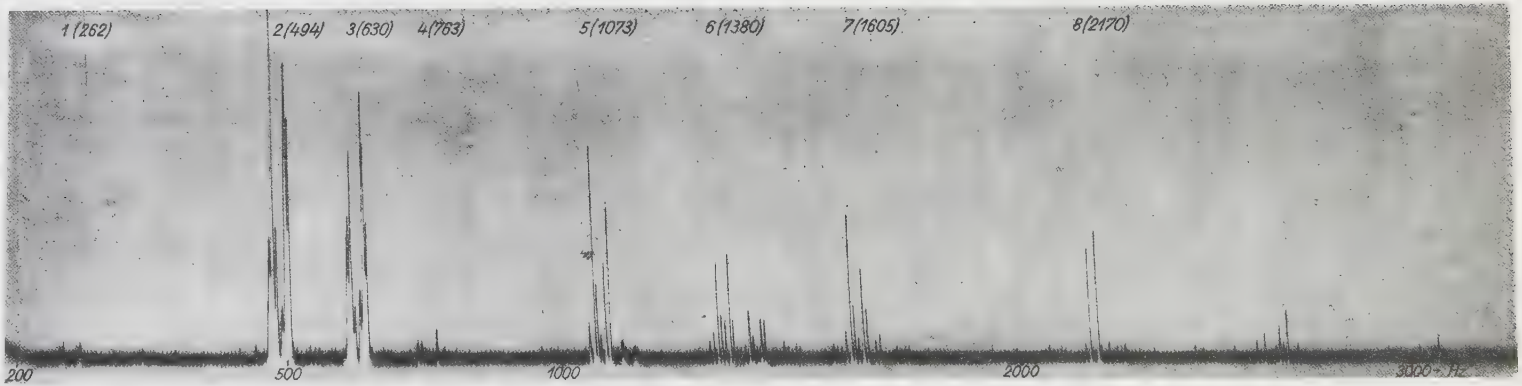

Figure 2. Sound spectrum of a bell.

always a residual amplitude shown on the indicator instrument during the frequency change of the search note in the vicinity of the partial. In the higher partials, the amplitude is present only for a short time after the strike; thus, the amplitude always goes down to zero. Figure 2 indicates qualitatively the various decay times of the partials. Starting with the upper octave, the decay times of the partials become significantly shorter. In order to investigate this further, two series of experiments were conducted. In one, an electrodynamic bone telephone receiver (as used in otology) was connected directly to the sound bow to excite the bell. The resonance curve of the vibration, driven with constant exciting force, was measured with an electromagnetic sound receiver. In the second experiment, with the same arrangement, instead of the resonance curve, the vibration amplitude was recorded on an oscillograph. Both experiments resulted more or less in the same findings; they gave dampings of 1.6 dB/s for the hum tone, 3.5 dB/s for the fundamental, 40 dB/s for the octave, and approx. 70 dB/s for the twelfth.[11] The resonance curves of some partials, depending on the place of excitation, also had double resonance curves. It has also been mentioned that the strike duration was examined by oscillographing the contact time. It amounted to 0.55 ms, that is, only a little longer than the half-period of the fifth partial. This time duration favors the excitation of the fifth partial, as Jones and Alderman[12] suggested.

In order to investigate the possibility of the origin of the strike note in a physiological way as a difference tone of the first order, the frequency difference between the individual partials must be examined. From this it is seen that the difference between the fifth and seventh partials with the frequencies 1072 and 1605 Hz is in fact suited to give the strike-note frequency. Through the research of Waetzmann[13] on carbon microphones, we have learned that these microphones possess a nonlinearity which is similar to that of the hearing organ. As is known, the ear applies to the received sound vibrations a kind of rectification, a property that the carbon microphone also possesses. This generates a difference tone of first order. Figure 3a shows a frequency analysis between the fundamental and the third conducted with the distortion-free condenser microphone. Figure 3b shows the same reception with an O. B. telephone receiver microphone. In both cases the microphone was positioned at a distance of one meter from the bell. It is clearly evident in the second instance how the difference tone follows with great intensity directly after the main note—its frequency is approximately 533 Hz. With this, the strike tone has been proven objectively, in a certain sense, for the first time. Receptions with the Reiss microphone, which as a carbon microphone possesses a nonlinear distortion, but to a minimal degree, brought the same results.

In the same way it is possible to determine objectively the time duration of the strike tone. For this purpose, after the microphone and amplifier, electric filter chains were inserted so that only the frequency

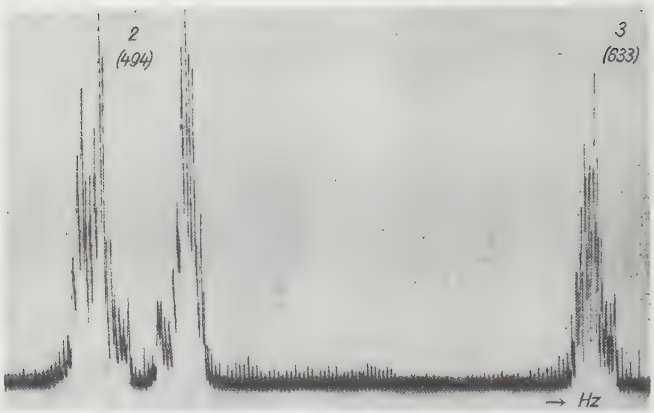

Fig. 3a.

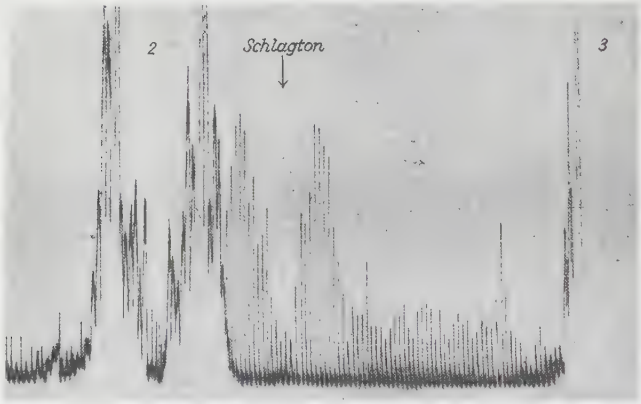

Fig. 3b.

Figure 3. Sound spectrum between the fundamental and the third a. with a condenser microphone; b. with a carbon microphone.

region 490–570 Hz (i.e. the region around the fundamental and strike tone) could be oscillographed. If this is done with the condenser microphone, we get the picture shown in Figure 4a, which shows the gradual decay of the main note after the strike. The small modulation of the fundamental at a frequency 136 Hz stems from the third (630 Hz) which, despite the effect of the filter chains, still comes through somewhat and creates vibrations together with the fundamental. However, the picture is totally different when the telephone-receiver microphone is used to receive sound (Figure 4b; the upper curve is the time signal). At the beginning of the oscillogram we receive very pronounced vibrations with an average frequency of 39 Hz; that is, aside from the fundamental at 494 Hz, there is also present the strike tone at approx. 533 Hz. After approx. 1.2 s, the strike tone has died off. These values agree with the subjective observations of Griesbacher, who gives one

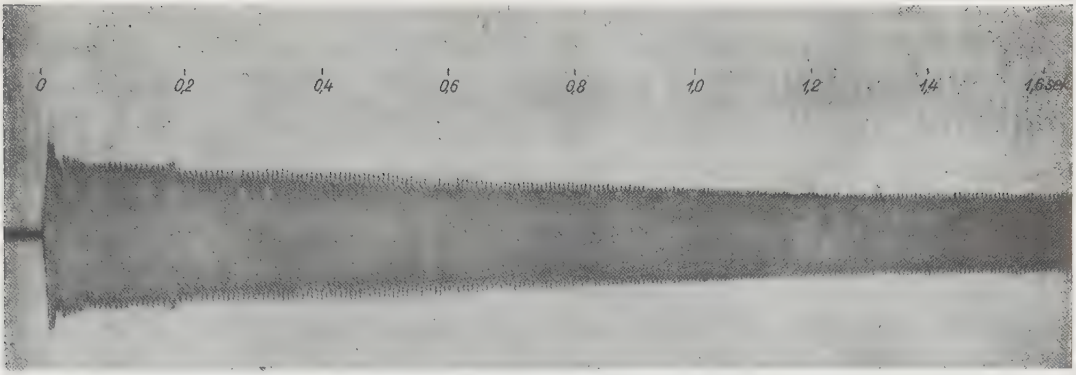

Fig. 4 a.

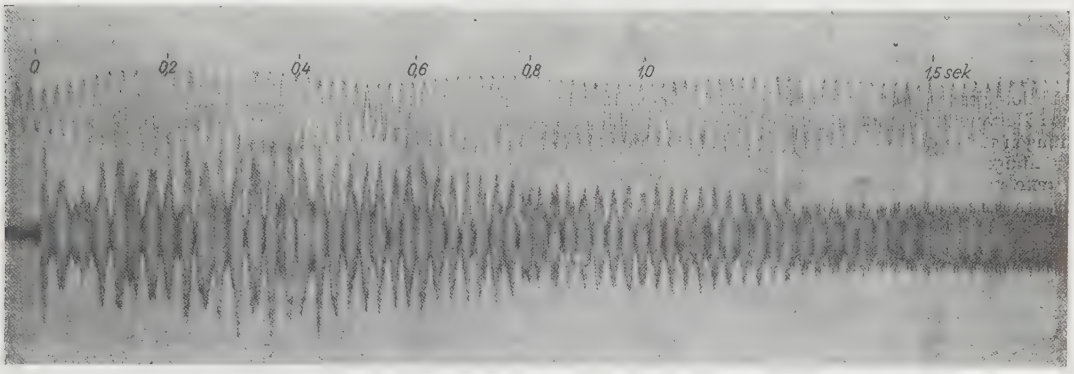

Fig. 4 b.

Figure 4. Decay curves of the fundamental a. with a condenser microphone; b. with a carbon microphone.

second as the strike note duration. Professor Biehle, the famous bell expert, was so kind as to measure the time duration of the present bell with a stopwatch. He determined it as approximately 5/4 s, without any knowledge of the above-mentioned oscillogram.

These experiments show sufficiently that in the present bell we can regard the strike note as the physiologically objective difference tone of two partials. Aside from the strike note, the reception with the carbon microphone further shows, of course, a series of newly originating partials, especially at low frequencies. Even if the frequency characteristics of the carbon microphone do not agree at all with the characteristics of the ear, and its nonlinearity agrees only incompletely with it, it is still to be expected that in the ear there will also appear other combinations tones aside from the strike note difference tone. Why they do not play a role is still debatable.

There is still a third possibility, that is to regard the psychological origin of the strike note as a sort of undertone of a partial, specifically, of the fifth partial (the octave). This is a real possibility, since the strike

tone has almost half the frequency of the octave (1073 Hz). In order to test this hypothesis, the bell sounds were reproduced with the help of a very good electric sound-transmitting apparatus.[14] This apparatus consisted of a condenser microphone, an amplifier, and an electro-dynamic and electrostatic loudspeaker. It was controlled by a special electrical network so that the frequency curve in the region of 30–15,000 Hz did not fluctuate more than ± 5 dB. The reproduced sound was thus exceptionally similar to the original. With the help of filter chains, the high frequencies of the series could be built up. Professor Biehle again was so kind as to give his expert opinion concerning this experimental series. Only the first two partials were presented to him in the first experiment; in the second, only the first three partials; in the third, only the first four, etc. The strike tone was not heard. Even in the most important case, when all partials up to the fifth (that is, up to and including the octave) were transmitted, the strike note was not perceived. Only when the seventh partial was also added did the strike tone appear with full clarity. This result of subjective observation shows, then, that in the case of the present bell, the strike tone does not originate psychologically as octave-deception from the octave, but rather that the basis is physiological. It originates as a difference tone from two primary partials. Of course, it should be noted that the strike tone undoubtedly receives substantial support in its audibility because objective partials exist in an almost harmonic relationship to it. The fifth and seventh partials are to it as the octave and twelfth, respectively. Herein apparently also lies the specific reason why the strike tone/difference tone is chosen from all other combination tones by the ear. It also follows that the octave is almost a harmonic of the strike note, in which case, the latter is situated approximately in the middle between twice the frequency of the hum tone and half the frequency of the octave.

That the octave alone is not decisive for the strike tone is also to be concluded from a statistical compilation given us by a foundry and a bell expert; it deals with over 500 bell analyses. All of them are from the last five years and are from bells of different foundries, partly also from old bells (fifteenth-sixteenth centuries). From a total of 322 bells examined by the foundry, in 49 of the bells there resulted complete agreement of the strike tone with half the frequency of the octave. For 251 of the bells, half the octave averaged about ¼ of a semitone higher; for 22 of the bells it averaged about ⅙ of a semitone lower. In addition, in 28 bells there appear differences of ½ a semitone and more, so that half the frequency of the octave lies above the strike tone. Thus, half the octave coincides with the strike tone in only 15.2% of the bells. Compilation of the results of experiments by bell experts shows the following relationship: The strike tone corresponds to half the octave in 38 (17%) of 233 bells examined. In 134 bells (60%), the deviation is greater than ¼ a semitone. In evaluating this result, it is important that the above analyses were conducted merely to evaluate the bells musically and were not meant to serve as any kind of

experiment on the position of the strike tone. Determination of the strike tone happened in the following manner: a tuning fork equipped with adjustable weights was set on the pitch of the strike tone felt by the ear. The numbers immediately contradict the supposition that the strike tone can be equated to half the octave.

In summary, the following results are observed from the examination of this specially prepared bell:

The strike tone is not physically observable. It originates in the present case as a physiological difference tone between objectively present partials, and as such can be determined also objectively in its origin, its size, and its time duration through appropriate arrangements. The strike tone has the advantage over other combination tones of similar origin in that it is psychologically prominent through its harmony with physically present partials. These results were derived from only one bell. They are supported by a statistical compilation of qualitative results on many bells. In order to make this a general result, it is necessary to examine an even greater series of bells, according to the methods outlined here.

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12. See Ref. 8.
13. E. Waetzmann, Ann. Physik, *42,* 729 (1913), Z. F. Physik *1,* 271, 416. (1920); compare also E. Meyer, Elektr. Nach-Techn. *5* 398 (1928).
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The Sound of Bells

Jottings from my Experiences in the Domain of the Sound of Bells

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(Received September 25, 1937)

THE STRIKE NOTE OF BELLS

THE strike note of bells has been discussed by A. T. Jones,[1] and it may be assumed therefore that the reader is acquainted with the strike note problem.

It is some ten years ago now that I wrote to the late P. Griesbacher as follows: "I wonder why you don't mention the seventh partial in *Glockenmusik*. Yet experience of the last few years taught me that this apparently unsignificant matter is of great importance for the exact determination of the strike note."[2]

From the moment that I noticed this phenomenon, originally strange but explainable, I have always, whenever an opportunity offered, noted the seventh partial together with the fifth partial with a view to be able to make out whether the mysterious strike note might be explainable as difference tone. But soon I verified that this could not be true, the tone-relations of the 5th and 7th partials being usually of such a nature as to exclude the possibility of the strike note being a difference tone.

In order not to take up too much space I submit three tables, which will sufficiently prove and explain my allegation. I confine myself to these three though I could add a dozen similar cases.

Table I shows the octaves below the fifth partials and the frequencies of the fifth and seventh partial of two bells in the tower at Maestricht, Limburg, the Netherlands; when these bells sound together, the two strike notes are heard in the interval of a minor third. Careful experimenting showed that the 5th and 7th of bell I are represented as d″ 8/16–a″ 10/16, respectively, the 5th and 7th of bell II as f″ 7/16–c‴ 5/16.

This table clearly shows that if the strike note should be originated by the fifth–seventh, the strike note as difference tone is excluded; should this be the case, not a minor third would be heard when the two bells rang together, but the major second d′ 13/16–f′ 0/0. The octaves d′ 8/16–f′ 7/16 below the fifth partials on the other hand are quite concordant with the minor third interval heard.

Table II shows the octaves below the 5th partials and the tone relations of the 5th and 7th of two bells in the tower of Vlokhoven, Noord-Brabant. When the two bells are rung together, a good major second interval is heard. Were the difference tones e♭′ 7/16–e′ 13/16 to decide of the strike notes, not a major but a minor second would be heard. So here too the octaves below the fifth partials are in favor of the strike note hypothesis.

The tone relations of the 5th and 7th in Table III are of two bells, of Breda, Noord-Brabant. When rung together, here too the major second is heard. Again, were the difference tones b♭′ 7/16–d♭″ 1/16 to decide of the strike notes, the minor third and not the major second would be heard.

Conclusion: "The results of this test appear to favor the octave hypothesis rather than the difference tone hypothesis."

I for one am of opinion that the fifth partial, both in the case of small bells and of big ones, originates the strike note. In defining the objectively nonexisting strike note, my only guide is the fifth; a guide fully to be relied on even in the most critical cases.

TABLE I.

	BELL I		BELL II	
	CYCLES/SEC.	NOTE*	CYCLES/SEC.	NOTE*
Seventh	901.6 =	a″ 10/16	1053.6 =	c‴ 5/16
Fifth	597.6 =	d″ 8/16	708.2 =	f″ 7/16
Difference tone	304.0 =	d′ 13/16	345.4 =	f′ 0/0
Octave below the fifth partial	298.8 =	d′ 8/16	354.1 =	f′ 7/16

* The fraction beside the name of the note is part of a half-step of an equally tempered tuning fork. The fraction on the left of the note-name points to a minus, the fraction on the right to a plus.

[1] A. T. Jones, J. Acous. Soc. Am. 8, 199–202 (1937).
[2] P. Griesbacher, *Glockenmusik* (Nachtrag, 1929), p. 43.

TABLE II.

	Bell I		Bell II	
	Cycles/sec.	Note	Cycles/sec.	Note
Seventh	935.2 =	b♭″ 4/16	1042.0 =	c‴ 2/16
Fifth	619.6 =	e♭″ 2/16	700.6 =	f″ 4/16
Difference tone	315.6 =	e♭′ 7/16	341.4 =	e′ 13/16
Octave below the fifth partial	309.8 =	e♭′ 2/16	350.3 =	f′ 4/16

TABLE III.

	Bell I		Bell II	
	Cycles/sec.	Note	Cycles/sec.	Note
Seventh	1421.6 =	f‴ 8/16	1618.8 =	g‴ 12/16
Fifth	948.8 =	b♭″ 8/16	1068.8 =	c‴ 9/16
Difference tone	472.8 =	b♭′ 7/16	550.0 =	d♭″ 1/16
Octave below the fifth partial	474.4 =	b♭′ 8/16	534.4 =	c″ 9/16

The late P. Griesbacher once asked me:[3] "Just tell me: How do you test bells higher than a″? As our tuning forks reach only to a‴, the method with the fifth leaves you in the lurch already at b″. *And is it your opinion too, that at that height the strike note somewhat loses its influence on the sound and that often the second and oftener still the first partial is predominant?*"

My answer: Last year I was sent two small bells for examination, bells of exactly the same diameter (pl.m. 40 cm) but formed after different profiles. The two bells were before me with mouths turned upwards. When struck respectively with a mallet there was no doubt about the strike note of the *A* bell being decidedly a quarter of a step below that of the *B* bell. After careful experimenting I obtained the tone relations as shown in Table IV. As the 5ths of the two bells were both of the same frequency the strike note concordance must be all right, I thought. At my request I had the two bells mounted for ringing and lo: when the two bells rang together it was as if only one bell was ringing, so perfect was the concordance between the two strike notes. So originally the distinct coming forth of the two first partials, differing about a quarter of a step, was misleading.[4]

TABLE IV.

	A Bell	*B* Bell
Fifth	c⁗ 12/16	c⁗ 11/16
Fourth	g‴ 12/16	g‴ 12/16
Third	e♭‴ 10/16	e♭‴ 12/16
Second	c‴ 5/16	2/16 c‴
First	2/16 c″	c″ 7/16

Is a Bell Tone, Obtained with a Fork, Tuned an Octave Lower, an Absolutely Reliable Picture of the Frequence Relation 1 : 2?

For many years I have been using a set of tuning forks (Edelmann) with a total range of F♯ to a‴ included, to investigate bell tones, of both church bells and carillon bells. As I could not afford to acquire the whole set at once on account of the great expense, I bought only one fork, the fork with a range of d♯′ to a′. As the range of the fork was rather small, the fork was of little use to me; the range of the tones of church bells and carillon bells being very extensive. Therefore I asked myself the question: Is it possible to bring forth the bell tones lying outside my tuning fork, as a harmonic overtone. And behold, by my first experiment I found that with a fork tuned an octave or duodecimo lower, the bell tone, which was an octave or duodecimo higher, reacted clearly and distinctly upon it. But a second question arose: Is the result arrived at in this way to be trusted; is the aliquot tone produced in this way, an absolutely reliable picture of the frequence relation: 1 : 2, 1 : 3?

Gradually I came into the possession of the whole set of nine forks. I thought now to be able, to prove the reliability of my method. I experimented on the bell in the belfry of our college, with a diameter of about 38 cm. This bell has the 4 following partial tones: first: c″ 8/16; second: b″ 15/16; third: e♭‴ 7/16; fourth: g♭‴ 15/16.

[3] Letter: 20, X (1930).

[4] My Dutch readers are informed that the firm of Petit & Fritsen, bell-founders at Aarle-Rixtel, N.B. found the case so interesting and convincing that they keep the two bells mounted in their show room in order to give those interested at any time an opportunity to verify the fact.

TABLE V. *The tones in column 1, were acquired with the normally pitched tuning fork. The fork tones in columns 2, 3, 4 and 5, bring forth as first aliquot tones, the tones of column 1.*

Fifth	c'''' 6/16	c''' 6/16	f'' 6/16	c'' 5/16	a♭' 8/16
Fourth	g♭''' 15/16	g'' 0/0	b' 15/16	g♭' 15/16	e♭' 0/0
Third	e♭''' 7/16	e♭'' 7/16	a♭' 8/16	e♭' 7/16	b 9/16
Second	b'' 15/16	c'' 0/0	e' 15/16	c' 0/0	a♭ 4/16
First	c'' 8/16	c' 8/16	f 10/16	c 10/16	A♭ 10/16
	1	2	3	4	5

When setting the fork an octave lower to the first, so to c' 8/16, the first partial very distinctly was brought out.[5] The second b'' 15/16 reacted very weakly on b' 15/16 but distinctly came out when the fork was tuned on c'' 0/0. The third e♭''' 7/16 clearly answered the setting e♭'' 7/16. The fourth g♭''' 15/16 on the other hand, only clearly came forth on a g'' 0/0 setting. Now I tried whether I could elicit the same partials (plus the fifth) also as 3rd, 4th and 5th aliquot tones. In this I succeeded perfectly, but, as Table V shows the results are not all in the exact frequence relations: 1 : 2, 1 : 3, 1 : 4, 1 : 5.

In all my subsequent experiments I observed the peculiar phenomenon, that sometimes a fork tuned a pure octave lower, produced the pure over-octave and sometimes not. How to account for it, that the results are so different? Is the cause to be found in the physical nature of the bell or of the tuning fork? (Table VI.)

AN UNKNOWN (?) BELL TONE

I am in the possession of a small bell (weight about 12 kg). One day as I was experimenting on this bell I suddenly noticed a tone I had not heard so far. Since that time I regularly meet with this unknown (?) tone in small bells. This tone has the peculiarity (a) it is very modest in its appearance, (b) it does not react upon the tuning fork. It is possible, however, to make it free of beats to respond to the tuning fork and consequently its pitch exactly definable. Its singing-out time is of the same length or nearly so as that of the 3rd partial and therefore I presume that it is closely related with the third partial.

[5] In order to arrive at a wholly reliable result especially in the case of the lower bell tones, such as the first and the second, it is desirable to put a piece of thin cloth on the soundbow, to prevent the fork from jumping up.

TABLE VI. *The partial tones in column 1 were acquired with the normally tuned fork; the fork tones in column 2 bring forth (with a fork tuned a pure (?) octave lower) the tones of column 1.*

		1.	2.
Bell I	Third	f♯'' 5/16	f♯' 5/16
	Second	d♯'' 5/16	d♯' 3/16
	First	d♯' 7/16	d♯ 8/16
Bell II	Fourth	g♯''' 11/16	g♯'' 11/16
	Third	e''' 8/16	e'' 5/16
	Second	c♯''' 7/16	c♯'' 4/16
	First	c♯'' 7/16	c♯' 7/16
Bell III	Fourth	g''' 8/16	g'' 8/16
	Third	e♭''' 10/16	e♭'' 8/16
	Second	4/16 c'''	2/16 c''
	First	c'' 6/16	c' 8/16
Bell IV	Fourth	g''' 11/16	g'' 10/16
	Third	e♭''' 9/16	e♭'' 7/16
	Second	c''' 4/16	c'' 4/16
	First	4/16 c''	1/16 c'
Bell V	Fourth	f''' 1/16	1/16 f''
	Third	d♭''' 7/16	d♭'' 4/16
	Second	b♭'' 1/16	b♭' 2/16
	First	b♭' 1/16	b♭ 2/16
Bell VI	Fifth	g♯''' 0/0	g♯'' 0/0
	Fourth	d♯''' 0/0	6/16 d♯''
	Third	b'' 0/0	b' 0/0
	Second	1/16 g♯''	3/16 g♯'
	First	1/16 g♯'	2/16 g♯

I should be inclined to call it the "acoustic tone" on account of its objective nonexistence. The first and second partial of my study-object are affected with beats. The third too has this defect. Now it is a well-known fact, that, if a partial sings out with beats, there are two sector groups, which each of them send out separate tones, only slightly differing in pitch.

As it happens, with my bell the sectors of the 1st and 2nd are quite congruent.[6] If I make the bell vibrate by means of the clapper in one of

TABLE VII.

SINGING-OUT SECONDS	*A* SECTORS	PARTIALS ACOUSTIC	*B* SECTORS	SINGING-OUT SECONDS
0.5	f'''' 15/16	Fourth	f'''' 15/16	0.5
3.	d'''' 4/16	Third	d'''' 3/16	3.
5.	b''' 1/16	Second	b''' 5/16	5.
3.	g''' 2/16	*Acoustic*	g♯''' 0/0	3.
15.	5/16 b''	First	8/16 b''	15.

[6] Incidentally I wish to draw the reader's attention to the possibly unknown fact that the position of the nodal meridians and specially that of the nodal circles is equally well definable by means of the adjustable tuning forks as with the resonators or any other instruments adapted to the purpose.

TABLE VIII.

	Bell 1		Bell 2		Bell 3		Bell 4		Bell 5		Bell 6		Bell 7	
P.A.	N.	S.S.	N.	S.S.	N.	S.S.	N.	S.S.	N.	S.S.	N.	S.S.	N.	S.S.
Third	c''' 0/0	9	e♭''' 10/16	7	e♭''' 9/16	7	e♭''' 1/16	7	f''' 0/0	6	d♭''' 7/16	11	0/0 b''	11
Second	1/16 a''	12	4/16 c'''	11	c''' 4/16	9	5/16 c'''	10	d''' 7/16	10	b♭'' 1/16	13	1/16 g#''	14
Acoust.	f'' 0/0	8	g'' 1/16	7	a♭'' 4/16	6	g'' 10/16	7	a♭'' 0/0	6	f#'' 2/16	11	e'' 1/16	10
First	a' 1/16	27	c'' 6/16	25	4/16 c''	26	3/16 c''	30	12/16 d''	22	b♭' 1/16	47	1/16 g#'	36

The initials P.A., N. and S.S. respectively indicate: partial/acoustic, note and singing-out seconds.

the *A* sectors, the elusive tone comes forth as g''' 2/16; if the bell is struck in the middle of one of the *B* sectors another tone presents itself, namely g♯''' 0/0. In the beginning you have to strain your ears to discern it from the other bell tones; once its acquaintance thoroughly made, it is rather easily discernable.

A synoptic table of the tone relations and singing-out-time is given in Table VII. For clearness sake of the above-said, I shall call the two sector groups *A* and *B*. Now I began experimenting on a 50 kilogram bell and discovered between the 1st partial c'' and the second partial c''' a tone of analogous character g'', which took up 4 seconds in singing-out. Since that time I have often been observing this "super" but have not succeeded so far in defining its origin. I met it not only in bells of Netherlands make, but also in many bells, found abroad.

I give by way of elucidation (Table VIII) the 1st, 2nd and 3rd partials, besides the acoustic tones with the singing-out times proper to them, of seven bells. May I have stimulated hereby to the study of this problem, of which, up till now, I have not been able to give a satisfactory explanation.

TEMPERATURE INFLUENCE ON BELLS

The late Griesbacher came by experiments[7] to the conclusion that the bell is not subject to temperature changes.[8]

Some years ago—it was an exceptionally hot August—I took in the evening a bell into a room and tuned the forks carefully i.e., free of beats to the 1st, 2nd and 3rd partial. The next day in the forenoon, the thermometer in the room where the tuning forks and the bell had spent the night, pointed to 18° Celsius. I controlled the fork tones together with the bell tones and the agreement was complete as the evening before. The forks were left in the room (18°C) and bell and thermometer were exposed to the sun for an hour. After an hour the thermometer pointed to 31°C. Now I brought the forks into contact with the bell one after another and counted

for the 1st partial 14 beats per 10 seconds
" " 2nd " 29 " " " "
" " 3rd " 33 " " " "

Some time afterwards I came to the same result on another bell:

for the 1st partial 10 beats per 10 seconds
" " 2nd " 20 " " " "
" " 3rd " 24 " " " "

These results clearly prove the influence of the temperature on the bell body and consequently on the tones produced. Incidentally I wish to draw my readers attention to the ratio of beats: 14, 29, 33, and 10, 20, 24 which is in agreement with frequence proportion: 1 : 2 (first–second) 1 : 2, 4 (first–minor third).

[7] If one wants to make this experiment by means of the tuning forks, it is necessary to handle it carefully as it is rather sensitive to temperature changes.

[8] Reference 2, pages 46–47.

14B

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The Sounds of Bells

Jottings from my Experiences with the Sound of Bells

The Secondary Strike Note

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(Received October 17, 1938)

INTRODUCTION

YEARS ago I took cognizance of an essay by P. J. Blessing[1] in which he expressed his opinion about the function of the bell-body in the formation of the first eight partial tones. What Blessing put forward was a revelation to the present writer (then a rather inexperienced novice in the field of bell-sounds). Mr. Blessing says: "In our bells occurs in one certain place a horizontal nodal line, round about the bell—a nodal circle, about one handbreadth above the soundbow." Blessing wrote of only one nodal circle, and this one only for partials that have even numbers.

Some ten years later I came into possession of a series of Edelmann's adjustable tuning forks, and it occurred to me that if a fork should be tuned to some partial of a bell and then placed on the nodal circle, the partial would not answer there. In this way I tested Mr. Blessing's statements on the largest of the bells in the municipal tower of St. Michielsgestel. The base-diameter of this bell measures 110 cm.

For the first partial I found that the bell reacted clearly to the fork at all levels. The second partial was quite mute about one handbreadth above the soundbow. One centimeter above this dead point the second partial was brought out again, though extremely weakly; and at two and three centimeters the sound revived more and more to regain its full strength higher up. Up to the shoulder I could not find another dead point.

For the third partial Mr. Blessing says there is no nodal circle. From the lip upward I found the third partial answering spontaneously as far as about two handbreadths above the soundbow; from there upward it became weaker and weaker, to fall quite silent about halfway. Higher up it came forth again clearer and clearer to regain gradually the lip intensity. So this did not tally with Mr. Blessing's experiment. In the same way I proceeded with other partials.

To locate the levels of the nodal circles I make use of a unit proposed by Müller.[2] He divides the diameter of a bell into fifteen equal parts, and he calls one of these parts a *schlag*. For bells which are in tune with themselves (or quite nearly so), and whose shoulder is at about 11.2 *schlags*, I find nodal circles on the outer side of the bell at about the levels given in Table I. From time to time bells are found in which certain nodal circles cannot be determined with certainty, so that I wonder if in many cases a nodal circle may be spoken of.

The levels given in Table I agree fairly well with those obtained on other bells by Rayleigh[3] and by Jones and Alderman.[4] The levels given in Table I are shown in Fig. 1.

It has been pointed out[5] that when a bell is struck at about the level of a nodal circle the corresponding partial is not heard or only faintly. So during the ringing or striking the

TABLE I. *Levels of certain nodal circles.*

Number of Partial	Pitch if the Strike Note is *c'*	Levels of Nodal Circles (*schlags*)	
2	*c'*	3.6	
3	*e♭'*		6.2
4	*g'*	1.9	
5	*c''*		5.5
6*a*	*e''*	2.2	
6*b*	*f''*	2.2,	6.0
7	*g''*		6.3
10	*c'''*		6.4
6*a*-octave	*e'''*	2.0	
6*b*-octave	*f'''*		5.6

[1] P. J. Blessing, *Harmonie der Glocke* (Gregorius-Blatt, 1906), pp. 4, 16, 27, 40, 52, 65, 77, 90, 109.

[2] Ludwig Müller, *Die Bronzewaaren-Fabrikation* (A. Hartleben's Verlag, 1877).

[3] Lord Rayleigh, Phil. Mag. **29**, 1 (1890).

[4] A. T. Jones and G. W. Alderman, J. Acous. Soc. Am. **4**, 331 (1933).

[5] A. T. Jones, Phys. Rev. **31**, 1098 (1928).

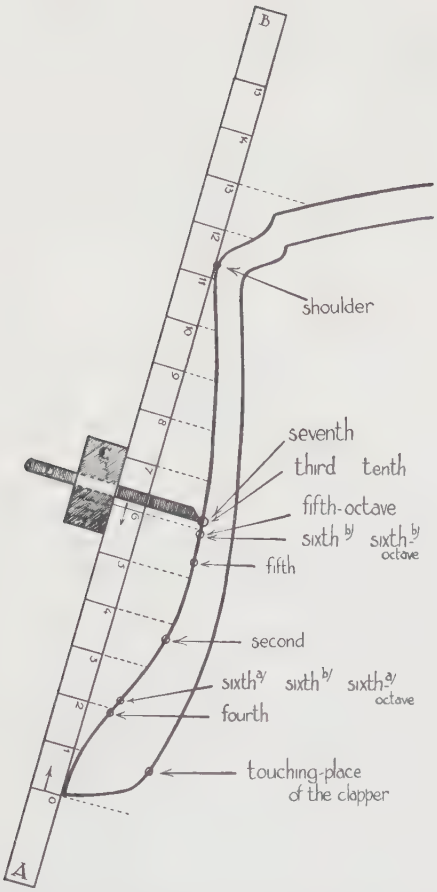

FIG. 1. Part of section of a bell. AB is a lath on which the diameter of the mouth of the bell is divided into fifteen equal parts. Along the lath moves the adjustable square CD.

partials 4, 6*a*, 6*b*, and 6*a*-octave will not be heard or very weakly.

THE SECONDARY STRIKE NOTE

Heavy- and middleweight bells (of about 800 kg and upwards) almost always send forth besides their strike note a tone which by its intensely clear, metallic character and timbre, its short duration (I did not come across any that took longer than eight seconds for dying out), in one word in its whole appearance, has all the characteristics of a strike note. In the case of some bells this tone occurs so predominantly that it equals or even supersedes the primary strike note. Because of its corresponding to the primary strike note I should be inclined to call it a *secondary strike note.* When attentively listened to and controlled, e.g., by means of a piano, it will prove to correspond with the tone which is a fourth above the fifth partial; thus being an eleventh above the primary strike note.

As the secondary strike note is apt to come out so strongly as to equal the primary strike note (though there are heavy bells in which it is not or hardly heard), it is obvious that this tone cannot be originated by the partial 6*b*, as the nodal circle of the partial 6*b* lies in the proximity of the spot where the clapper touches the bell. I presume that in the same way as the primary strike note owes its origin to the partial that is a pure octave higher,[6] the secondary strike note is due to the partial that is a pure octave higher; namely, the 6*b*-octave partial.

I have come to this conclusion after repeated and varied experiments. One of these was as follows. From the street I listened to three bells and defined their pitch by means of adjustable tuning forks; then I climbed up to the bells to investigate their 6*b* partial and 6*b*-octave partial. In all three cases the secondary strike note pitch noted in the street, exactly agreed with the 6*b*-octave partial, whereas the pitch of the 6*b* partial considerably deviated. These three bells are in the steeple of St. John's Cathedral at Hertogenbosch. Table II gives the tone relations.[7]

I sometimes came across bells where the secondary strike note was not a fourth but a major third above the fifth partial. Such bells, however, are rare, as far as I know. In such

TABLE II. *Secondary strike notes.*

FOUNDER AND YEAR	JACOB NOTEMAN 1641	F. AND P. HEMONY 1644	A. VAN AERSCHODT 1872
WEIGHT (kg)	5500	2350	1980
6*b*-octave	15/16 *c'''*	*f'''* 4/16	*g'''* 3/16
6*a*-octave	28/16 *b''*	*e'''* 6/16	23/16 *f#* . . .
6*b*	11/16 *c''*	*f''* 0/0	*g''* 1/16
6*a*	none	*e''* 0/0	13/16 *f#''*
Sec. strike note	15/16 *c''*	*f''* 4/16	*g''* 3/16
5	6/16 *g'*	2/16 *c''*	5/16 *d''*

[6] Jan Arts, J. Acous. Soc. Am. 9, 344 (1938).

[7] The fraction beside the name of the note is part of a half-step of an equally tempered tuning fork. The fraction on the left of the note-name points to a minus, the fraction on the right to a plus.

cases might the origin of the secondary strike note lie in the predominating character of the 6*a* partial or the 6*a*-octave partial? I think not, as both of these partials have their nodal circles on the soundbow. According to my experiments it must also be due to the presence of the 6*b*-octave partial; the fourth of this being too small in this case. If Jacob Noteman's bell is listened to when ringing or striking, the secondary strike note is not heard as a pure fourth above the fifth partial but as a slightly sharp major third which is quite in accord with the 6*b*-octave partial.

Conclusion

Whether the secondary strike note is heard as a tenth or eleventh (measured from the primary strike note) in both cases the 6*b*-octave partial is responsible for it.

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THE STRIKE NOTE OF BELLS

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The physical sound spectrum of a bell consists of a large number of characteristic eigenfrequencies of partials, which can be distinguished as belonging to various groups determined by the number and position of circular nodal lines. For each group, the eigenfrequencies are determined by the number of meridian nodes S, of which the minimum number is two.

Within each group the eigenfrequencies as a function of S lie on a curve having a certain hyperbolic character (Fig. 1).

When one hears a bell, one naturally ascribes to the sound a certain pitch, *the strike note*. With some musical skill one can hear approximately the lowest eight partials by paying attention to the respective tones. These are given in Table 1 for groups I and II, based on a tuning in C. If one concentrates on these partials when listening to a ringing bell, it is a remarkable experience to hear them with every strike. The minor third is quite conspicuous and it diminishes only slightly in loudness between strikes. The lowest partial is more difficult because one has to "look for it so low," and because it goes on almost continuously. For this reason the English correctly call it the hum note. Physically, this can be explained by the fact that the sounding period of a partial is approximately inversely proportional to the frequency, and therefore differs by a factor of two per octave. Listening to the twelfth (Dutch: duodeciem) is a musical surprise. It lies so much higher and sounds so much thinner than the strike note that one can hardly imagine that it really belongs to the bell's sound. Once one has noticed it, however, one hears it in every strike. In Table 1 we have indicated a strike note that has the same pitch as the fundamental (Dutch: priem). At present, this is striven for deliberately by means of appropriate tuning,[1] but in older bells this is by no means a fixed rule.

Here we touch on the core of the problem of the strike note. Rayleigh[2] observed that none of the eigenfrequencies of the bell correspond to the pitch of the strike note. This has since been confirmed by many investigators and we are therefore confronted with a very curious phenomenon: *the strike note does not occur as a partial in the physical spectrum*. Rayleigh also proposed the following empirical rule: *the pitch of the strike note is one octave below that of the fifth partial (the octave)*. This "octave rule" has been shown to be a good approximation to this day.[3,4,5] It was, however, a thorny problem for later generations of acousticians. In what other cases do we encounter the phenomenon that

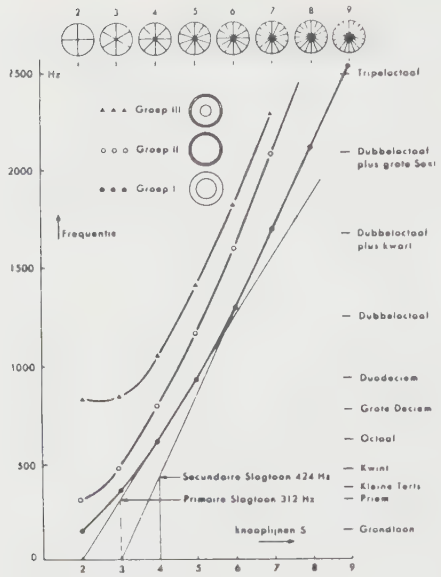

Figure 1. Eigenfrequencies of the partials as a function of the number of nodal lines for groups I, II and III (according to Lehr) and the construction of the primary and secondary strike note of group I. The octave and the twelfth lie on a straight line which intersects the horizontal axis at the point S = 2. They are thus harmonic in the ratio 2:3, and they produce the primary strike note of 312 Hz. The partials I-7, I-8 and I-9 lie on a straight line which intersects the horizontal axis at the point S = 3. They are harmonic in the ratio 4:5:6 and result in a secondary strike note of 424 Hz. The double octave I-6 is somewhat too high for both strike notes.

Table 1. Summary of the lower partials of the first and second groups.

Pitch	*Name in Dutch*	*Name in English*	*Group* I	*Group* II *or* III	*Strength*
c (C_3)	grondtoon	hum note	I-2		strong
c^1 (C_4)	priem	fundamental		II-2	strong
es^1 (E_4^b)	kleine terts	minor third	I-3		very strong
g^1 (G_4)	kwint	fifth		III-2	weak
c^2 (C_5)	octaaf	octave	I-4		strong
e^2 (E_5)	grote deciem	tenth		II-4	mod. weak
g^2 (G_5)	duodeciem	twelfth	I-5		very strong
c^3 (C_6)	dubbeloctaaf	double octave	I-6		strong
c^1 (C_4)	slagtoon	strike note			dominating

a tone with a certain frequency is heard one octave too low? Why should it exactly be the fifth partial that leads to this phenomenon? Meyer and Klaes[6] did discover surprisingly that a sound consisting of only the lowest five partials does not give this strike note sensation. However, adding the twelfth was sufficient to hear the strike note. Their conclusion was that the strike note, which was not present in the objective sound, had been formed by nonlinear distortion in the inner ear as the difference tone between the strong octave and the twelfth. Since then it has become apparent that the pitch of the strike note corresponds much better to the improbable octave rule than with the physically more attractive difference tone rule. On the other hand, everybody could agree (and this was also demonstrated by using bandpass filters) that the strike note does not have a causal connection with the lower partials but is related to the higher partials, particularly the octave and the twelfth. Also in connection with this, Blessing[7] has shown the timbre of the strike note to be totally different from that of a partial of similar pitch. In the literature this is often called a metallic timbre. We agree with Lehr,[1] that this is not entirely correct. Our own analysis has shown that when one listens to the total sound of a bell it has the following three characteristics:

the strike sound has a metallic timbre of metal on metal, is atonal and dies out very quickly;
the strike note has a sharp timbre and dies out fairly quickly; and
the fundamental (or second partial) has the soft timbre of a pure tone, a pitch approximately equal to the strike note, and continues to sound longer than the strike note.

With many old bells, apart from those cast in the Low Countries, one can observe that the pitch seems to fall or rise slightly during its ringing. This is caused by the fact that attention is drawn first by the region around the pitch of the strike note, in which after its dying out the somewhat lower or higher fundamental continues to sound.

SUBJECTIVE SOUND ANALYSIS. THE RESIDUE.

In 1938 and 1939, the first author studied the problem of subjective sound analysis under the direction of R. Vermeulen at the Philips Research Laboratories. If one listens to a sound consisting of a large number of harmonics, one can usually recognize separately the lowest eight harmonics, each with its own pitch and loudness.[8] Higher harmonics cannot be heard separately. In itself, this is very understandable because the frequency analysis of the inner ear has only a limited resolving power which can be taken to be approximately one whole tone. It turned out that the remainder of the sound appears to our ear as a separate tone with a sharp timbre and a pitch approximately equal to that of the fundamental frequency.[9]

This remainder was called the residue. The residue, therefore, is the

collective perception of a number of closely spaced frequencies which cannot, or only partially, be analyzed by the ear. If these frequencies are almost harmonic, then the residue, within a certain area of existence,[10] is tonal with pitch equal to the fundamental frequency, which might not be objectively present. If the frequencies are strongly inharmonic, then the residue is atonal. For many sounds that we know, like the sound of a violin and many vowels, the loudness of the residue dominates far above the simultaneously present lower harmonics. Also, it should be mentioned that in telephone acoustics frequencies below 300 Hz are not transmitted. This is possible without complaints from customers because the pitch that we attribute to a voice is not determined by the objective fundamental frequency but is determined by the residue of higher harmonics that are transmitted.

Earlier investigators had already noticed that a periodic sound in which the fundamental frequency is absent is still perceived as having a pitch corresponding to the fundamental frequency. The two Nestors of modern acoustics, Fletcher[11] and von Békésy,[12] thought they could prove that in this "case of the missing fundamental" the culprit was the nonlinear distortion in the inner ear, to the effect that the objectively absent fundamental frequency was still heard as a difference tone of the equidistant harmonics.

The first author thought he could refute the experiments of Fletcher and von Békésy and opposed these with the hypothesis of the residue.[13] As for the theoretical bases, he started with the limited frequency resolving power of the inner ear and its linear behaviour for sound levels that are not too high.

The perception of pitch was attributed to a measurement of the periodicity of the products of the analysis in the inner ear; that is to say, of the time elapsing between repeating patterns of vibration. The hypothesis of the residue has been turned over and over by many people in the past 25 years. Especially two compatriots have been concerned with it: Hoogland[14] thought he could furnish proof that the residue did not exist, and de Boer,[15] also in connection to this, wrote a dissertation in which the existence of the residue was confirmed. Moreover, de Boer also confirmed an effect with quasi-periodic signals as discovered by the first author, and added a new effect to it. This led again to serious controversies, but the new effect has meanwhile been confirmed at the Institute for Perception Research IPO.[16]

Although many investigators now recognize the existence of the residue and of the role of time interval measurement in pitch perception, von Békésy, for example, still has not conceded.[17]

THE STRIKE NOTE AS A RESIDUE

Given the residue hypothesis, one has to assume that if in a sound consisting of a number of arbitrary frequencies some frequencies form a harmonic series, these harmonic frequencies, if strong enough, will be

perceived together as a tonal residue with a sharp timbre and with a pitch corresponding to the fundamental frequency of the harmonic series. This would mean that the strong partials I-4, I-5 and I-6 of the first group, being the octave, the twelfth and the double octave with the frequency ratios 2:3:4, together should be audible as a residue (see Figure 2a).

One sees therefore that historically the problem of the strike note in bell science runs completely parallel to that of the "case of the missing fundamental" in acoustics. That is to say, there may be a very pronounced subjective sound component with a sharp timbre, and a pitch which does not correspond to any of the frequencies present in the objective spectrum. One hypothesis is that this connection is based on linear distortion in the inner ear. The refutation of this hypothesis and the suspicion that both phenomena are caused by the residue will now be discussed.

In 1939, exactly at a time that these thoughts began to take form, Mr. J. Rodriguez de Miranda asked the first author to tune for him an electronic Westminster chime. This chime consisted of four steel springs whose vibrations were picked up electromagnetically. In spite of the careful design of the loudspeakers, this chime, which sounded loudly from the so-called 'Lichttoren' in Eindhoven, created much annoyance among people passing, because of its false tuning. Upon investigation, it turned out that the pitches perceived from these springs did not correspond to any of the (inharmonic) eigenfrequencies. However, for a number of springs the third, fourth and fifth eigenfrequencies were exactly two, three and four times that of the strike note (Fig. 2b). The following may serve as

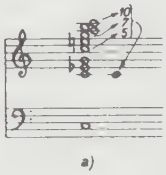

Figure 2a. Sound components of a carillon bell. The strike note is not among the eigenfrequencies of the bell. However, its pitch does lie very close to that of the second partial. The 5th, 7th and 10th partials form a more or less harmonic series with frequencies in an approximate proportion of 2:3:4. The strike note can be conceived as a residue of these, and therefore lies one octave below the 5th partial.

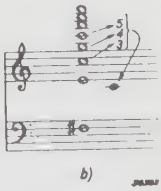

Figure 2b. Components of the sound of a steel spring from a kind of Westminster chime. The strike note is here the residue of the 3rd, 4th and 5th eigenfrequencies. The frequencies are very well in the proportion of 2:3:4. The strike note, therefore, lies one octave below the 3rd partial.

a characteristic example of the sacred element in the use of bells, as discussed by Lehr:[1] the first author almost naturally gave names to the springs, in line with an old bell tradition. These names were Jumbo, Jericho, Johannes, and Judas. It turned out that the spring Judas was not incorrectly tuned in the sense of being too high or too low, but had an ill-toned strike note, which was connected to the nonharmonic position of the third and fifth partials.

CRITICISM OF THE RESIDUE HYPOTHESIS

Originally, the residue hypothesis for the strike note of bells, so elegant to the first author (13b and 13d), had even less success than the hypothesis of the residue itself. Van Heuven[18] could not confirm the harmonic position of the octave, the twelfth, and the double octave, and preferred the hypothesis that the strike note was caused by an interaction of bell and clapper. Van den Berk and Lehr[5] rejected both hypotheses and also rejected the difference tone hypothesis of Meyer and Klaes.[6] Their experience confirmed over and again that the octave rule was the best approximation.

We had occasion several times to discuss with Prof. Thienhaus of Hamburg his extensive measurements of a large number of old German bells. It turned out that the spectra of these old bells were very capricious, and were not of much help.

In 1959, we came into contact with the Viennese bell founder Joseph Pfundner, who defended the residue hypothesis, which was at that time scarcely known in the bell world.[19] He did this with so much enthusiasm that he was almost "plus royalist que le roi." To support the hypothesis, he selected, from a bell measured by Tyzzer, all eigenfrequencies from a total of 50 that could be considered integral multiples of the strike note. Lehr correctly raised the objection that many of these partials are so weak that they are hardly able to contribute to the occurrence of a strike note. Nevertheless, Pfunder must be given credit for again reviving the residue hypothesis.

RECENT INVESTIGATIONS

When we received the request to give this presentation about the strike note to the Acoustical Society, we were forced to recognize that the embroidery of the residue hypothesis, however attractive in itself, was still disfigured by a number of loose threads. Putting together this presentation, therefore, was a new challenge to bring some clarity to this rather obscure matter.

Unfortunately, in science, challenges do not always guarantee solutions. In this case however, we have been fortunate as it turned out that the solution was not to be found in the direction of new concepts but in a critical comparison and reevaluation of existing concepts.

In the first place, a recording with the sound spectrograph (see Fig. 3) confronted us again with the fact that, apart from the lowest partials, all

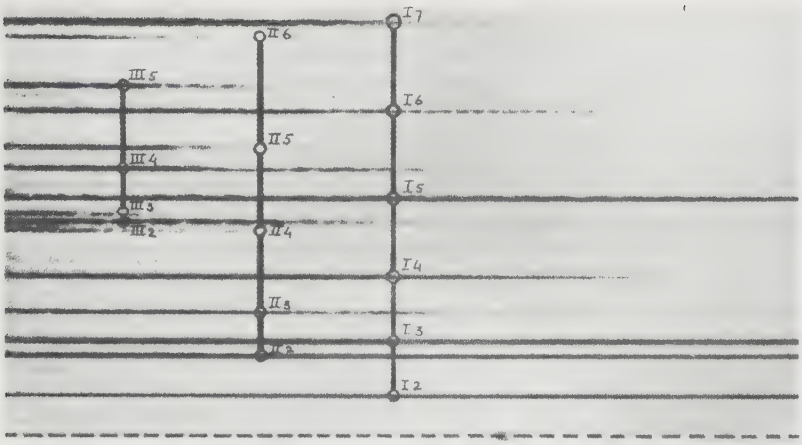

Figure 3. Spectrogram of the bell of Figure 1. For the partials of groups I, II and III, their origin is indicated.

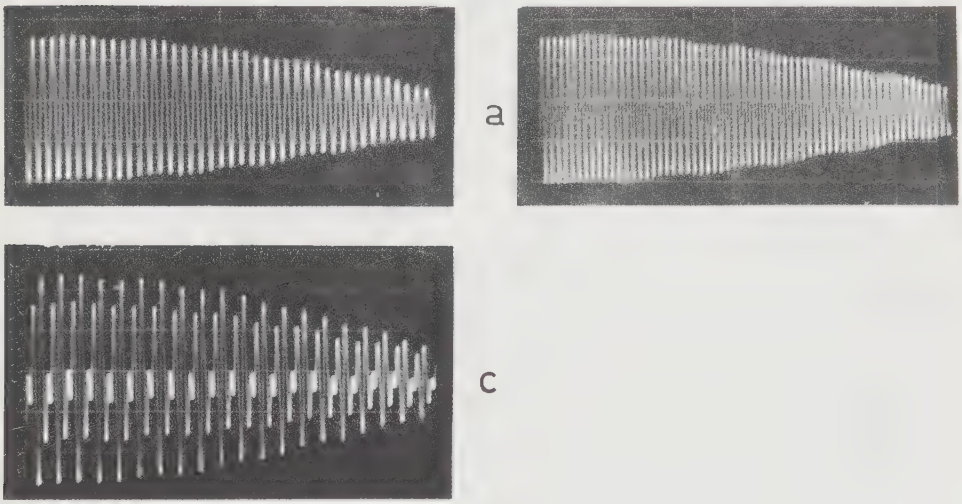

Figure 4. Picture of the wave forms shaped by an electronic gate, to create an artificial strike note. A. A pure tone of 400 Hz; B. A pure tone of 600 Hz; C. The sum of A and B. The periodicity of 200 Hz is clearly visible. This is heard as a strike note of 200 Hz.

the others can safely be ignored, with the exception of those of group I. For this group only I-4, I-5 and I-6 have a harmonic or almost harmonic relationship to the strike note.

Next we carried out the following crucial experiment: The output of a number of tone generators which had been tuned to the relative frequencies 2, 3, 4, 5, etc., was passed through an electronic gate (see Fig. 4). This gate caused the sound to commence abruptly and die out exponentially.[20] If only one frequency is fed through this gate, one hears a ping, like

a mandolin. When one takes the octave and the twelfth, one hears already the pitch of the fundamental frequency. If the double octave is added, the effect becomes even clearer and it increases in clarity when higher harmonics are added. In conclusion, we can say that *three or even two harmonics are sufficient to cause the perception of the fundamental tone.*

After many wrong tracks (among others about a potential collective behavior of all partials, and about a possible particular periodicity at the onset of the bell tone), with this experiment we have finally returned to the simple hypothesis of 25 years ago.

As a bell scientist, Lehr was particularly interested in this demonstration on account of the fact that octave and twelfth together already give a clear strike note perception, and that the double octave added to this does contribute to the prominence of the strike note without having much effect on its pitch.[21] Experimentally, it appears that the pitch (h) rises both when the octave $f_2 \approx 2h$ is tuned upwards and when the twelfth $f_3 \approx 3h$ is tuned upwards. This agrees with the empirical rule proposed by us earlier that the pitch of the residue corresponds to the highest common factor of the contributing harmonic frequencies, and that when the deviations from the harmonic positions are small, the pitch corresponds roughly to the best approximation of this highest common factor. For small deviations from the harmonic position, we suggest:

$$h = \frac{\alpha}{2} f_2 + \left(\frac{1-\alpha}{3}\right) f_3 \qquad (1)$$

This formula satisfies the requirement that for harmonic frequencies a proportional increase leads to an equal proportional rising of the pitch. The weight factor is a constant which has to be determined. When $\alpha = 1/2$ the rule of the highest common factor is expressed. Preliminary measurements as well as the experience of bell scientists justify the hypothesis that $1/2 < \alpha < 1$.

If we now compare this with the difference tone hypothesis, then we have to express this hypotheses as:

$$h = -f_2 + f_3 \qquad (2)$$

In consequence of this, an increase of the octave f_2 would lead to a lowering of the pitch, which is contrary to the observed behavior of the strike note. Also, it turns out in our experiments that the strike note occurs only markedly and is clearly tonal for small deviations from harmonicity of f_2 and f_3, whereas for difference tones that limitation would not exist.

In conclusion, one can state that in the course of the years all investigators have agreed that the octave and the twelfth are the main originators of the strike note and that the pitch is predominantly determined by the octave. The hypothesis of distortion in the inner ear (Ref. 2)

would give rise to an incorrect dependence of the strike note pitch on the position of the octave. Moreover, perception theorists have meanwhile disproved of this hypothesis. The hypothesis of the strike note as a residue according to Formula (1) appears to lead to a correct dependence. Experiments with artificial strike notes confirm the experience of bell scientists. Moreover, the knowledge about the residue was enriched by the insight that in many cases even two harmonic frequencies will give rise to a clearly tonal perception.

SECONDARY STRIKE NOTES

Apart from a normal strike note some bells exhibit a second strike note (so-called "metaalkwart" in Dutch, or "metal fourth") which as far as its behavior is concerned is just as elusive as the normal strike note.

Vaz Nunes[23] believed that he perceived a pitch fourth above the strike note. Later this was confirmed by Griesbacher.[24]

The name "secondary strike note" was coined by Arts,[25] and we think it is more precise because the pitch does not always lie a fourth but sometimes a major third or a tritone (augmented fourth) above the primary strike note. The secondary strike note is a menace to all bell founders because it can discord with the other tones in a very annoying way.

Arts already suspected a relationship to exist between the secondary strike note and the partial I-7 which lies two octaves higher (see Table 2: the partials of group I according to Lehr). The first author proposed earlier that when given a spectrum within which several harmonic groupings are possible, more than one residue will occur. This exactly appears to be the case with bells, because I-4, I-5, and I-6 are the second, third, and fourth harmonics of the primary strike note based on c^1 (C_4) while I-6, I-7, I-8, and I-9 are the third, fourth, and fifth, and sixth harmonics of the secondary strike note on f^1 (F_4). Consequently the secondary strike note is the residue of partials between I-6 and I-9 when these are sufficiently harmonic.

However, we have to add one limitation: Van den Berk and Lehr[5] have

Table 2. Summary of the partials of group I according to Lehr[1].

Rank	*Pitch*	*Name*
I-2	c (C_3)	hum note
I-3	es^1 ($E^\flat_4$)	minor third
I-4	c^2 (C_5)	octave
I-5	g^2 (G_5)	twelfth
I-6	c^3 (C_6)	double octave
I-7	f^3 (F_6)	double octave + fourth
I-8	a^3 (A_6)	double octave + major sixth
I-9	c^4 (C_7)	triple octave
	c^1 (C_4)	primary strike note
	f^1 (F_4)	secondary strike note

already indicated that the secondary strike note occurs only for bells that are heavier than 800 kg. This gives a strike note of about 370 Hz and a range between the double octave and triple octave of 1500–3000 Hz.

Here Pfundner[19] saw a connection with the hearing sensitivity curve. We would like to amend this since the residue has an existence region which is limited towards the higher frequencies (Ritsma[10]). As a consequence, a small bell with partials between I-6 and I-9 that are approximately harmonic will have no audible secondary strike note, but a secondary strike note will become audible when a recording of such a bell is played back at half the speed. It is clear that in principle even a tertiary strike note can be expected. If one assumes a gradual increase of the frequency spacing of the partials then with a secondary strike note of a major third, a fourth, or a tritone, a tertiary strike note of a fifth, a sixth, or a seventh can be expected, respectively. Also, it is not impossible that some partials of group II are involved in a strike note. For this a harmonic situation is needed, and the bell must be struck in such a(n) (unusual) way that the partials of group II become the strongest.

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20) For the quick development of this circuit we thank Mr. W.H. Noordermeer of our laboratory.
21) In practice, the double octave is usually a little too high. The eigenfrequencies of a group lie on a concave line (see Fig. 1), so that the frequency differences increase monotonically.
22) Although there are, as mentioned earlier, very strong indications that the pitch is not determined by frequencies but by a time interval, the second order deviations between observed pitches and calculated physical time intervals are still not explained. The simple rule of the highest common factor therefore remains of practical value although it does not link up directly with a physical model.
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16

ON THE STRIKE NOTE OF BELLS

J. Pfundner

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1. INTRODUCTION

The problem of the strike note of bells is determined by the fact that even the strongest note, whose pitch is determined by the position of the bell in the ringing, is not a physically observable objective note, but rather a subjective sound perception. Researchers such as Rayleigh, Arts, Jones, E. Meyer, Klaes, and Van Heuven could not find a solution to the problem. Schouten, who drew up the theory of the "residue," remarked that the strike note could be a residue phenomenon. In this article it will be proven, on the basis of a selection of examples from a great number of experimental results, that the frequencies of the strike note arrived at on the basis of Schouten's theory coincide indisputably with those that were ascertained aurally, and that the residue can explain several phenomena in the realm of bell ringing.

2. THE PROBLEM OF THE BELL STRIKE TONE

The sound of a bell consists of a metallic sharp strike note of great sound intensity but short duration, to which may be added, under certain conditions, one or more secondary strike notes of similar tone colors, as well as of several soft sum tones of lesser sound intensity but relatively long duration. While it can be indisputably shown that the sum tones from the vibrating body of the bell exist objectively as partial tones (physical reality) (Tyzzer, [1]), the strike note and the secondary strike notes can only be aurally established as subjectively perceived note impressions (musical reality). Hence, the problem of the strike note lies in that: (1) we hear a note that does not exist objectively; (2) the intensity of the merely subjectively perceived strike note is so great that it covers the objectively existing partials; (3) the strike note, despite its great intensity, is only of short duration, while the less loud partials continue to echo for a long time.

3. RESEARCH SITUATION UP TO 1939

Lord Rayleigh [2], who could not prove the strike note physically, but who had heard it unequivocally and had recognized its importance for

bell ringing, set up the empirical rule that the strike note always lay one octave lower than the fifth partial of the bell, the so-called upper octave. This rule is not generally applicable, but only in the case of pure octave bells in which the "prime," the second partial of the bell, has the same pitch as the strike note. Rayleigh may have had only such bells at his disposal for experimenting. Other researchers, such as Griesbacher [3], Jones [4], and later Van Heuven [5], who examined the same types of bells, came to the same conclusions as Rayleigh in regard to the dependence of the pitch of the strike note on the upper octave. In regard to the character of the pitch, they were, however, of differing opinions. The musician Griesbacher, who considered the strike note the most important note of the bell and a musical reality, was also convinced of the physical reality of the strike note and believed that he had even found a tuning fork reaction. Jones thought that in the strike note there was present an octave delusion, while Van Heuven assumed a synchronization effect from the prime. Blessing, for whom the strike existed also as a musical reality, but who could find no frequency pertaining to the strike note in the musically perceived partials, was one of the first to advance the theory that the strike note was the sound impression from several higher partials. Biehle [7], who had available mainly seventh bells, whose strike notes do not appear as significant as those of pure octave bells, called the strike note an "imaginary" note creation. E. Meyer and Klaes [8], who had only a bell with a lowered prime at their disposal, concluded that the strike note depended on the difference frequency between the twelfth and the upper octave, and assumed that the strike note was a difference tone. They were strengthened in their conclusions also by the observation that in the tone structure of the bell sound from the partial notes up to the higher octave there was no impression of a strike note; however, when the twelfth was added, the strike note was aurally perceivable. Weissenbäck and Pfundner [9] subsequently examined a great number of bells and determined that in the specimens examined the difference tone calculated through the twelfth and higher octave frequencies did not coincide with the strike-note pitch. Until the beginning of the Second World War, the strike-note problem had not been solved, and some kind of progress was achieved only when the prevailing opinion became that the strike note had to be the result of higher partial notes. Indeed, the fundamental research begun before 1938 and ended in 1940 left the problem still unsolved.

4. THE STRIKE NOTE—A RESIDUE PHENOMENON

Schouten [10–13], in his report entitled "The Perception of Pitch," revealed the results of this research on the problem of the missing fundamental and created a new concept of the "residue." What he called residue is the subjective impression of a sharp note which, through the

simultaneous realization of several whole-number harmonics, originates in the ear as the pitch of the fundamental of these harmonics, even when the fundamental is not present as an objective sound mixture. Among other things, it was mentioned in this report that the residue offered an explanation for the phenomenon of the strike note, and that it was also possible in the presence of two or more groups of whole-numbered multiples of basic vibrations in the bell spectrum to create secondary strike notes.

This work, which came to my attention in 1952, seemed to show the way to the solution of the strike-note problem. The difficulty in confirming the theory of the residue with bell sound lay in that with the tuning forks normally used to determine pitch, one was able to determine the high pitches in the range of two to three octaves above the higher octave only in big bells. Still, it was possible to find an explanation for the second, fourth, and three-tone strike notes as residue phenomena in Schouten's theory by using a few large specimens through the evidence of the presence of the several integral variables of the basic frequencies of aurally perceived strike notes. A further substantiation followed through the production of magnetophone pictures. During the conference of bell experts in Nürnberg in 1956, M. Grützmacher presented the note construction of a bell sound. It was shown that in the construction of the first experimental series, which started with the lowest partial note and then added the higher partial notes, only when the twelfth note was added (as in the case of the bells studied by Meyer and Klaes in the year 1933) did the impression of the strike note appear, and it became stronger with the addition of further overtones. In the second experimental series, whose construction began with the highest overtones and to which lower partial tones were added gradually, the impression of the strike note was clearly audible already in the range of the fifth above the second octave, and when partial tones were added down to the range of the higher octave this impression was strengthened. This presentation, which applied the theory of Schouten without any other commentary or reference, strengthened my belief in the correctness of this theory and led me, after obtaining an electronic counter, to undertake experiments in great numbers that all had positive results. Only through the application of the counter was it possible to determine the frequencies of the high partial notes of smaller bells. In the following tables (1–4) are shown some of the results of these experiments, which testify to the correctness of the theory of the "residue" in regard to the sound of bells. The cases were especially chosen, because in their case neither the empirical rule of Rayleigh, nor the assumptions of an octave illusion, nor the assumption of the construction of difference tones would have led to a correct, nonaural determination of the strike-note pitch, let alone to an explanation of the phenomena.

Table 1. Bell 1

Partial	Residual tone	Pitch	Frequency	Multiple of the partial: U	(S)	P	T
1. Unterton = U		g/1 + 1,5	392	1			
2. Prime = P		g/2 − 4	753			1	
	(Schlagton) = (S)	(g/2 − 1)	(770)		(1)		
3. Terz = T		h/2 ± 0	974				1
4. Quint		d/3 − 4	1127				
5. Oberoktave		gis/1 − 3	1632				
6.			1889				
7.			1942	5			2
8.			2086				
9.			2303			3	
10.			2328		3		
11.			2426				
12.			2574				
13.			2893				
14.			2964				3
15.			2992			4	
16.			3048		4		
17.			3128	8			
18.			3441				
19.			3540	9			
20.			3787			5	
21.			3807		5		
22.			4221				
23.			4284	11			
24.			4681		6		
25.			5374		7		

The bell shows pronounced major third character, which is attributed to the low position of the strike note, a residual tone structured from the 10th, 16th, 21st, 24th, and 25th partials. This bell does not approximate Rayleigh's rule, since the upper octave is not used for the strike note.

Table 2. Bell 2

Partial	Residual tone	Pitch	Frequency	Multiple of the partial: U	(S)	P	T (Durterz)
1. Unterton = U		C + 5	67	1			
	(Schlagton) = (S)	(c + 1.5)	(130.6)		(1)		
2. Prim = P		c + 4	133.1			1	
3. Terz = T		es + 5	159.5				1
	(Durterz)	(e + 5)	(169)				(1)
4. Quint		g + 5.5	201.5	3			
5. Oberoktave		c/1 + 2.5	263.4		2		

(continued)

Table 2. Bell 2 *(continued)*

				Multiple of the partial:				
Partial	*Residual tone*	*Pitch*	*Frequency*	*U*	*(S)*	*P*	*T*	*(Durterz)*
6.		d/1 + 4	299					
7.		e/1 + 2.5	335	5				2
8.		f/1 + 5	358					
9.		g/1 — 3	379.4					
10.		g/1 ± 0	387.5		3			
11.		a/1 ± 0	435					
12.		h/1 + 2.5	497.3					3
13.		c/2 + 2	524.8		4			
14.		c/2 + 4.5	534.5	8	4			
15.		d/2 + 3	593.6					
16.		g/2 + 1.5	783.4		6			
17.		gis/2 + 3	838.9					5
18.		c/3 ± 0	1034.4					6

Bell with pronounced major third character, where the strike note (130.6 Hz) appears as a residual tone as well as the major third (169 Hz). The minor third (159.5 Hz), existing as only the 3rd partial, stands against the major third which is structured from the 7th, 12th, 17th, and 18th partials.

Table 3. Bell 3

				Multiple of the partial:				
Partial	*Residual tone*	*Pitch*	*Frequency*	*U*	*(S)*	*P*	*T*	*(Quart)*
1. Unterton=U		G + 6	101.7	1				
	(Schlagton) = (S)	(g + 5.5)	(201)		(1)			
2. Prim = P		g + 6	202			1		
3. Terz = T		b + 6	240				1	
	(Quartschlagton)		(268)					(1)
4.			323					
5. Oberoktave			403		2	2		
6.			479				2	
7.			538					2
8.			603	6	3			
9.			773					
10.			836					
11.			963				4	
12.			1089					4
13.			1166					
14.			1319	13				
15.			1356					5
16.			1580					6
17.			2009		10			
18.			3099				13	

Minor-third bell, which in addition to the strike note (201 Hz), also has a strike note fourth (268 Hz) built up as a residual tone (from the 7th, 12th, 15th, and 16th partials).

Table 4. Bell 4

				Multiple of the partial:			
Partial	*Residual tone*	*Pitch*	*Frequency*	*U*	*(S)*	*P*	*T (Tritonus)*
1. Unterton							
= U		d	145	1			
	(Schlagton) = (S)	(d/1—1)	(288.1)	2	(1)		
2. Prim = P		d/1	290			1	
3. Terz = T		fis/1—2	360				1
	(Tritonus)	(gis/1)	(412.2)				(1)
4. Quint		a/1—1	432	3			
5. Oberoktave		d/2	580	4	2		
6.			690				
7.			766				
8. Duodezim			854		3		
9.			977				
10.			1148	8	4		
11.			1234				3
12.			1495				
13.			1630				4
14.			1848				5
15.			2078		7		5
16.			2297		8		6
17.			2743	19			
18.			3085	21			
19.			3664				10 9
20.			4127				10
21.			4579		16		11

The three-tone strike note, recognized by bell experts through hearing, was unknown to bell founders, because it does not appear in the spectrum of partials. After I heard this tri-tone and compared it to the measurements (of partials 11, 13, 15, 19, 20, and 21), the founders acknowledged it.

5. THE DURATION AND SOUND INTENSITY OF THE STRIKE NOTE

Against a theory that overtones are supposed to build the strike note as difference tones, many musicians as well as physicists (e.g., Van Heuven) propose as their main argument that overtones of weak intensity could not summon such a strong impression as that of the strike note. But in the case of the residue, it is not only two notes, but more often a great number that create the intensity of the sound impression. If one examines the sound intensity curves of Fletcher and Munson, one can see that it depends very much in which pitch range the partial notes lie that are supposed to create the residue. Then it becomes clear why the fourth strike note appears only in a certain bell size, and why it becomes perceivable more or less clearly if we move the sound spectrum up or down by one octave. It also becomes understandable that a residue is possible only up to a certain pitch, because when the note pitch of the high partials goes beyond the range of the sensitivity of the auditory

nerves, a residue impression can no longer arise. This explains the phenomenon that high carillon bells do not exhibit a typical chord effect in their bell sound, but rather give the impression of a simple note. The brevity of the strike-note impression is determined by the limited duration of the high partial notes. The residue note cannot have a longer duration than the components that produce it.

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Part V

BELL MATERIAL, TONAL BALANCE, AND WARBLE

Editor's Comments on Papers 17, 18, and 19

17 SCHAD and WARLIMONT
Acoustical Investigations of the Influence of the Material on the Sound of Bells

18A BIGELOW
Part I

18B BIGELOW
Part II: The Need for Testing Representative Medium and Treble Bells, and the Apparatus and Method Used

18C BIGELOW
Part IV: Establishing the Lines of Proportion and Profile

18D BIGELOW
Part V: The Completely Balanced Carillon in All Its Acoustical and Physical Aspects

19 PERRIN and CHARNLEY
Group Theory and the Bell

Most high quality bells are cast of bronze made up of about 80% copper and 20% tin. Steel bells are considerably less expensive and mechanically stronger, but their tonal characteristics are generally considered inferior. In particular, the sound of a steel bell is louder and of a shorter duration than that of bronze bells of the same pitch (see Sections 45 and 46 in Paper 9C). Some brass bells are found, especially in Poland.

One of the most extensive investigations of the tonal characteristics of bell bronzes as a function of composition is reported in Paper 17. Good bell material should have sufficient toughness, rigidity, and resistance to corrosion, and it should have as little internal damping as possible. In addition, the speed of sound should be as high as possible, because a given pitch can then be reached with a relatively small and heavy bell which will be damped less by radiation and air friction. In this paper it is concluded that variations in the

dimensions and mass of the bells (due to porosity) influence their sound more strongly than considerable changes in alloy composition or impurity content.

Because of the lower cost of steel bells, Paper 9 discusses means for decreasing the damping in steel bells. An idea attributed to C. W. Kosten is to drill holes in the bell to decrease the damping due to sound radiation. It is also suggested that using steel for the bells of highest pitch might improve the tonal balance in a carillon.

Paper 18 consists of excerpts from a book by Bigelow addressing the question of tonal balance in a carillon. Although most carillons employ one logarithmic scaling throughout (the diameter, height, and thickness are nearly inversely proportional to the frequency of the bell), Bigelow argues for upscaling the treble bells in order to increase their relative strengths.

Also included in Paper 18 is a careful study of the effects of clapper weight and travel distance on tone. In order to improve tonal balance, the treble clappers should be increased in mass, and the travel distance of bass clappers should be decreased, preferably by moving their pivot point away from the bell axis.

Bell warble occurs because modes of vibration occur in nearly degenerate pairs of doublets, the nodal meridians of one coinciding with the antinodes of the other. The two components of these doublets beat together to produce the warble. Selecting a strike point near the node of one component or cutting the bell at the proper point reduces this beating, but reducing the beating in one partial may increase it in another mode.

Paper 19 is a study of bell symmetry that uses group theory to classify all possible modes of vibration (including modes that radiate no sound). The authors propose to reduce warble by casting the bell slightly eccentric to increase the spacing of the doublets and to cause the antinodes of one component of each doublet to conincide with the strike point. These ideas are expanded in subsequent papers by the authors.

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17

ACOUSTICAL INVESTIGATIONS OF THE INFLUENCE OF THE MATERIAL ON THE SOUND OF BELLS*

C.-R. Schad and H. Warlimont

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SUMMARY

Sound analyses were made on five experimental bells in a systematic investigation to determine the tonal qualities of bell bronze as a function of the alloy composition. It was established that variation of the dimensions and the mass of the bells influenced their sound more strongly than considerable changes in alloy composition or a higher impurity content.

1. INTRODUCTION

In order to describe the influence of the material on the sound properties of the bell, the velocity of sound ($\sqrt{E/\rho}$, E = Young's modulus, ρ = density) and the internal friction (decrement θ) of the material are essential as physical properties that determine tonal pitch and tonal duration. In a previous work [1], their dependence on Sn-concentration, on porosity, and on the addition of third elements, regarded as detrimental or favorable, were examined. In the present work, the results gathered from cylindrical rod-shaped specimens are compared to the sound analyses of five experimental bells. Through comparison with other sound-determining quantities, the significance of the influence of the material is discussed.

As a musical instrument the bell is regarded as a self-sounding idiophone. It is a shell of rotation whose characteristic shape is established through the geometry of the rib (Figure 1). During the sound decay of a struck bell, the bell body executes free bending vibrations whose damping consists of three parts: material damping (internal friction), friction damping (boundary-layer damping) and radiation damping (acoustical damping). Every individual vibration has a characteristic frequency, that is a partial role and a vibration form [2,3] classifiable through its number of nodal circles and nodal meridians. Because of its special shape, the bell possesses, in addition to several inharmonic partial notes, a series of partial notes with a harmonic frequency ratio; the partial note scale in most other shells is in general purely inharmonic [4]. However, the laws of

*based on the Dr. Eng. dissertation of C. R. Schad, University of Stuttgart, 1969.

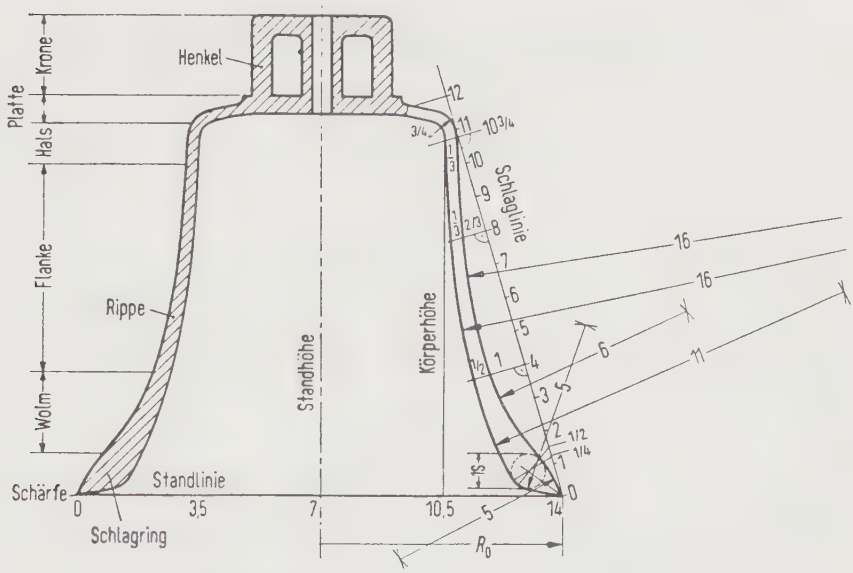

Figure 1. Construction and Nomenclature of a bell (to scale).

simple shells can be applied in analogy to bells. Thus, Rayleigh [5], for example, deduced the following approximate formula for the natural frequency of the bending vibrations of a circularly cylindrical shell (free ends):

$$f = \frac{1}{2\pi}\sqrt{\frac{E\,d^2}{12\,\varrho\,R_0^4(1-\mu^2)}}\;\frac{n(n^2-1)}{\sqrt{n^2+1}} \tag{1}$$

R_0 = average radius of the shell
d = wall thickness
μ = Poisson's constant (or ratio)
$2n$ = number of nodal meridians

Aside from the recognition that these vibrations of a cylindrical shell do not possess a harmonic frequency relationship, this relation shows that the elastic modulus, the density, the velocity of sound ($c=\sqrt{E/\rho}$), and Poisson's constant are the material properties that determine the frequency. Through them, the absolute pitch of the partial notes, but not their frequency relationships, is established. When only one partial note is considered, Equation (1) can be simplified:

$$f = \text{const.}\,\frac{d}{R_0^2}\sqrt{\frac{E}{\varrho}\,\frac{1}{1-\mu^2}} \tag{2}$$

This equation is also called a bell formula, since it can be used to derive some important relationships for practical bell construction. For the

common case of bells being manufactured in different sizes from the same material and the same shape, the following equation is true, according to the laws of similarity:

$$\frac{f_1}{f_2} = \frac{S_2}{S_1} = \left[\frac{G_2}{G_1}\right]^{1/3} \tag{3}$$

S = stroke (see Fig. 1)
G = bell weight

If we use different materials for the same shape, then we have the following relationships from Equation (2):

$$\frac{f_1}{f_2} = \sqrt{\frac{(E/\varrho)_1}{(E/\varrho)_2}}\sqrt{\frac{1-\mu_2^2}{1-\mu_1^2}}\,. \tag{4}$$

It is suggested then that in examining the influence of the material, bells of the same size and shape should be compared.

The value of a bell is measured by the quality of its sound. For this reason, bell examination by experts is mainly concerned with the musical impression of the bell. Thus, various sound characteristics, described in the "Definitions for bell examination" [6], are evaluated according to the "Guidelines for determining new bells" [7]. However, in general, only the partial note frequencies are subject to an objective measurement.

A totally objective description of the acoustical characteristics of bells was attempted by several authors. Stüber and Kallenbach [8] report on measurement methods and show how it is possible to have a more complete analysis of these complicated sounds through the development of modern electroacoustic instruments.

Bell sound is characterized by its pitch, its timbre, its steady state, and its sound decay. The pitch, which is crucial for the musical characteristic of a bell, is determined by the strike note. The frequency of this loud bell note, which is heard at the moment of the stroke and then ebbs away very fast—in reality it is a sound, can be established only through subjective comparison with a note of known frequency, but not by physical measuring methods. Since the bell cannot be stimulated into a vibration mode of this frequency, the strike note is not a partial note of the bell. Its origin has been examined in many works [9], and it is best explained through the "residue theory" of Schouten [10]. The timbre of a bell is influenced by the number of its partial notes as well as their frequency, sound pressure amplitude immediately after the stroke, and their decay rate. Since these are accessible to objective measurement, they are included in the physical description of the bell sound. Because of their great initial amplitude and relatively slow decay, the deep partial notes (principal range) contribute the most to the musical value of the bell. Therefore, they are examined on the basis of their harmonic frequency relationship with the strike note, which they are supposed to have. Corresponding to the desired intervals, they are designated as follows (octave bell):

First partial note	Lower octave (hum note)	
Second "	"	Prime (fundamental)
Third "	"	Third (major or minor)
Fourth "	"	Fifth (quint)
Fifth "	"	Upper octave (nominal).

While the tenth, eleventh, and twelfth are sometimes included in the principal range, the double octave and all higher partial notes belong to the mixture range. In contrast to the partial note frequencies, which are solely determined by the shape and the material of the bell, the initial amplitudes of most partial notes depend on the manner of stimulation, the so-called intonation. Next to the excitability of the partial notes and the location of the stroke, the most important factors to be considered are the shape, the material, and the stroke velocity, and closely connected with it, the contact duration of the clapper [3]. In measuring the sound-pressure amplitudes, the form of the sound field should be considered [11,12], which in turn is influenced (in practical ringing situations) by the acoustical conditions of the room (belfry) and the pendulum movements of the bell.

During the sound decay, the timbre of a bell that has been struck changes continually because of the different decay rates of the partial notes and the unavoidable beats [13]. In the process, the amplitudes of the high frequencies decrease very fast so that the bell sound is reduced to the first partial note in a matter of seconds. Therefore, it is customary to use the decay time of the lower octave (hum) as a simple measure for the sound duration of a bell. The decay time of the partial notes is a criterion for the sound quality of a bell, and in the case of comparable bells, a measure of the material damping.

Next to sufficient hardness, strength, and corrosion resistance, the most important properties required for bell material are those that provide for long decay time. The bell material should have as little material damping as possible. In addition, if one ignores the insignificant variations in Poisson's constant, a low velocity of sound and a high density are advantageous. Thus, a given tonal pitch (Equation 4) can be obtained with a relatively small and heavy bell, which is damped less through sound radiation and air friction than a bell which by nature of its material is larger and relatively lighter. In reality, these requirements are fulfilled especially well by bell bronze. Thus, it is not only the advantages in the metallurgical and technological sense that have made this alloy the most preferable bell material from the earliest bells of China (ca. 1100 B.C.) up to the present.

At present in Germany a Cu-Sn alloy of the following composition is specified for the manufacture of bronze bells [7]: 78-80% Cu, 20-22% Sn; Cu + Sn $\geqq$ 98% (99% in Baden-Württemberg). Lead can make up at most half of the impurities. In the metallurgical part of this work [1] the texture and the tone-determining and mechanical properties of bell bronze were examined using test rods of this alloy composition range, which were

cooled according to the cooling procedure for bells. The tin content in one series was varied between 14% and 27% by weight; in a second series the tin was replaced gradually by antimony; and in a third test series, alloying elements were added at a constant copper-tin ratio up to 10% lead, 5% silver, 2% aluminum, and 1% phosphorus, magnesium, and bismuth, respectively. It was concluded that the damping was influenced not only by the composition but also to a considerable extent by the density, which varied relatively widely because of porosity variations. The influence of the third alloying element was examined systematically, and it was determined that the velocity of sound and the damping were substantially more influenced by the elements P, Al, Mg and Bi than by Pb, whose content has been the only one especially limited in the specifications up to now.

On the basis of these metallurgical results, we will proceed in this report by looking closer at the acoustical experiments.

2. EXPERIMENTAL STUDIES

The five experimental bells (weight 50 kg, pitch b'' or $B_5^{\flat}$) were prepared according to the usual procedures[1]. Their composition was chosen according to the following criteria: On the one hand, the tin content was to be systematicallly varied; on the other hand, the addition of 3% Pb, which in measurements on test rods had proven sound-neutral, was to be studied for its effects on the sound of a bell and the possibility of a substitution of tin with antimony. Table 1 shows the alloy composition of the experimental bells.

In order to evaluate the pitch-determining material properties, the velocity of sound $\sqrt{E/\rho}$ and the Poisson constant μ, the intrinsic frequencies of the longitudinal, transverse, and torsional vibrations of circularly cylindrical rods were measured with an "Elastomat" [14]. Normally, samples were used that had a length $l = (130 \pm 0.05)$ mm and a diameter $d = (6.5 \pm 0.005)$ mm. When other dimensions had to be used, the ratio $l/d = 20$ was maintained.

The frequency measurement was evaluated according to the following relationships:

Table 1. Composition of the Experimental Bells by Weight

Bell No.	*Cu*	*Sn*	*Sb*	*Pb*	*Rest*
1	80.9	18.5	—	—	0.6
2	78.6	20.8	—	—	0.6
3	76.2	23.4	—	—	0.4
4	75.3	21.1	—	3.2	0.4
5	78.4	10.1	10.8	0.3	0.4

[1]By the Karlsruhe Bell Foundry

velocity of sound:

$$\sqrt{E/\varrho} = 2\,l\, f_{1\,\mathrm{long}}\, K_{1\,\mathrm{long}}$$
$$K_{1\,\mathrm{long}} = 1 + \underbrace{\frac{2}{16}\frac{d^2}{l^2}\mu^2}_{K'} \tag{5}$$

Poisson's constant:

$$\mu = \frac{1}{2}\left[\frac{f_{1\,\mathrm{long}}}{f_{1\,\mathrm{tors}}}\right]^2 - 1 \tag{6}$$

l, d = length and diameter of the rod,
$f_{1(\mathrm{long})}$, $f_{1(\mathrm{tors})}$ = frequencies of longitudinal and torsional vibrations,
K_1 = correction for change in cross section after Rayleigh [5].

With the rod dimensions used and an average value for Poisson's constant of $\mu = 0.30$, the correctional factor $K' = 4.29 \times 10^{-5}$ becomes negligibly small. The relative error in the velocity of sound is smaller than 5×10^{-4}. However, it is doubled if we allow a temperature variation of $\pm$ 3° C, as may be seen from the measurements presented in Figure 2.

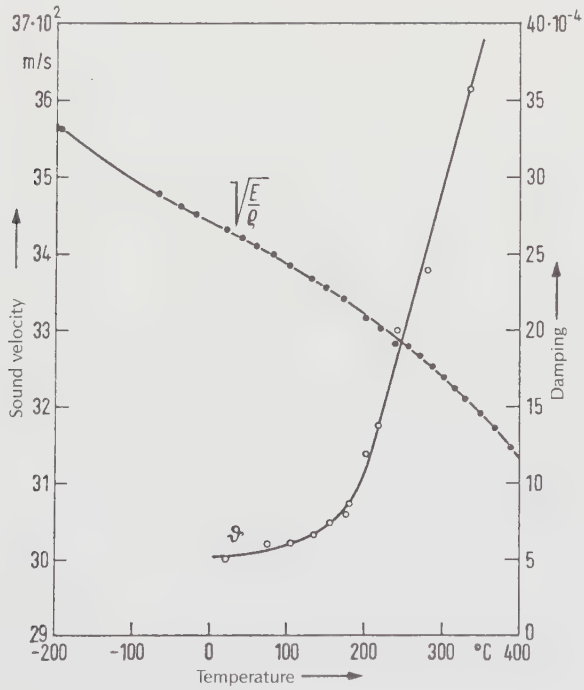

Figure 2. Temperature dependence of sound velocity and damping of bell bronze with 20.7% Sn.

The procedure for deriving Poisson's constant from the longitudinal and torsional intrinsic frequencies according to Equation (6) is the most exact of the indirect, dynamic methods of measurement [15].

In order to measure the damping, the rods were originally excited at the frequency of their basic transverse vibration. After switching off the energy supply, the number N of the vibrations of the rod was counted during the exponential part of their free decay in amplitude. The connection with the physical damping measurements is:

$$\frac{1}{N} = \vartheta = \ln \frac{A_n}{A_{n+1}} \tag{7}$$

In general, the natural logarithmic decrement θ is used in this article. The additional damping which stems from the support is very small, when the rod is held by thin wires near its vibration nodes [14]: it is at most on the order of 10^{-6} [16,17]. The damping which is caused by the coupling of the rod to the system that stimulates the vibration or the one that picks it up is also of this magnitude if there is no friction between the wires and the surface of the rods.

The damping of the vibrating rod by the surrounding atmosphere is composed of a friction part and a radiation part. Its magnitude can be calculated approximately using the treatment proposed by Kneser [16]. For the transverse basic vibration of a bronze rod we have:

$$\begin{aligned}\theta_{\text{air}} = \theta_{\text{friction}} + \theta_{\text{radiation}} &\cong 1.8 \times 10^{-5} + 0.2 \times 10^{-5} \\ &\cong 2.0 \times 10^{-5}.\end{aligned}$$

Comparative measurements at atmospheric pressure and in vacuum, which were carried out with the very sensitive apparatus of Zuckerwar [17], showed that the influence of the atmosphere on the damping of rod vibrations of higher frequency is substantially greater than on the basic vibration (Figure 3), because radiation damping increases with increasing ratio of the rod circumference $2\pi R$ to the wave length λ of the radiated sound ($2\pi R/\lambda < 1$). In order to determine the characteristics of the material that determine sound duration, the basic transverse vibrations of the test rods were always used, while external damping could be neglected in view of the expected material effects and the reproducibility of the damping measurement on using the Elastomat. For the same reasons, and because the frequency of the basic rod vibrations varied only between 1050 Hz and 1300 Hz in the various copper alloys, the frequency dependence of the damping was not given further consideration. Dependence of the damping on the vibration amplitudes, as they are stimulated with the Elastomat, could not be determined in any case. In the case of the damping measurements, which were carried out with especially slim rods vibrating transversely, thermoelastic damping, caused by macroscopic heat flow, should be deducted from the total damping. This was not

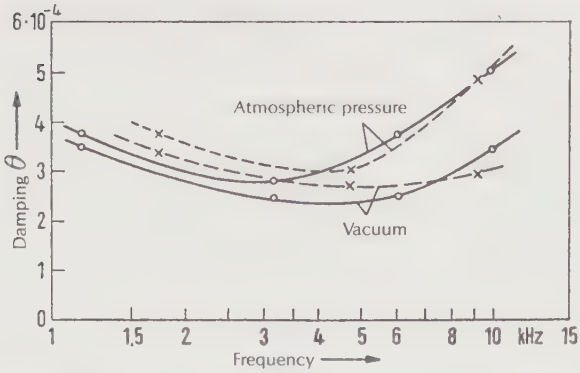

Figure 3. Frequency dependence of the damping in transversely vibrating rods in air and in vacuum.

o—o Bronze 79% Cu, 21% Sn

x—x Steel C 15

necessary in the present investigations because the frequencies of the basic rod vibrations are much greater than the frequency for the maximum damping due to this mechanism. A rough calculation according to the relationships of Zener [18] showed that this damping is only on the order of 10^{-6}.

The basic transverse vibrations were included not only for the damping measurements but also for a cross-check on the results from the longitudinal vibrations. It was fortunate that the tolerance of the test rods was so precise that no audible differences in the intrinsic frequencies occurred. Thus, it was possible to judge directly by ear the changes in the sound properties caused by varying the rod material.

In measuring frequency, initial amplitude, and decay rate of the bell's partial notes, the bell stood with the crown down on the floor which was covered with a soundproof mat. For the excitation and detection of the intrinsic bell vibrations, there were two softly bedded electromagnetic transformers, each of which was positioned opposite a mu-metal plate of 0.3 g glued on the bell; however, the transformers did not touch the plates. Figure 4 presents the experimental arrangement schematically. The vibration transducer on the left side was energized by an RC generator whose frequency was continually varied until it coincided with that of a bell frequency. The exact adjustment of the resonance conditions was made using a Lissajous figure on a cathode-ray oscilloscope. Since the generator frequency was finely adjustable and stable, the partial note frequency could be determined with an electronic counter to $\pm$ 1.5%. Thus, the precision of the measurement was better than the analytical ability of the human ear.

In bell acoustics it is customary to designate tonal pitch by its musical identification (related to A_4 = 435 or 440 Hz) and not by the frequency. For deviations from the scale of equal temperament, and for the note

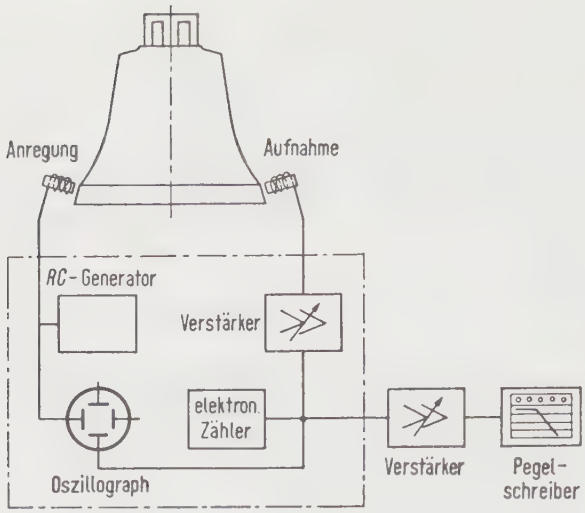

Figure 4. Arrangement for determining the frequency and damping of bell partials.

pitch intervals in general, fractions of the tempered semitone (HT) are used as interval measurements; thus, one-sixteenth of a semitone is also used for practical reasons along with the cent (= 1/100 HT). The following relationships hold:

$$\text{Intervall} = 3986 \log_{10} \frac{f_2}{f_1}, \quad f_2 > f_1 \tag{8}$$

$$1/16\ \text{HT} = 6{,}25\ \text{cent} \rightarrow \Delta f/f = 3{,}62‰ \tag{9}$$

The damping measurements on the bells were carried out exactly as on the test rods, except that the damping of the partial notes was not calculated from the number value N, but rather from the tracings of its decay. The logarithmic damping θ thus derived ($\theta = \theta_{\text{material}} + \theta_{\text{radiation}} + \theta_{\text{friction}}$) is not used in bell acoustics to indicate the damping; instead, the damping is expressed as the rate at which a partial note decays, measured in dB/s [6]. The two definitions are related by the relationship:

$$V = 8{,}686\, f\, \Theta\,, \qquad V \text{ in dB/s}, f \text{ in Hz.} \tag{10}$$

According to the definition chosen here, V is the decay rate, f the frequency, and θ the damping of the observed partial note.

Experiments to determine the initial amplitude of the partial note sound pressures were carried on in an open field. The excitation of the bell,

which again stood on its crown, was achieved by striking the outer sound bow with an actual clapper corresponding to the size of the bell. In order to maintain reproducibility in the striking force, the position of the hanging point of the clapper remained the same, and the drop height of the downward-swinging clapper also remained constant.

The sound radiated by the bell was recorded on tape by means of a condenser microphone placed 4.5 m away at the height of the bell mouth, and an amplifier. This test arrangement is shown schematically in Figure 5. In linear measurement the total sound pressure at the moment of the strike was about 85 dB.

Five individual strokes were averaged for each bell. Between strokes, the bell was turned on its axis so that the stroke points were at irregular intervals over half of the bell circumference. Thus, the highly structured sound field was averaged, and at the same time, a chance accentuation or suppression of a partial note was prevented.

Only those tape segments were evaluated on which the sound of the first 0.5 s, measured from the respective stroke, was maintained. These tape segments were spliced together into an endless ribbon and played on the same tape recorder used for recording them. The recurrent sounds were analyzed with an automatic search-tone analyzer and traced with a level recorder over the frequency of the bell partial notes as a spectrum of the pressure amplitude.

3. RESULTS

First the results from the test rods on the dependence of the sound-determining properties on the tin content (Figs. 6 and 7), the lead content (Fig. 8), on the substitution of tin with antimony (Fig. 9), and on the density (Fig. 10) are presented briefly. While the sound velocity and the

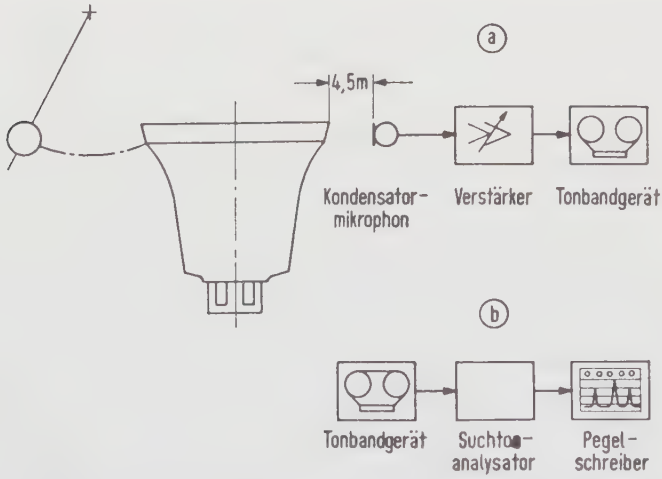

Figure 5. Arrangement for measuring the sound pressures of the partials of a bell: (a) recording; (b) analysis.

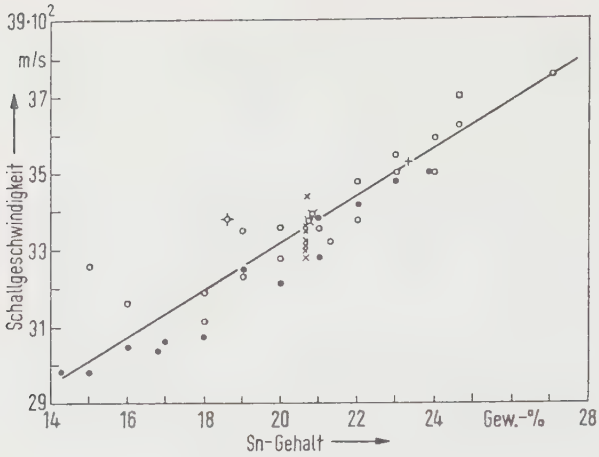

Figure 6. Influence of tin content on the sound velocity in Cu-Sn alloys.
ooo Test rod
xxx Test bell, 200 kg
+++ Experimental bell, 50 kg
ooo Porosity < 4%
Cabaret, Guillet, LeRoux

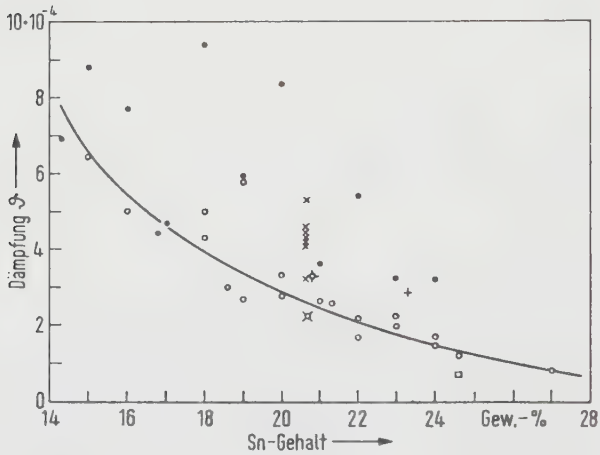

Figure 7. Influence of tin content on the damping of Cu-Sn alloys (same symbols as Figure 6).

damping are systematically dependent on the tin content. Figure 11 shows no systematic dependence for Poisson's constant. The dispersion in the sound-determining properties is mainly due to the variable porosity; therefore, in the corresponding diagrams, those measured points that were established with tests of a porosity smaller than 4% are accentuated.

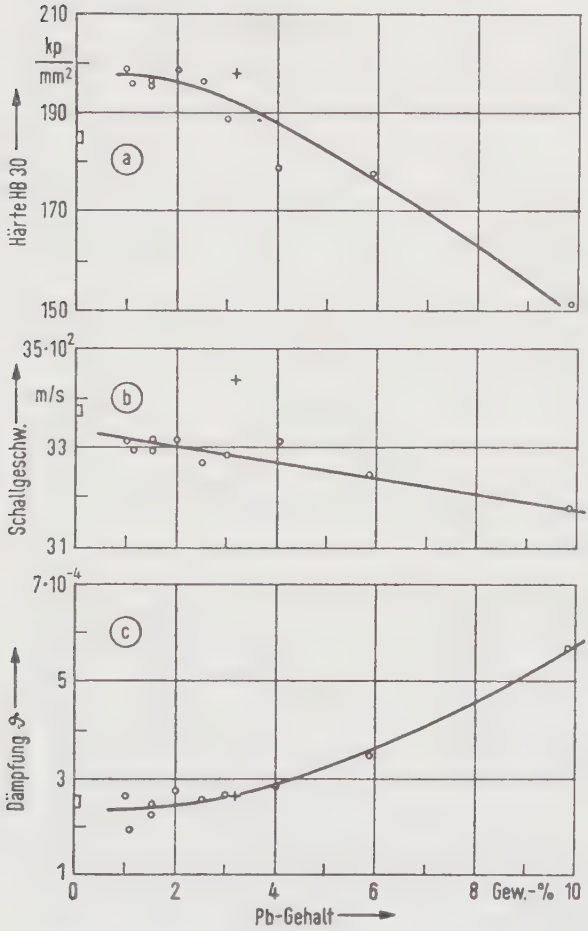

Figure 8. Influence of lead content on: (a) hardness; (b) sound velocity; (c) damping; of Cu-Sn alloys (+ = experimental bell).

From Figures 8 and 9 it can be deduced that the values of the elastic modulus and the damping correspond largely to those of pure bell bronze in the alloy composition of the experimental bells 4 and 5 (Table 1), according to the measurement on test rods. Moreover, it was to be expected that the pitches of the experimental bells change in proportion to the velocity of sound of their alloys. Thus, the pitches of bells 1–3 should increase linearly by approximately $^{12}/_{16}$ semitone (Fig. 7), and the pitches of bells 4 and 5 should lie approximately $^{7}/_{16}$ and $^{3}/_{16}$ semitone lower, respectively, than that of bell 2 (Figs. 8 and 9).

The pitch of the bells was established aurally by comparison with the pitch of adjustable, calibrated tuning forks. In Table 2, the deviations of

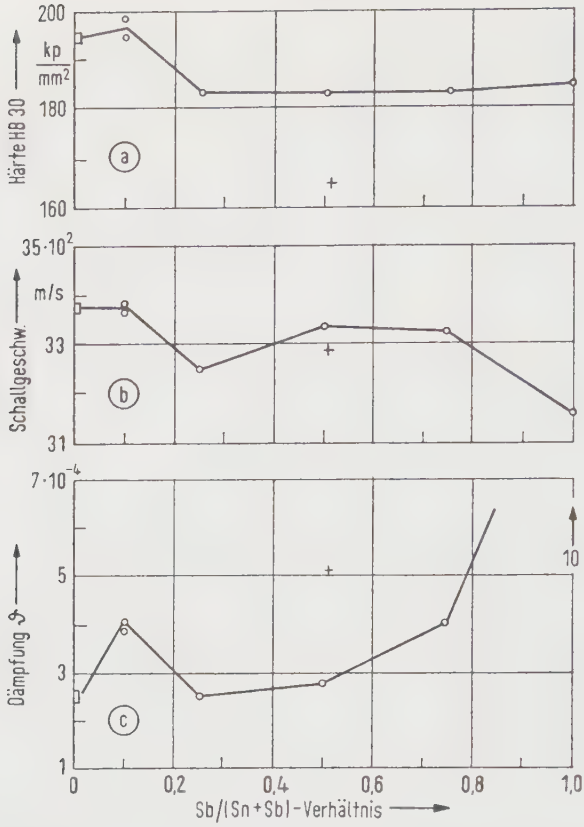

Figure 9. Influence of antimony fraction on: (a) hardness; (b) sound velocity; (c) damping; of Cu-Sn alloys (Cu proportion constant 79% by weight). (+ = experimental bell)

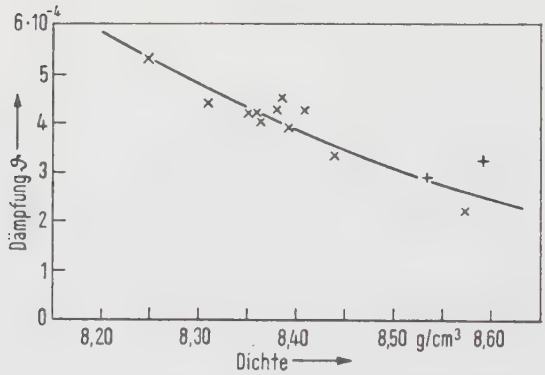

Figure 10. Dependence of damping on the density of bell bronze with 20.7% Sn. (x = test bell, 200 kg, + = experimental bell, 50 kg).

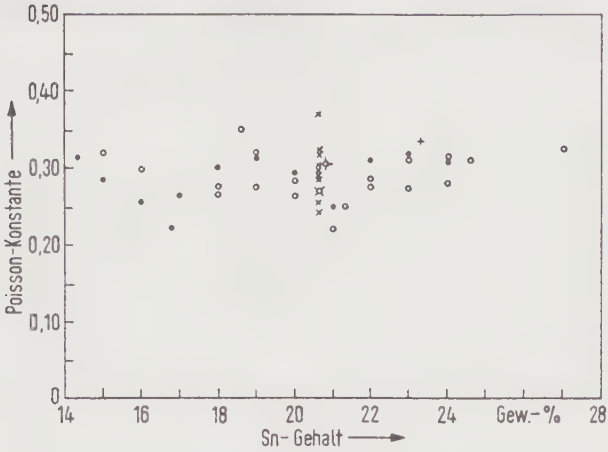

Figure 11. Influence of tin content on the Poisson constant of Cu-Sn alloys (same symbols as Figure 6).

Table 2. Strike Note Pitches of the Experimental Bells

	Deviation of Pitch from b'' (B_5) on Tempered Scale		*Vibration Frequency*	
Bell No.	*in* 1/16 HT	cents	*Subjective* (Hz)	½ *Upper Octave* Frequency (Hz)
1	− 3/16	− 18.75	912	913
2	+ 4/16	+ 25	935	935
3	+ 4/16	+ 25	935	935
4	− 1/16	− 6.25	919	920
5	− 20/16	− 125	858	856

the pitches from the normal pitch of tempered tuning are given in sixteenths of a semitone and in cents. The reference note for the fork calibration was $A_4 = 435$ Hz; thus, the pertinent frequencies could be calculated with the relationships (8 and 9). The last column of the table gives the half vibration numbers of the upper octaves, which should, according to a rule first established by Rayleigh, coincide with the strike note frequencies in these minor-key octave bells (octave bells with minor third). Actually, the deviations between the corresponding vibration numbers of +0.3/16 HT and −0.6/16 are smaller than the accuracy of measuring the subjective strike notes.

In the amplitude spectra (Fig. 12), amplitudes within 50 dB of the largest partial note are recorded together. The differences in sound pressure level in dB between the most important partial notes and the strongest partial note were deduced from the spectrum of each bell.

The plotting method of Thienhaus [6], used in Figures 13, 14, 15, 16, and 17, provides vivid presentations that serve to describe objectively

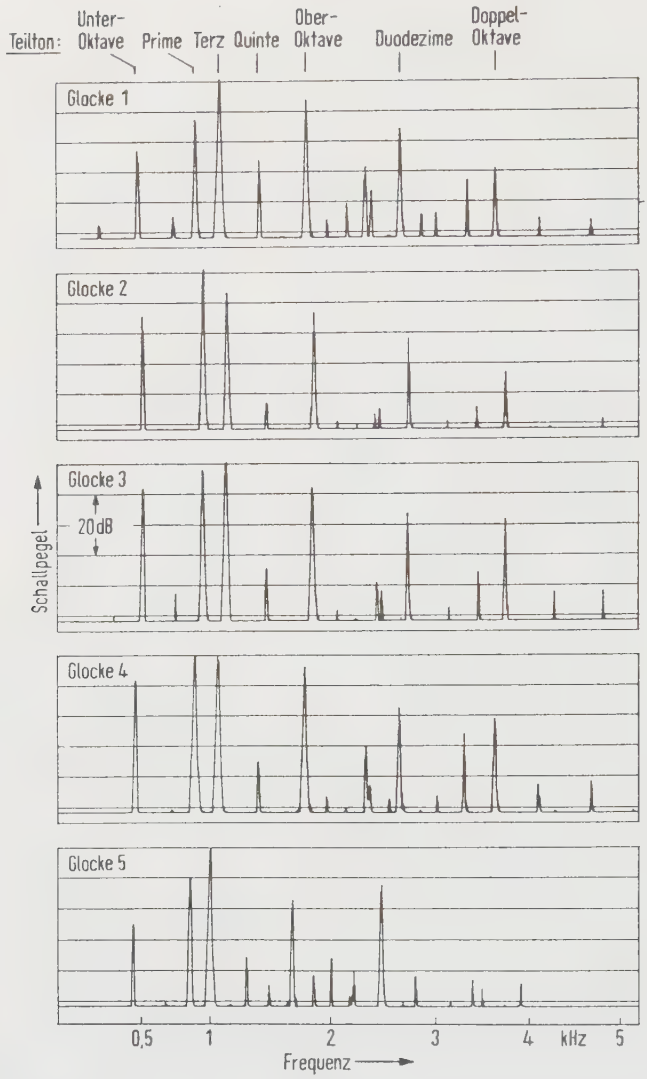

Figure 12. Partial tone spectra of the experimental bells.

the sound properties of a bell. On the horizontal axis, the partial note pitches are given in cents compared to the strike note. The vertical heights (in the amplitude-frequency plane) of the partial notes indicate their strengths directly after the strike, in relation to the strongest partial note. A reciprocal scale for expressing damping is shown in perspective.

A number of subjective properties for the bell sound can be deduced from the sound spectra. The smaller the deviations of the partial notes from the cent-values that correspond to intervals of pure tuning with the strike note, the better the inner harmony. The smaller the decay rate of a

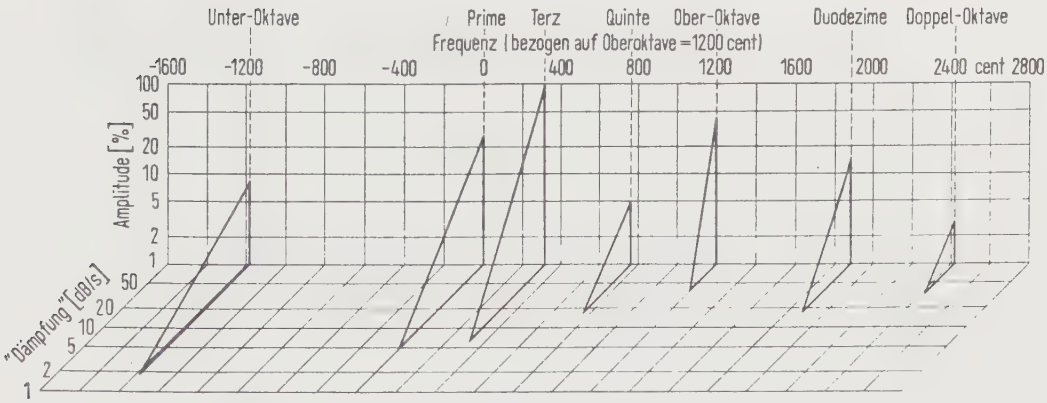

Figure 13. Sound spectrum of experimental bell 1, strike note b'' — 18.7¢ (913 Hz).

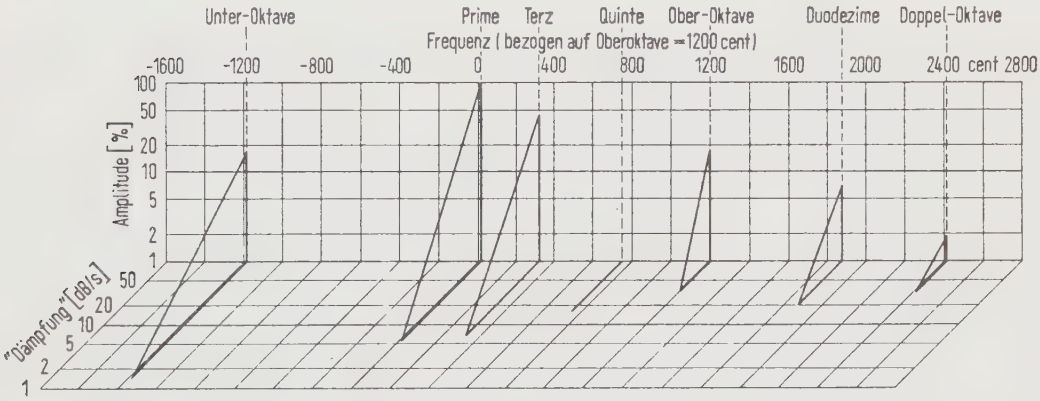

Figure 14. Sound spectrum of experimental bell 2, strike note b'' + 25¢ (935 Hz).

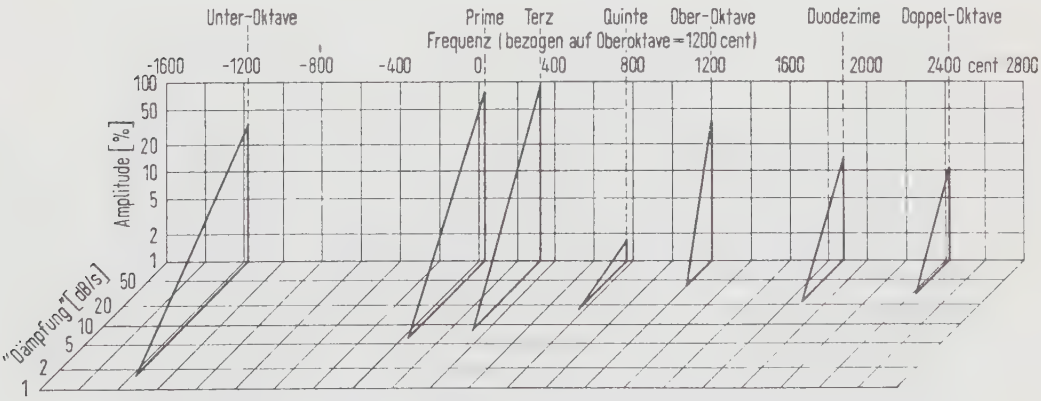

Figure 15. Sound spectrum of experimental bell 3, strike note b'' + 25¢ (935 Hz).

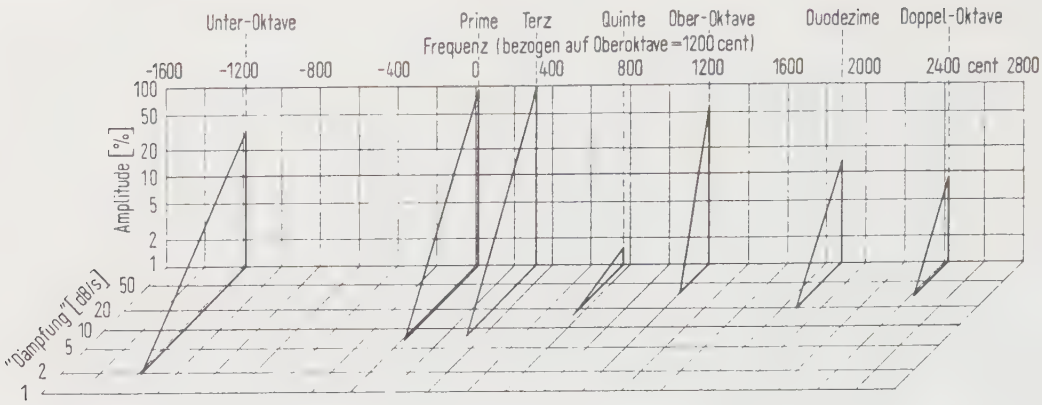

Figure 16. Sound spectrum of experimental bell 4, strike note b'' — 6.2¢ (920 Hz).

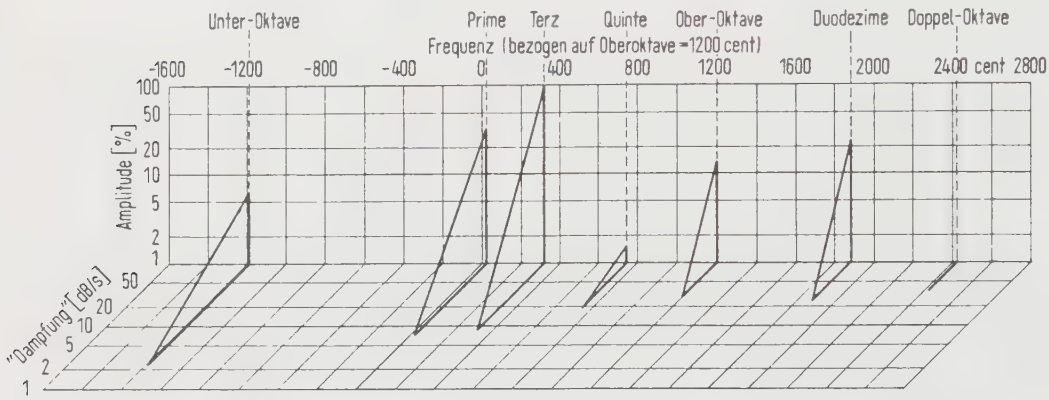

Figure 17. Sound spectrum of experimental bell 5, strike note b'' — 125¢ (856 Hz).

partial note in dB/s ("damping"), the greater their decay time, that is, the time, measured from a powerful strike, until the sound become inaudible. This time is, of course, also dependent on factors that are not properties of the bell, such as the strength of the stroke, the hearing ability of the observer, and the presence of distracting sounds. A partial note becomes much more pronounced the greater its amplitude and the lower its decay rate. In the sound spectra, the triangular areas that are sketched for every partial note can be seen as characterizing this "resonance." Finally, we recognize proportions in the amplitudes and decay rates of all important partial notes from the triangles. This "sound development" is an important criterion for a well-balanced timbre and for the carrying ability in the bell sound.

4. DISCUSSION

The sound analyses revealed that the pitches of the experimental bells did not have the expected values. This initially surprising result points

to the strong influence of factors aside from the material composition that determine the pitch of the bells. Next to the influence of the state of the material, the dimensional accuracy of the bells should be mentioned here primarily.

How great the shape influence is can be evaluated through an error estimate on circular cylindrical shells. In the case of a shell of average radius $R_0 = 200$ mm and wall thickness $d = 15$ mm, the dimensional error must not exceed $\pm$ 0.2 mm, if the frequency error is not to exceed $\pm$ 8/16 HT [Equation (2), $R > d$, Equations (8) and (9)].

In order to obtain an indication of the exactness with which the desired pitches can be attained in practice, the test records for 92 untuned bells were evaluated. Among those, 45 bells are within the manufacturing specifications identical with experimental bell 2, the remaining ones are different only in terms of size (100..... 1000 kg). The results of this evaluation are presented as histograms in Figures 18a–e. The bar heights indicate the percentages of deviation from the desired pitch (abcissa value 0).

In Figure 18a the frequency distribution of the strike note pitch about the nominal value is given for uncorrected bells. In the upper part of the diagram, the position of the strike pitches of the experimental bells is included. In view of the width of the distribution, it is understandable that the strike note pitches of the experimental bells do not have the expected relation to each other.

Figures 18b–e show the frequency distribution of the partial note intervals of the 45 and 92 examined bells, respectively, and the partial note frequencies of the experimental bells. In these presentations the abcissa value 0 means that the pitch of the respective partial note and the strike note pitch correspond exactly to the designating partial note interval in tempered tuning. Here, too, the deviation range of bells of the same material is so great that no dependence on the material composition can be deduced from the inner harmony of the experimental bell.[3] However, a substantial material influence is unlikely, since the difference between the experimental bells are relatively small. This conclusion agrees with the fact that the intrinsic frequency relationships of a cylindrical shell are not dependent on the material, to a first approximation (Eq. 1), and to closer approximation are dependent only on Poisson's constant of the material [4].

It can also be concluded from the sound spectra that the differences in the decay rates between similar partial notes of the experimental bells 1–3 are minimal. The dependence of the damping of the bell bronze on the tin content was not effective here. This result, as well as the unexpected high partial note dampings of test bells 4 and 5, can be traced to the strong influence of the porosity.[4]

[2]From the Karlsruhe Bell Foundry

[3]It is notable that all bells evaluated here can be tuned in such a way through supplemental change of the bell shape, that they meet the Limburg specifications (7).

[4]An examination of the test bells by an expert indicated that in the case of all five bells the sound duration meets that of the Limburg specifications.

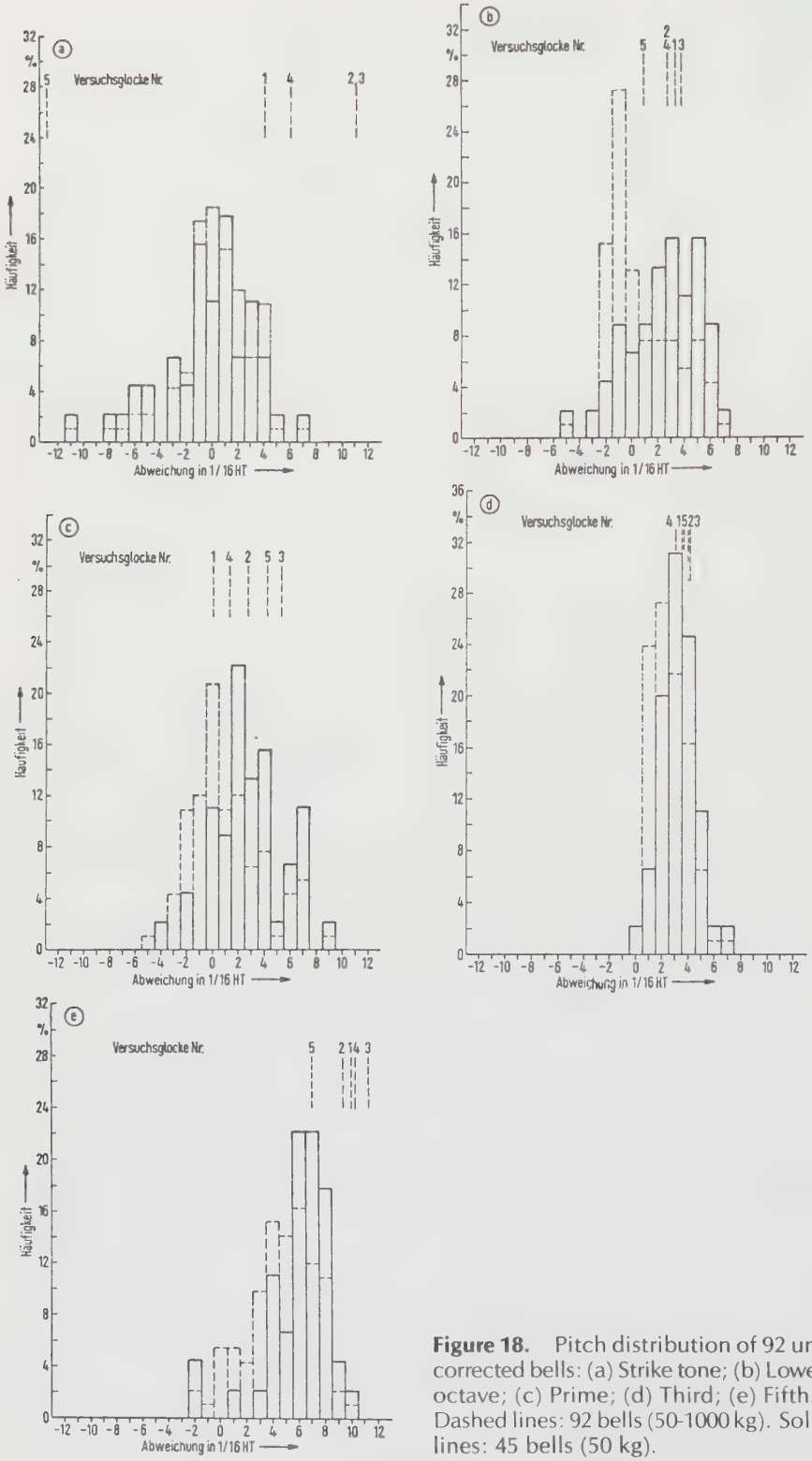

Figure 18. Pitch distribution of 92 uncorrected bells: (a) Strike tone; (b) Lower octave; (c) Prime; (d) Third; (e) Fifth. Dashed lines: 92 bells (50-1000 kg). Solid lines: 45 bells (50 kg).

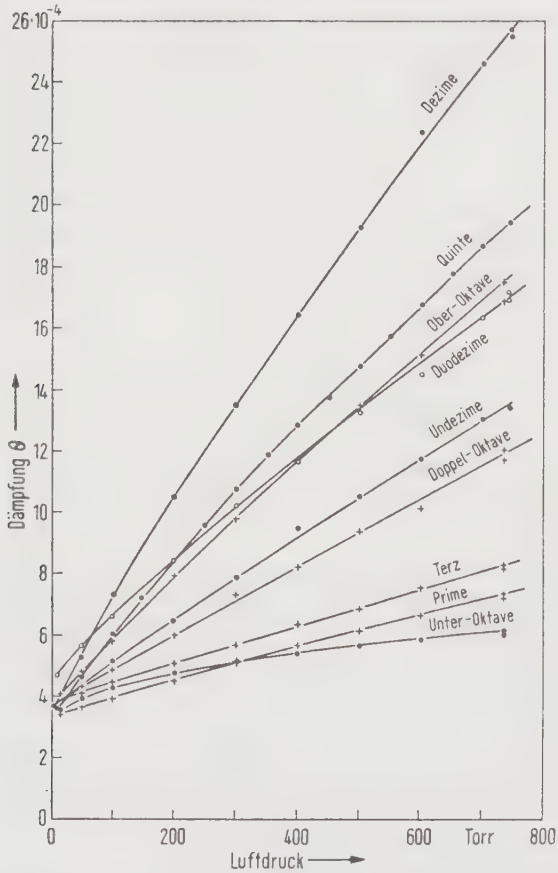

Figure 19. Dependence on air pressure of the partial tone damping in a bell.

In all the experimental bells, the lower octave, fundamental, and third are weakly damped; the fifth and the upper octave, on the other hand, are strongly damped. The material damping is not very dependent on the frequency in the range of the partial note frequencies discussed here (Figure 3). Also, it was not possible to measure the unavoidable energy loss of the bell to its support, which could depend upon the vibrational configurations of the different partial notes. Therefore, it was assumed that the variation in partial note damping of a bell was caused mainly by differences in the radiation and friction damping. This supposition was confirmed through measuring the partial note damping of a smaller bell (17 kg, stroke note e''' or E_6) in relation to the atmospheric pressure (Fig. 19).[5] In comparing the curves drawn in Figure 19 it becomes apparent that the damping of the lower octave is, within the measuring uncertainty, proportional to the square root of the atmospheric pressure. This explains

[5]The experimental arrangement was as illustrated in Figure 9; however, the bell and the electromagnetic transformers were in a soundproof enclosure that could be evacuated.

a great part of the friction damping which in the other partial notes decreases below the radiation damping.

From the curves that describe the dependence of the damping of some partial notes on the atmospheric pressure, the damping of the bell material can be determined through extrapolation to 0 torr. Here it amounts to $\theta_0 = \delta = (3.6 \pm 0.3) \times 10^{-4}$ for the customary bell bronze. Its part in the tonal damping of the lower octave, the fundamental, and the third at atmospheric pressure (θ_{760}) amounts to approximately 60, 50 and 40%. Thus, the material damping is not small compared to the external damping parts, as is occasionally assumed [3].

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18A

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PART I

A. L. Bigelow

In the development of any musical instrument there are periods of definite advance and periods where development seems to remain stable. This we can only judge, of course, in looking back over the centuries, comparing the different examples which have come down to us: their tonal and physical aspects, the music written for them at the time, and their ability to render it.

During each period, the instruments built are considered adequate by the majority of those using them. A later generation, however, uses its own standards to judge earlier instruments, to determine what it needs for the performance of music written during its own period and to evaluate the quality and tonality that it has learned, from previous experience, to appreciate and to require. If we are able to execute the music of today upon an instrument of another age -- the instrument meeting our standards, or most of them -- then we consider it a good instrument.

When an instrument maker continues to better his product, musically and physically, giving it more tone and versatility, musicians come to accept it and compose new works employing these advances; as a result, the music often acquires new form and new expression. The musicians themselves demand instruments of certain characteristics from their makers. And thus an instrument develops.

Progress in the development of the carillon comes not so much through the desire of bellfounders in general to change their product, as perhaps through the desire of a musically educated public to expect and even to demand, when possible, certain considerations in a set of bells. Their musical experience tells them what would please them in a carillon, in its tones and in its performance; if certain requirements could be met, their musical pleasure would be augmented. This would also result in a more general appreciation and acceptance of the instrument.

There is another side to this question. Those who know the carillon are used to the sound of bells, used to their natures, musical and physical, and have accepted the instrument and all its effects, not

requiring it to meet certain musical standards which other instruments must meet. They argue that it is not necessary to compare it to other producers of tone; it must be taken for what it is and its particular charm allowed to manifest itself.

Though there is something to be said for this, and no doubt there are those who appreciate the sweet-sour music from a quaint and probably out-of-tune carillon and do not seek the performance they would require from another instrument, there is a legion of discerning musicians and the musically-minded who have come to know and to love the sound of this instrument of percussion, and know that it would stand to gain if only certain aspects could be improved. Some founders, also, are of this opinion; they have sought, in the measure of their knowledge and their possibility, to meet this sentiment and to improve upon their product. Other founders have declared their bells perfect and have resisted, even adamantly, any change.

Yet there still remains general agreement -- mostly unexpressed in so many words -- that the carillon's performance is deficient, due principally to its tonal imbalance between bass and treble, and to the question of the bell's minor-third partial. There are still other considerations beyond these, but of lesser importance. If there were a way to achieve tonal balance, and at the same time reach some sort of solution to the minor-third problem, the carillon would overcome two great handicaps, handicaps which hinder its general musical acceptance.

In entering upon this work we cannot study enough the carillons of past founders. Everything they can reveal will help establish a manner in which to proceed. Basing our observations upon what they have or have not done, the results they obtained and the nature of the music of their carillons, we can then perform some experiments of our own, hoping to establish some law, or at least the rectification and further application of already existing laws, proceeding toward a more highly developed carillon, in every respect a fully acceptable musical instrument.

As already stated, the question of tonal inadequacy in the higher bells has not seemed, for the most part, to occupy carillon founders to any great degree. Let us see just how the founders' series of bells correspond to the AMP line, and note any deviations there might be in favor of greater tonal balance, if any. We will take them chronologically, chart and analyze the data, to measure any possible progress through the centuries that man has been making carillons.

Starting with the greatest founders of the era before the French Revolution, consider first three carillons by the Hemony brothers, founding at Amsterdam:

1. The Oude Kerk in Amsterdam, cast in 1658;
2. The Dom Toren, Utrecht, cast in 1663;
3. The Heilige Sulpiciuskerk, Diest (Brabant), cast in 1670.

These three carillons embrace a range from the B-flat below middle C to the F 3 1/2 octaves above. It will be seen that the lower bells of the first 1 1/2 octaves adhere to the AMP line. While the height holds to the AMP line for a full 2 octaves, the great and small diameters leave it after 1 1/2 octaves.

At C_5 the great diameters deviate from the AMP line by 4.5%; at C_6 by 19%, and by extension, at C_7 by 65%.

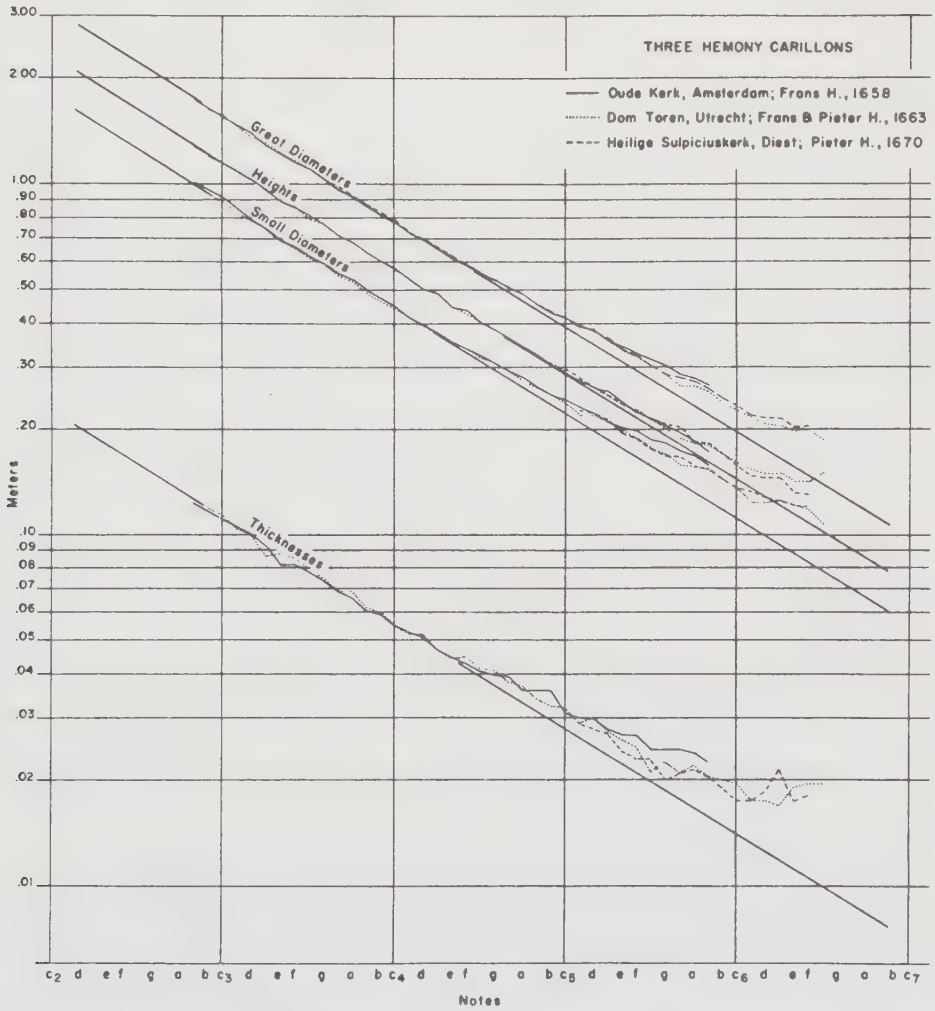

Figure 6. Chart of the Measurements of the Bells of Three Hemony Carillons

The ratios of the parts of the bells to their diameters are:

	$A^{\#}_2$	C_3	C_4	C_5	C_6	F_6
Great Dia.	1	1	1	1	1	1
Small Dia.	.570	.570	.570	.570	.600	.600
Height	.735	.735	.735	.700	.640 to .740	.630 to .660
Thickness	.072	.072	.072	.077	.085	.100

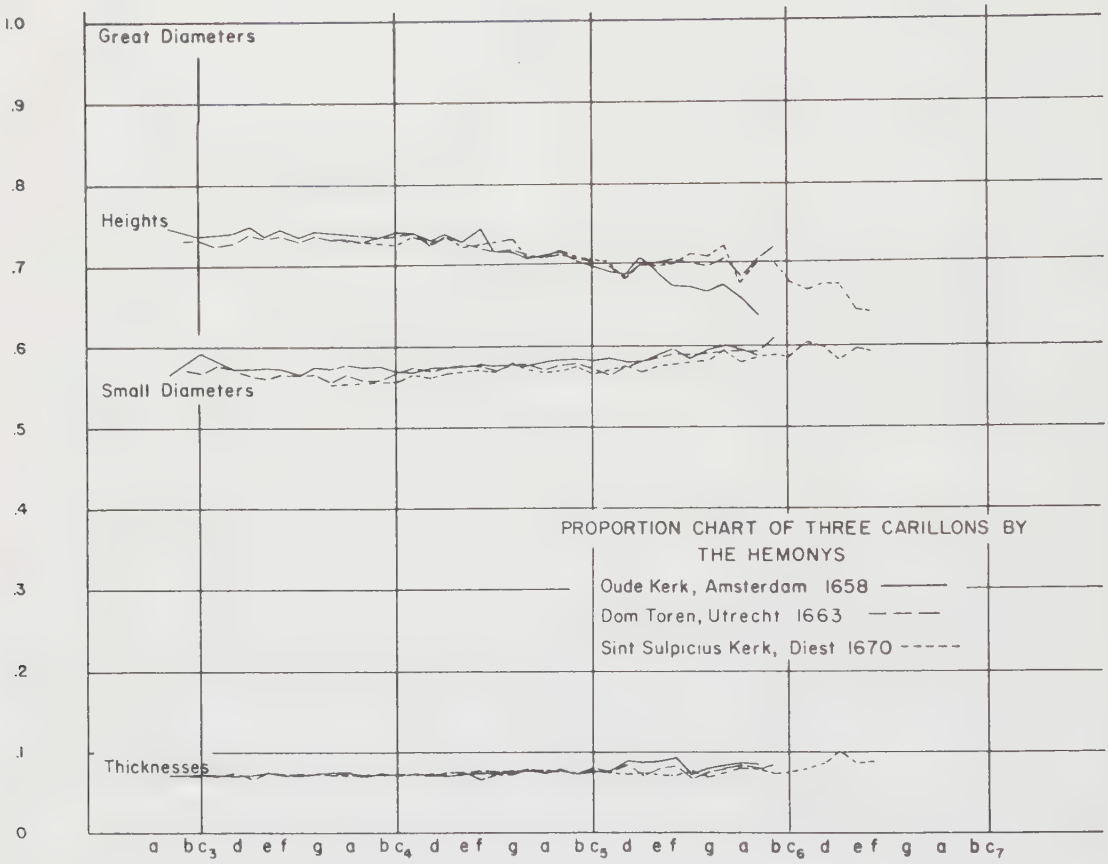

Figure 7. Proportion Chart of the Three Hemony Carillons

Beyond C_6 the small diameters and heights tend, at first, to approach each other, but do not quite reach the point where we might say that the bell loses the general proportion of the lower ones.

The Hemonys did not cast carillons extending to the end of the

fourth octave above middle C. The carillons at Mechlin and Diest show us their thinking only as far as F_6. The Hemony bells at Utrecht extend only to B_5 and at the Oude Kerk to $A^{\#}{}_5$. The Hemonys' bass and medium registers form an instrument of very satisfying tones, possessing good resonance and a bright "ring". Beyond the medium register, the tone of the bells becomes thinner, less brilliant, maintaining its dignity but losing its resonance. There is no evidence here that the Hemonys had analyzed the question of tonally balanced carillons, even though they did tend to cast their bells of somewhat thicker proportions in the higher register.

Consider now three carillons by the Van den Gheyns:

1. St Gertrude, Louvain (Brabant), cast in 1730, a 4-octave instrument based upon B below Middle-C, of which the bells are almost entirely by this founder;
2. Veere, Zeeland, 1735, an instrument of large and medium bells;
3. Steenokkerzeel, (Brabant), 1735, a carillon of medium and small bells.

If these instruments are tabulated, a continuous series of well over four octaves is realized. The lines of proportion of the instruments compare closely enough so that an average can be taken.

For the first two octaves, from C_3 almost to C_5, the bells adhere very closely to the AMP line. At C_5 the Great Diameters deviate from the line by 5%, at C_6 by 20%, at C_7 by 77%, and at $F^{\#}$ in this fifth octave above middle C, by 142%. The Van den Gheyns' highest bells are thus approaching 2 1/2 times the size they would be if they adhered to the AMP line.

The ratios of the bells to their diameters are:

	C_3	C_4	C_5	C_6	C_7	$F^{\#}{}_7$
Great Diameter	1	1	1	1	1	1
Small Diameter	.57	.57	.57	.595	.62	.63
Height	.74	.735	.725	.705	.675	.66
Thickness	.075	.075	.080	.085	.125	.175

A quick comparison between the Van den Gheyn and Hemony series shows a marked difference. The Van den Gheyn instruments, cast many decades after the Hemony, show that their founders were quite aware that increased scale was necessary in the treble for balance of tone, and that they had gone a long way to assure it. What they did not understand, however, was that increased scale alone was not enough to match acoustically the upper register with the lower. Another two centuries had first to intervene.

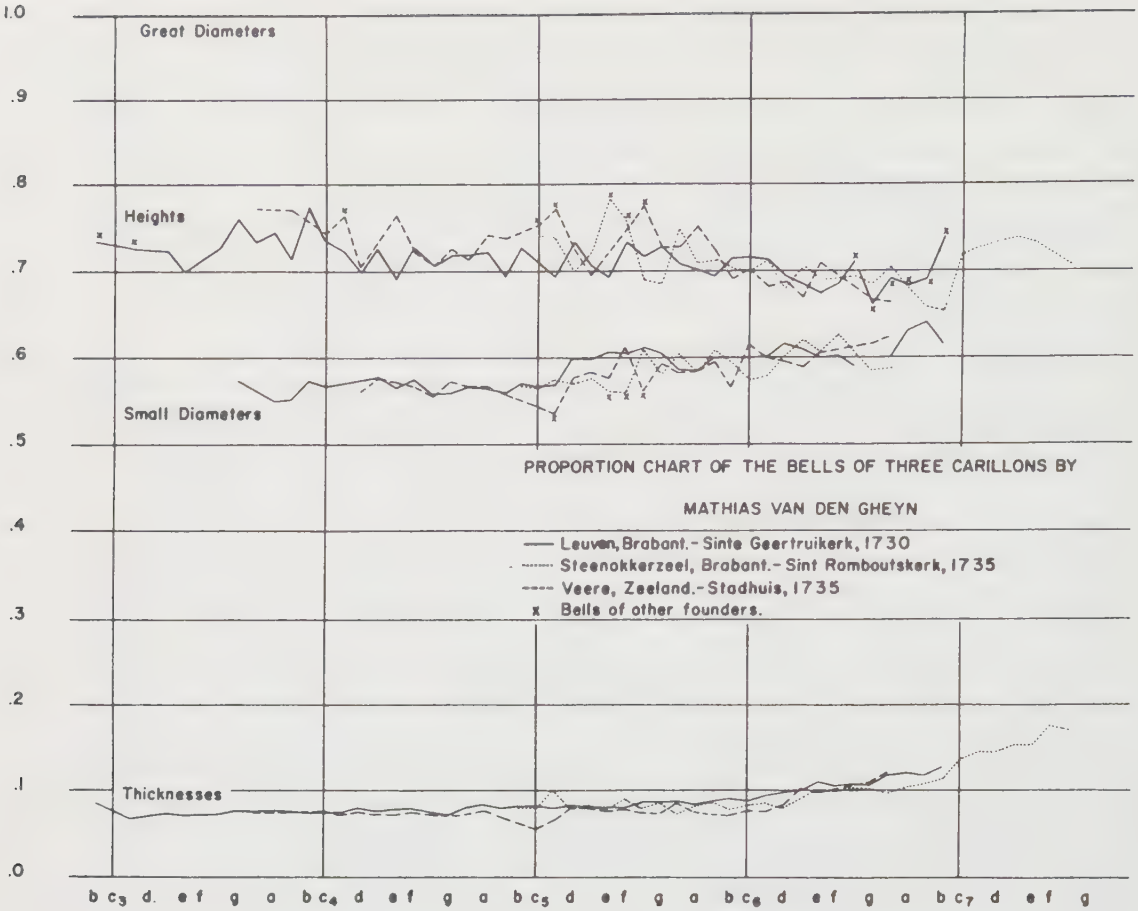

Figure 8. Proportion Chart of Three Carillons
by Van den Gheyn

Another of the earlier "greats" was Dumery of Bruges whose existing instruments date from 1743 to 1772. Dumery came a bit later than the Hemonys and was a contemporary of the later Van den Gheyns. Although he cast fewer carillons than the other two, his surpassed theirs in size. The largest bell Hemony cast was a B-flat, a bell of 4 tons. Dumery's bourdon at Bruges is a deep G, weighing close to 6 tons. His highest trebles, those at Tielt in West Flanders, reach as high as $F^{\#}_7$ (as do those of Van den Gheyn at Steenokkerzeel). Together, Dumery's instruments at Bruges and Tielt span a range of 5 full octaves less one half-tone.

As with the other founders, Dumery holds to the AMP line in his lowest bells, starting to deviate after the G_4, in the second octave above Middle-C. At C_5 his diameters increase by 5%, at C_6 by 26%, at C_7 by 65%

and at F_7 by 100%, the last bell just twice the size it would have been if it had been cast to the AMP line.

The ratios of the parts of the bells to their diameters are:

	G_2	C_3	C_4	C_5	C_6	C_7	$F^{\#}_7$
Great Diameter	1	1	1	1	1	1	1
Small Diameter	.570	.570	.570	.580	.610	.625	.630
Height	.725	.720	.715	.710	.675	.750	.630
Thickness	.072	.072	.076	.085	.100	.135	.550

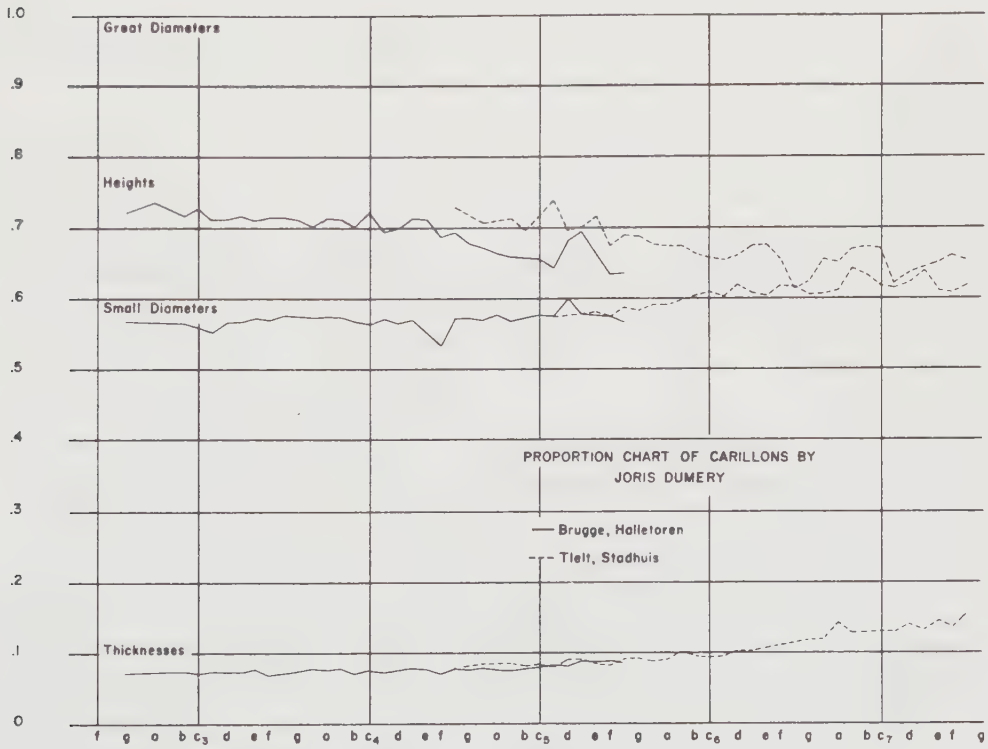

Figure 9. Proportion Chart of Three Carillons by Joris Dumery

Dumery's bells are full and rich in the lower and medium registers. But he had not preoccupied himself to any extent with the tonality of his trebles. While the tiny treble bells are sweet and melodious, their tone is weak and short-lived, quite unable to balance that of the basses. The top octave of the Bruges carillon was replaced, in 1937, with slightly

larger and thicker bells in an attempt to remedy the situation.

The study of bells by other founders of this period, erroneously termed the "Golden Age" of carillon founding, reveals the same results: instruments full-toned in their bass, of adequate tonality in their medium register, but woefully inefficient in their trebles.

We now bridge the period of the French Revolution and the disturbed political and economic times which followed it. It was mid-19th century before a series of bells was again attempted, when several French firms were active for the first time, trying their hand at designing and casting a complete series. While their success was limited, there were some definite signs of progress.

Hildebrand in Paris was doing the most commendable job. In his basses some of the partials were tuned. His mediums were less successful, but his trebles are well worth studying.

Consider the carillon of 3 1/2 octaves, C_3 to F_6, across from the Louvre in the tower of St. Germain L'Auxerrois.

For well over 2 octaves, his bells follow a regular diminution, falling almost exactly on the AMP line. Seeking to give his upper bells more tonal importance, at F_5 he deviates sharply from the AMP line and the model for Bell F_5 is retained for the entire highest octave, with the same diameter and profile used for all the remaining bells. Making each one thicker than the foregoing, he realized higher and higher pitches. The last bell of the series, F_6, deviates from the AMP line by 122%, much more than the bells of any other founder at this pitch. If the series were so continued -- if it were possible to continue it -- at C_7 the deviation from the AMP line would come to 320%. Carrying this series to C_7 is, of course, hypothetical, for such a bell would be so thick that it could not vibrate. As it is, Hildebrand's highest trebles, inasmuch as their mouths are absolutely flat, give the impression of heavy bronze cylinders with holes bored into them.

Of two bells of the same diameter, the thicker one will have the higher note; thickness causes rigidity, thus greater frequency and higher pitch. It is this idea which was later employed in the development of a tonally matched series of carillon bells.

Hildebrand had noticed the deficiency of tone in the upper bells of a carillon, and took definite steps to correct it. He augmented the intensity of the upper register, and his highest bells certainly have much more to say than if they were cast closer to the AMP line.

It is only the Hum-Tone in the highest bells which has occupied the founder. The other partials are neglected. The extraordinary thickness

(the thicknesses of the highest bells are 1/3 their diameters) has very much attenuated the other partials. The Strike-Tone has disappeared. The Prime, where there is any, seems to have been totally ignored by the founder. The bells have little ring, and no distinctive timbre, because there are no partials to give them any.

We can learn from this series of bells, imperfect though they are, that trebles can deviate much more from the AMP line than had those of any founder up to that time, and with beneficial result.

The English next enter the field of carillon founding. Long a country of pealing bells only (pealing is the national tradition), the people of the "Ringing Isle" have been slow to accept the carillon, calling it a "foreign import". Not requiring their bells to play tunes, and certainly not to play in harmony, there was no desire for such an instrument. In all of Great Britain today the carillons number less than a dozen.

It was only 35 years ago that her bell founders, quite reluctantly, gave up the form of the traditional pealing bell with its Hum-Tone something less than an octave, and accepted the Continental form with its tuning. Tuning has always been looked upon with much disfavor in England, her founders arguing that to touch a bell was to rob it of its original sweet tone. (This reasoning may have prevailed because they did not know how to tune, and were also not set up to tune in their shops.) But more recently the English saw a ready market for their product and soon learned to tune with as much accuracy as any of the Continental founders of the two or three previous centuries. English carillons exported to the rest of the world outnumber still the instruments by all other founders.

They extended the range of the carillon down to C_2, with its Strike-Tone an octave below Middle-C, a bell weighing in the neighborhood of 20 tons. In the treble they extended the range up to $C^{\#}_8$ over 6 octaves above. Here, if anywhere, we can compare the balance of the tone of the trebles to that of the bells of the medium and lower register.

The bells of a founder no longer casting will be considered here. His bells will be taken as representative, even though there are some differences in size and weight, both slightly greater or lesser, when compared to those of other founders from the same country.

In studying the sizes of English bells, we find that there are sometimes two distinct series, a heavier and a lighter. These are what are sometimes termed "heavy profile" and "light profile". This is particular to the bass register, while both heavy and light profiles join in the medium and treble registers. One founder may have both a lighter and a heavier series, choosing what he believes to be the best for each particular installation. While lighter bass bells can sometimes serve in

smaller and lower belfries, up to a certain point in the series, there is one noticeable case where lighter basses first installed in such a belfry were later removed to a larger and higher one, causing their tone to be quite inadequate in their new position.

Whether of light or heavy profile, the series of bells by English founders are in one aspect quite comparable to the Hemony series: they hold to the AMP line from their basses almost to C_5.

At C_5 the Great Diameters deviate from the AMP line by 5%; at C_6 by 15%; and at C_7 by 57%.

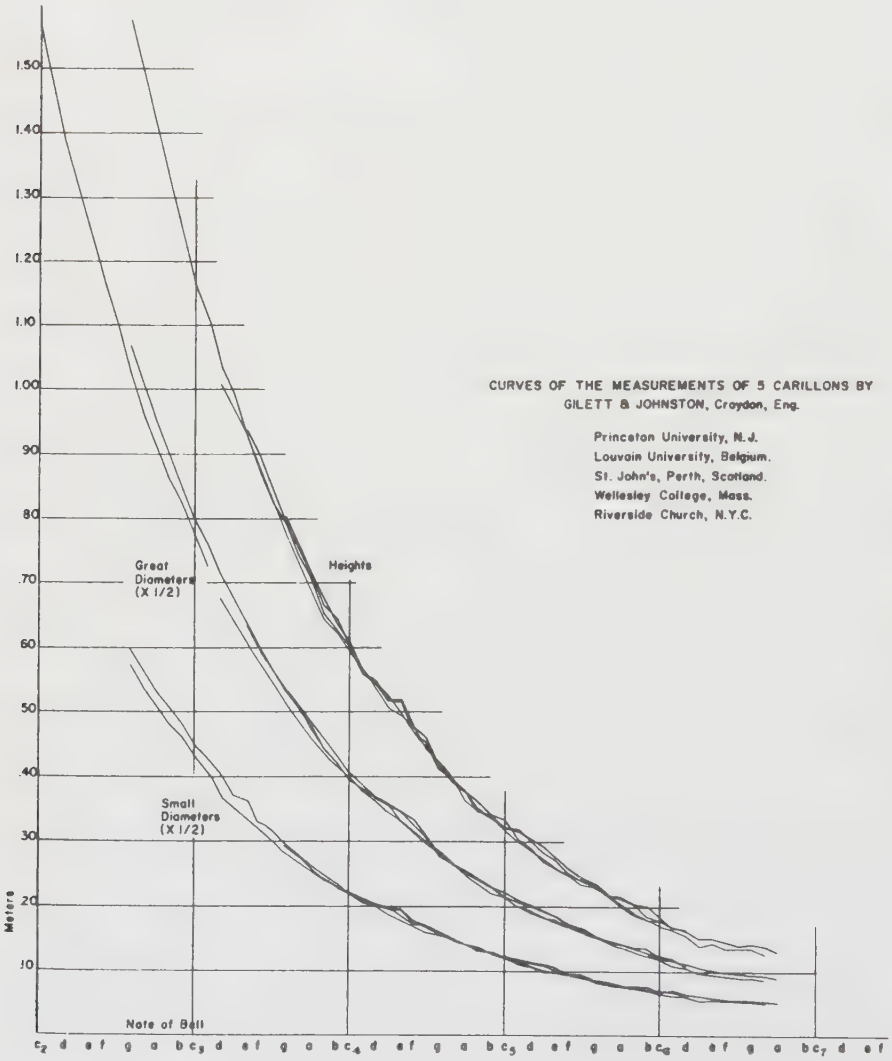

Figure 10. The Measurement Curves of Five Carillons by Gillett and Johnston

The ratios of the parts of the bell to their diameters are:

	C_2	C_3	C_4	C_5	C_6	C_7	C_8
Great Diameter	1	1	1	1	1	1	1
Small Diameter	.55	.56	.55	.56	.56	.58	
Height	.73	.73	.74	.75	.74	.74	
Thickness	.07	.07	.07	.075	.08	.105	

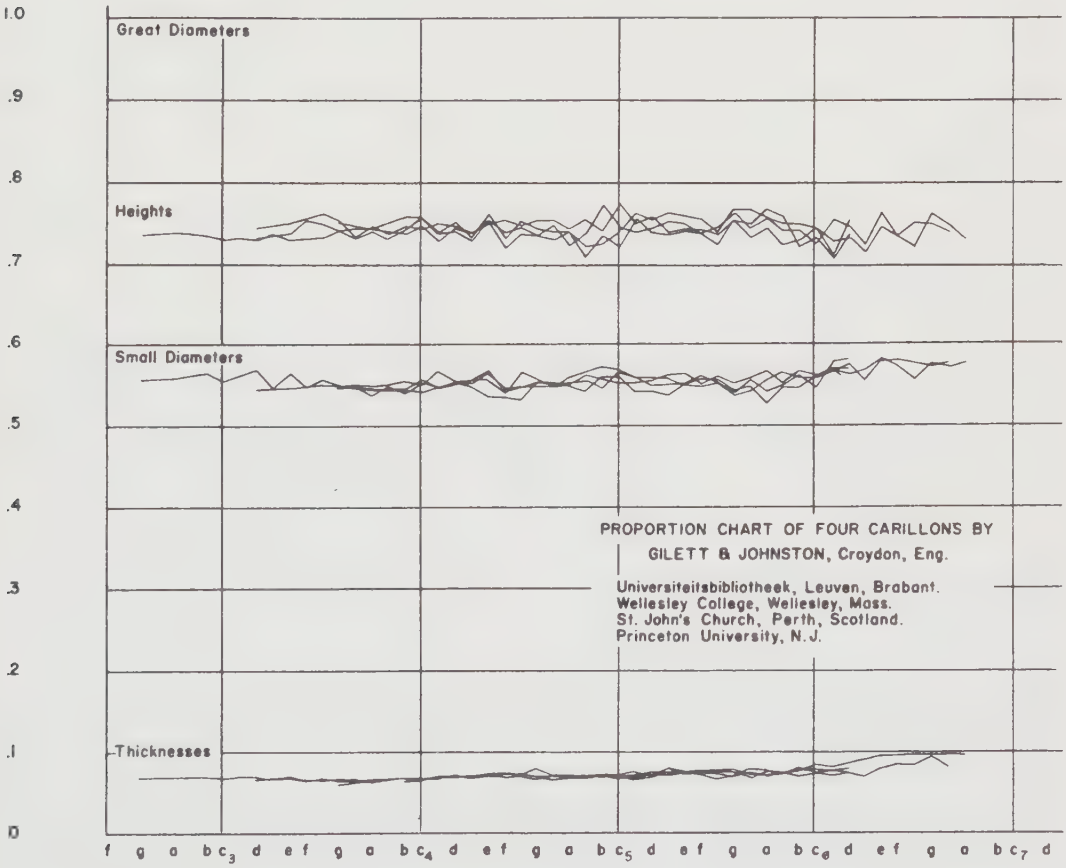

Figure 11. Proportion Chart of Four Carillons by Gillett and Johnston

The proportions of the bells are surprisingly constant throughout the range. This is for two reasons: first, because they do not deviate markedly from the AMP line, and hence can keep their proportions; and second, because their heights are kept in proportion even though this is

not necessary. When bells become smaller and leave the AMP line, they become proportionately thicker. This automatically reduces the height of the bells and increases the small diameter. But the bells discussed here are cast with added material above their effective heights -- their shoulders are thickened upward -- thus allowing the small bells to have the same appearance as the larger ones, and hence the same proportions.

The bass and medium bells of these instruments, where they are not too thin, possess the body and intensity that bells of these registers normally have. When bells are too thin, a disturbing Fourth, at the interval of a 4th above the Strike-Tone and one tone above the Minor Third, intrudes and distorts the tonal picture. This tone is of the same nature as the Strike-Tone, and indeed tends to take over the role of the latter if the bell is too thin. It is very disconcerting when descending the scale into the basses to hear "Fa" when the last note of the series is expected to sound "Do".

The upper registers of these sets are light, lacking body, and inclined to jangle. Thus no tonal balance exists between basses and trebles: their tone has an almost instantaneous decay, devoid of all resonance.

The advent of these instruments clearly demonstrated that the range of the carillon could be extended to over six full octaves. Large and small bells had been cast before, but never in a complete series by one founder. With this innovation, the carillon gained added respect. It was now quite far removed from the traditional chime of from 10 to 12 diatonic bells, common to English and American belfries, or even from a carillon of 2 or 3 octaves of more-or-less matched bells. And, except when the minor-third partials were strong -- often the case in thinner bells -- these carillons sounded in tune.

What can we learn from such instruments as these? Their thin upper bells are quite lacking in intensity and in resonance. Just after the middle of the series, their tones seem to lose all body and become just points in a musical scale. In an effort to realize added tone from them, bellmasters play them in octaves, more often than not using a continuous tremolando, with a quite unmusical result. If the tonality of the trebles were matched to the tonality of the basses -- provided the basses are of at least normal thickness and thus relatively free from the disturbing Fourth -- then they would have gone a long way toward proving that a carillon can become that instrument which, for the most part, has remained only an ideal.

Summing up this chapter, we can establish:

A. The bass register of a series of carillon bells can be founded following an AMP line whose C_3 bell (Middle-C) measures around

62" in diameter; the proportions of the bells are roughly

Grand Diameter	1
Small Diameter	.55 to .57
Height	.72 to .74
Thickness	.071 to .072

The carillons measured and tabulated here, all instruments of acceptable tonality at Middle-C, generally agree to these figures. There is, of course, a vast difference in the timbre of the several series, due to their individual profiles, exterior and interior, the form of lip, the curve of the waist, the tuning and voicing, the metal used, and the many other considerations which govern the tonal quality of the bell.

B. Bass registers following an AMP line whose C_3 bell is less than 61" to 62" in diameter are inadequate in performance and tend to lose their fundamental pitch tone, possessing a Fourth as a dominant tone rather than the note for which the bell was designed.

C. With one or two exceptions, the treble registers of the carillons adhere more or less to the AMP line. This results in inadequately toned upper registers, incapable of matching the intensity of the bass bells to form a homogeneous instrument of equal tonal importance throughout. In the exceptions that exist, where the treble bells have been cast to a larger scale, the resulting trebles did not form the desired balanced instrument because certain other conditions were not fulfilled.

It is the task of this work to:

1. Develop and determine a deviation from the AMP line for the sizes of the treble bells which, if cast to this line, would correct the tonal imbalance between the upper and lower registers of the carillon;

2. To develop a bell whose proportions and profile will give it added body, clarity, and increased musical performance, once its tonal output has been augmented by deviation from the AMP line;

3. To study the complete balance of the carillon, tonal and mechanical, and devise a means of matching its registers, acoustically and physically.

18B

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PART II
THE NEED FOR TESTING REPRESENTATIVE MEDIUM AND TREBLE BELLS, AND THE APPARATUS AND METHOD USED

A. L. Bigelow

In attempting to establish corrected lines of proportion for medium and treble bells, it is beneficial to study a number of existing examples, noting their positions in the series and analyzing the intensities of their more important partials. With this as a background, experimental bells can be cast and tested to compare their tone and characteristics to the existing bells. Therefore, bells no longer serving in their carillons, bells still serving in carillons, and bells cast specially for these studies, were used in the experiments. (One result of these experiments was the replacing of some bells from carillons tested, for their performance was found lacking due to inadequate tuning or unequal intensity.)

A testing frame was designed in which to hang the bells and assure controlled striking (see Plate No. 1). To give the necessary insulation, the frame, made of welded angle-iron, is mounted on wooden sleepers. A wooden beam firmly attached to the frame, from which the bells are hung, reproduces as closely as possible the situation in the belfry.

Whether to strike the bells on the inside by clappers, according to normal carillon play, or on the outside by "hammers" was a matter of some concern. Striking a bell from the outside is not at all foreign to the carillon. When a bell is used both as a note in the carillon and as a swinging bell in a peal, then it is imperative that it be furnished with an outside hammer for carillon play. The clapper must be free to swing when the bell is pealed. In some belfries abroad, no outside hammers are used. Each time the instrument is to be played, the clappers are hitched to the carillon mechanism. As soon as the playing is ended, they must be unhitched. This is often quite complicated, especially when the clappers are heavy and access to the bells difficult.

Even though it may be argued that there is some slight difference

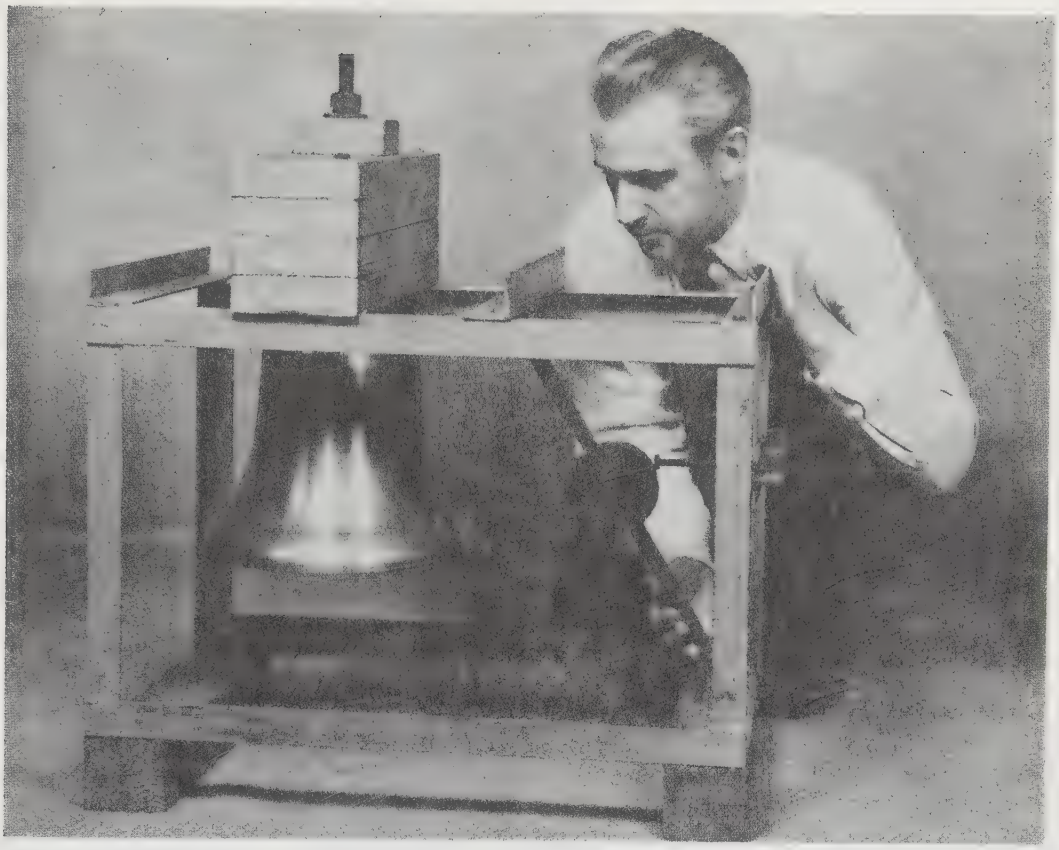

Plate 1. The Testing Frame

in tonality when the bell is struck on the outside as opposed to the inside, it was felt, for the purposes of this study, that the outside striking should be adopted. Because of the possibility of absolute control of the striking through gravity, the outside hammer was found simpler -- and possibly more accurate -- than attempting to set up a mechanism to control the strike of the inside clapper by some electrical, pneumatic, or purely mechanical means -- a means necessarily quite complicated if uniform and measurable strikes were to result.

The frame supports an adjustable angle from which the hammer is suspended. The hammer is fixed to a light stem of negligible weight, so that it will not unduly influence the force of the stroke. This stem is welded to a horizontal yoke suspended on point bearings.

The hammers are of steel, similar to that from which clappers are made, and are furnished with set-screws by which they can be fixed at

a given distance along the stem, according to the size of the bell being tested. The force of the strike can be regulated by holding the hammer against an adjustable screw at any distance from the bell, at the start of the stroke. This simple mechanism thus assures a maximum uniformity of strike with minimum complication. To test the bell, the operator has merely to hold the hammer against the stop-screw and then release it, allowing it to strike the bell once, catching it on the rebound. This seemingly crude set-up proved its value in results obtained; readings made from it were surprisingly uniform.

It was desirable that the bells be struck with hammers proportional to their weights, and that the stroke distances be varied for lighter to heavier blows. This was accomplished by four hammers chosen to simulate, grosso modo, the effect the clappers striking the bells. With this series of hammers, weighing 2 1/2, 3 1/2, 4 3/4 and 9 3/4 pounds, it was possible to strike each bell with a light, medium, heavy or extra-heavy hammer, depending upon the proportional weight of the hammer to the bell being tested. (Plate No. 2)

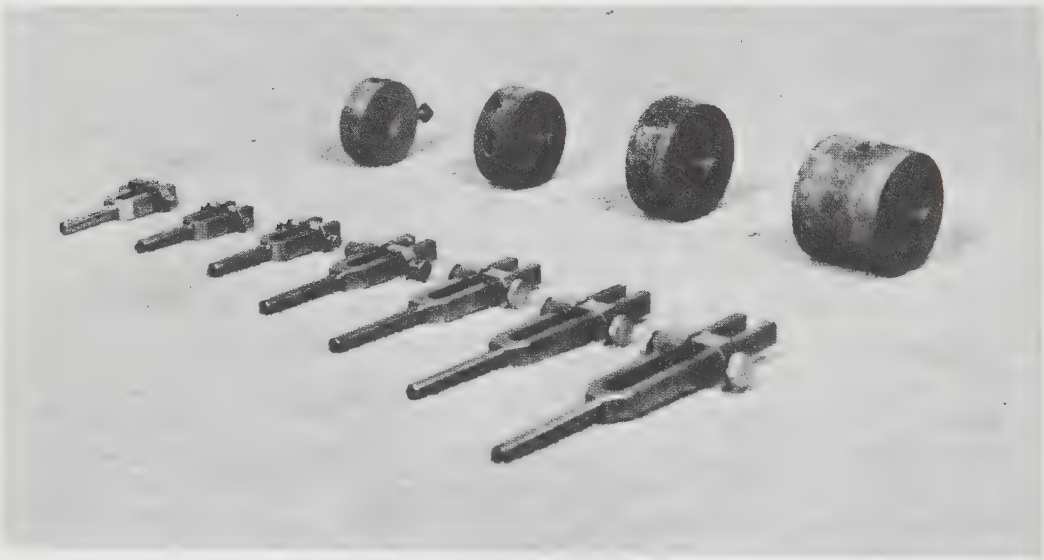

Plate 2. The Hammers and the Tuning Forks

In general, three hammers were used per bell: light, medium, and heavy. The hammers were adjusted so that they would strike at the very center of the sound-bow, where the clapper normally strikes.

For the most part, three distances of strokes were adapted for each clapper:

A 2" stroke to simulate a light blow;
A 4" stroke to simulate a medium blow; and
A 6" stroke to simulate a heavy blow of the clapper.

Nine different strikes were obtained in this manner and, except in the case of the smallest bells where it was impractical, each bell was subjected to these nine strikes.

The testing frame was clamped firmly to the floor of a field-free room. An R.C.A. 44 B.X. microphone, with a flat frequency response, was set at a distance of 4 feet in front of, and on a level with, the center of the bell. While one operator worked in the field-free room adjusting the mechanism and striking the bells, another worked with the electronic equipment in the laboratory outside.

For this study, testing the bell involved measuring the intensities of the several basic partials which contribute to the bell's over-all intensity and to its timbre. These partials were tested:

(a) Hum Tone
(b) Prime
(c) Octave
(d) Minor Third (noted as "Third")
(e) Fifth

The partials whose notes correspond to the pitch name of the bell are referred to as "octave partials". Thus the Hum Tone, the Prime, and the Octave, separated by octave intervals, are "octave partials", since they sound the same note.

At the start of the test, the exact pitch of the partial was measured by a calibrated tuning fork held to the bell. When struck, the fork caused the bell to vibrate in sympathy. The fork was then taken to the test lab and a Hewlett-Packard Wave Analyzer set to the frequency of the fork. The operator at the bell caused the hammer to strike, and the operator at the analyzer set the exact bell frequency and recorded the readings. (Figure 12)

In the larger bells, three hammers were used and each of the five partials was measured according to the nine different strikes. As the series being tested extended into the treble, two hammers were used, and then only one. Similarly, as the partials of the upper bells entered the higher frequency range, not all tones were tested. Thus for the upper trebles there are fewer readings, though the ones taken are the most significant.

As the tests were based on the tonal comparison of the partials, the entire tone was studied from its initial intensity, at the impact of

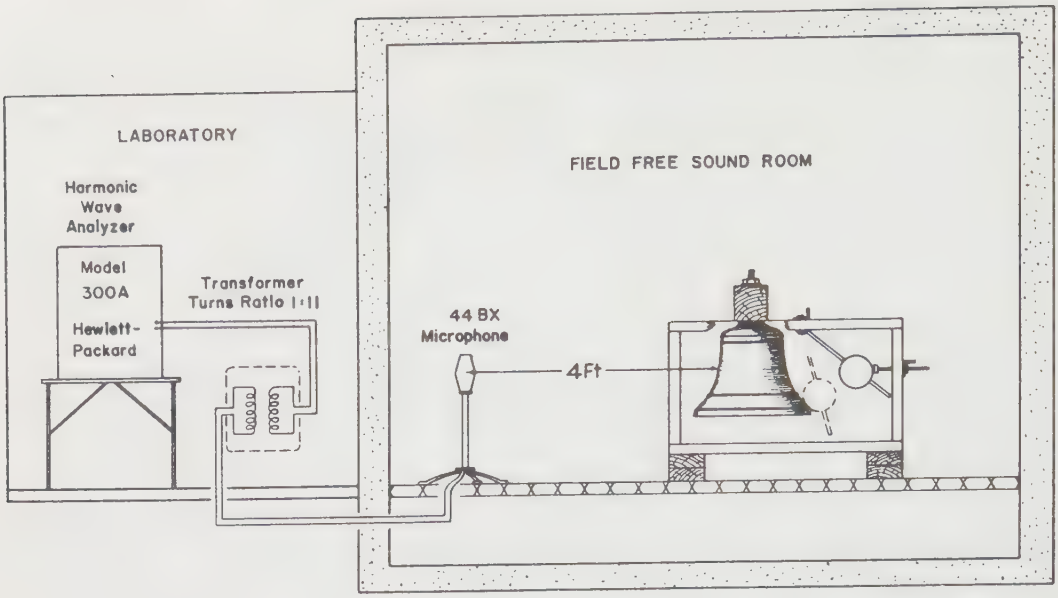

Figure 12

the clapper, to the end of the decay. Readings were, therefore, taken at impulse, at 2, 4, 6, and 10 seconds. Only in the case of the Hum-Tone, and this in the larger bells, was a reading possible at ten seconds. Theoretically, there is no amplitude at the very moment of impact; at this moment the bell just takes on motion and only a split second afterward produces tone. The tone so closely follows the blow of the clapper upon the bell that this infinitesimal delay was neglected and the maximum intensity charted at 0 seconds, or at "impulse".

The results were first tabulated on a 10 × 10 grid with sound pressure in decibels on the ordinate against time in seconds on the abscissa. The sound pressure in decibels was obtained in the following manner: when the bell is struck by the blow of the hammer, sound waves are transmitted to the microphone and measured in millivolts on the analyzer. As shown in Figure 12, a transformer having a step-up of 11 to 1 was also included in the circuit.

$$\text{Volts output of Microphone} = \frac{\text{Volts Measured at Analyzer}}{11} .$$

The R.C.A. 44 B.X. microphone was previously calibrated; its output sensitivity was 200 microvolts per dyne per square centimeter.

Therefore:

$$\text{Sound pressure (dynes/cm}^2) = \frac{\text{Reading of analyzer (millivolts)} \times 10^3}{11 \times 200}$$

$$= \frac{\text{Reading of analyzer (millivolts)}}{2.2}$$

Converting to db. (absolute):

$$0 \text{ db} = .0002 \text{ dynes/cm}^2$$

$$\text{db} = 20 \log \frac{\text{sound pressure in dynes/cm}^2}{.0002}$$

Sample calculation:

With an impulse of .0081 volts or 8.1 millivolts:

$$\text{Sound pressure } p = \frac{8.1 \times 10^3}{11 \times 200}$$

$$= 3.68 \text{ dynes/cm}^2$$

$$\text{db} = 20 \log \frac{3.68}{.0002}$$

$$= 20 \log 18400$$

$$= 20 \times 4.264$$

$$\text{db} = 85.28 \text{ where } 0 \text{ db} = .0002 \text{ dynes/cm}^2 .$$

The value of 0 db = .0002 dynes per square centimeter is the standard reference for sound pressure and is sometimes referred to as decibels "absolute". The sound pressure of .0002 dynes per square centimeter corresponds to the threshold of hearing for normal human ears as given by the frequently used Fletcher-Munson* curves for equal loudness contours for pure tones.

The three charts on the following page illustrate the tonal pattern of an F_5 bell.

It was felt that musician and layman would have greater facility in interpreting the tonal patterns of the bells studied if the results of the experiments were plotted on an arithmetical rather than logarithmical grid. Therefore, rather than converting the readings in dynes/cm^2 to decibels, the "loudness", or intensity, of the partials was kept in dynes/cm^2, and represented as sound pressure.

* Fletcher and Munson, Jour. Acous. Soc. America, Vol. 5, No. 2, p. 82, 1933.

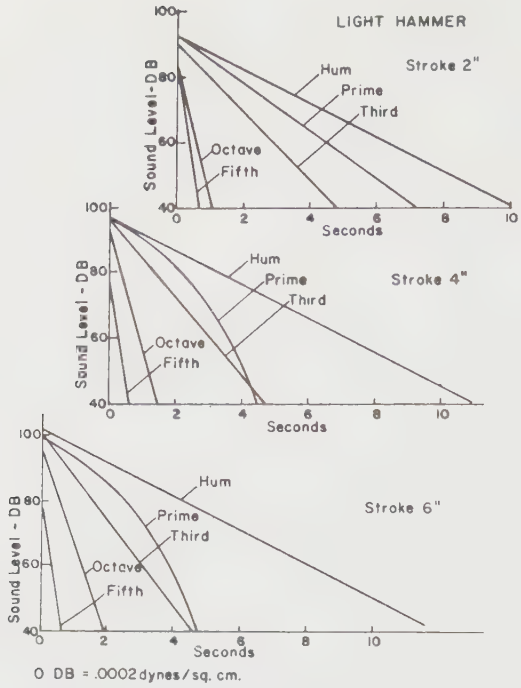

Figure 13

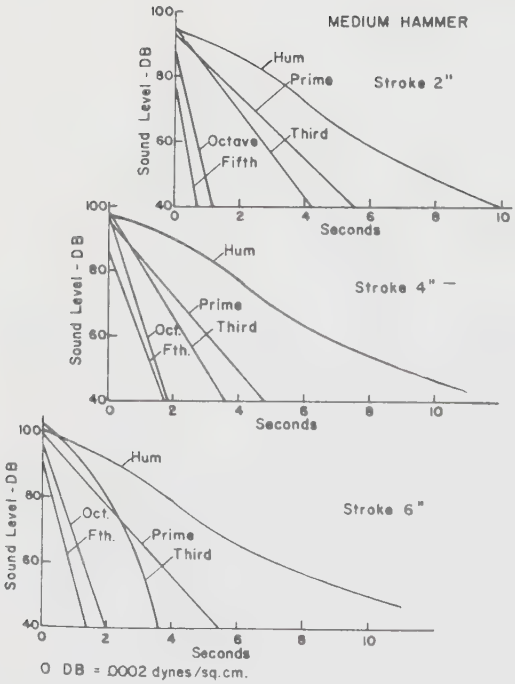

Figure 14

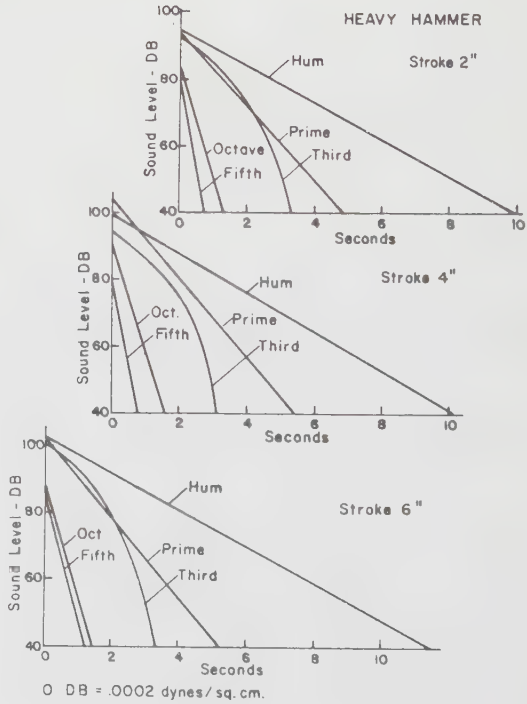

Figure 15

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PART IV
ESTABLISHING THE LINES OF PROPORTION AND PROFILE

A. L. Bigelow

This chapter deals with two operations: determining the sizes of the bells, the lines of proportion for the great diameters, heights and thicknesses for the improved series; and determining the shapes, the profiles, both outer and inner, of the bells themselves.

The two operations are mutually dependent. Two bells of different sizes, both sounding the same note, will have different profiles. While in the bass register the difference in the form of the two may be negligible, little by little as the treble is approached, especially when the scale of the treble is increased, the "cut" of the highest bells is noticeably different from that of the larger ones. The bass bells will be long, svelte, with sweeping lines and curves. The trebles, to the contrary, will tend to be short, stubby, squat, with wide mouths and thick lips, departing visibly from what we have come to recognize as "carillon bell shape". This difference does not take place all at once at some particular point in the series, but asserts itself so gradually that if any bell were taken and compared to those on either side of it in the series, no difference could be remarked.

The increased intensity of the medium and treble registers results, to a great extent, from the increased thickness of the walls of the bells. But walls cannot be thickened without an increase in the sizes of the bells: an increase in the scale. In designing the new series, it is necessary to establish lines of proportion controlling the measurements of each bell, and then it is necessary to design the shape of the bell, the profile, to assure not only the desired increase in intensity, but also the increased ring and improved timbre. Both observations are based on the foregoing experiments.

NOTE: "Improved timbre" need not imply "improved quality", though this might follow. "Quality" is often regarded as strictly a matter of personal

taste: a particular tone which appeals to one person might not appeal to another, and so on. "Improved timbre" might be expressed as "improved performance". In improved performance, the bell would sound fuller, devoid of harshness and tinniness; it would be richer in its octaves tones, thus adding interest and clarity; it would sound louder, ring longer, and thus be able to take its place in the balanced series. All these attributes improve its performance and even, it is believed, its quality, making the bell more pleasing to listen to.

Performance, timbre, quality are here closely related.

Therefore, with the results obtained in Part III, we can set about to design the new series.

THE LINES OF PROPORTION

A series of bells of "medium profile" will be established, and the Line of Musical Proportion for the diameters based upon those bells which exhibited strong tone, sonority, and purity. Using the C_3 as a logical representative starting point, the bass bell of a classic instrument of four octaves, let us take a bell whose Great Diameter measures 5' - 2", or 62", a bell weighing over 2 1/2 tons. This is neither an especially light nor heavy bell for this note; it is rather a good average, possessing a full tone of adequate intensity. It might, perhaps, be slightly on the heavier side, compared to most carillon bells of the same note.

On a semi-log graph, we lay off 72 equal spaces on a horizontal base line, representing the notes of the carillon. The measurements of the bells will be laid off vertically on the log cycles. Starting with C_2, we note the series all the way to C_8. This 6-octave range will allow for the extension of the lines of proportion beyond the four octaves in consideration - C_3 to C_7.

We then lay off the diameter of the C_3 bell, 62", on the C_3 line of the chart. The AMP line is then established by pointing off one-half the diameter of the C_3 bell, or 31", on the C_4 line; one-quarter the diameter, 15 1/2", on the C_5 line; one-eighth, 7 3/4", on the C_6 line, and so on. A line through these points will give the desired slope of the AMP lines used in this chart; the AMP lines of Small Diameter, Height, and Thickness will be parallel to it.

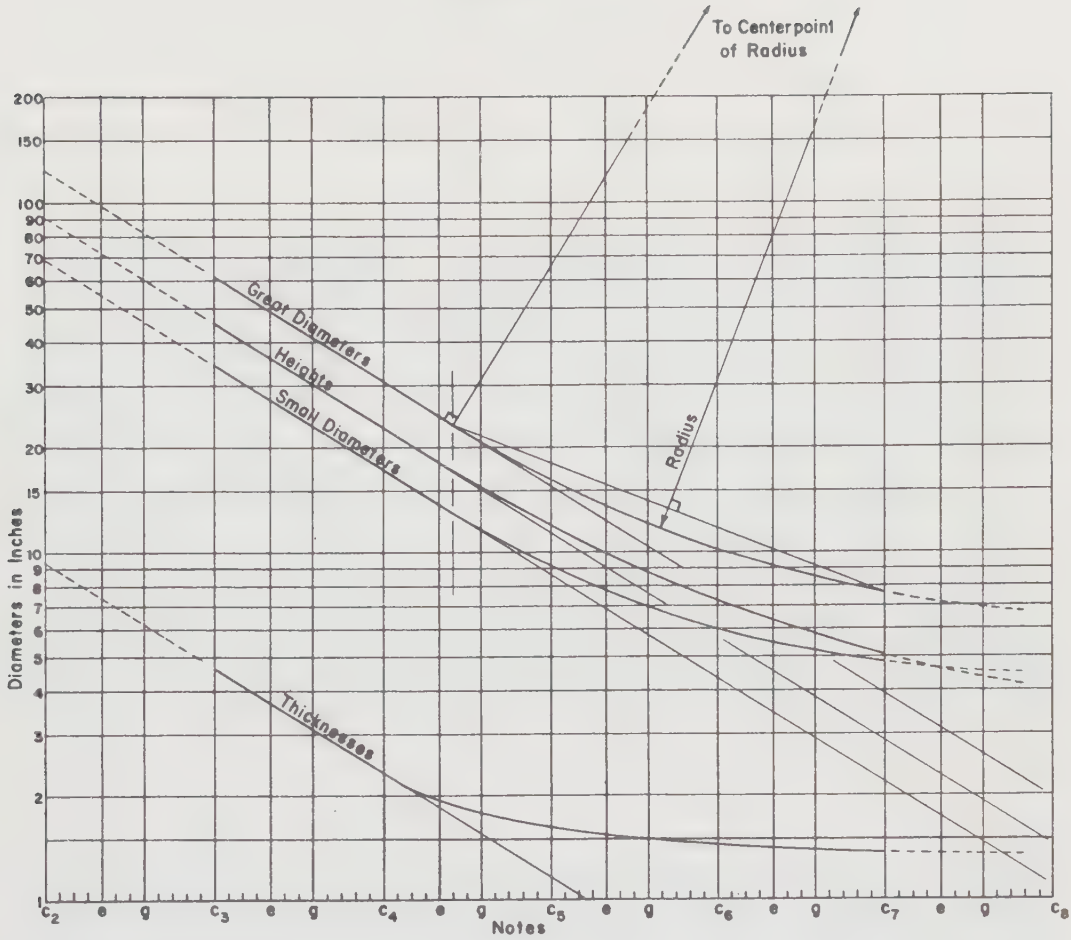

Figure 40. Establishing the Lines of Proportion for a Carillon of Improved Performance

When we determine these other AMP lines from the proportions of the C_3 bells studied, we find them to be approximately

G.D.	S.D.	Hgt.	Thick.
1	.56	.74	.075

This will give

62" 34.72" 45.88" 4.65"

for the other dimensions. These lengths will be laid off on the C_3 line,

and through these points lines will be drawn parallel to the AMP lines for the Great Diameters. Or, the lines may be determined by dividing the octaves 2, 4, 8, and so on, as was done in the case of the AMP line for the Great Diameters.

These four lines determine the measurements for a series of bells homogeneous in character, graduating in like proportion from the bass to the upper registers. If the lines should be extended downward to C_2, the lower octave would be determined; bells cast to the dimensions thus obtained would be of correct weight and timbre, comparable in performance to the octaves just above -- as far as C_5, for instance.

Above the C_5 bell, however, we have found it necessary to correct the AMP line; if we did not, we would end up with trebles from 1" to 2" in diameter, bells tonally incapable of holding up their end of a range of several octaves. Here, then, we deviated from the AMP line by plotting on the chart the dimensions of the several bells found by experiment to possess a greater intensity than this range of carillon bells usually has, and fitting a curve to them.

Although this method is adequate, another, and more scientific, way would be to use the "anchor point" method. The last bell of the series, in this instance C_7, is designed in accordance with the experiments performed for increased tonality. The diameter of this bell will deviate from the AMP line from 90 to 100%. Its diameter is then laid off on the C_7 line, and an arc drawn from this point tangent to the AMP line several tones below C_5 -- at F_4 for instance. If a higher point in the range is taken, there will be too small an increase at C_5, so closely does the arc follow the AMP line at first.

This arc may be extended into the octave above C_7 for a few tones only, because the diameters would soon increase out of measure after C_7. Also, if only a few notes above C_7 are desired, the bells could be made to the same diameter as the C_7 bell, and the heightened pitch obtained by increasing the thickness proportionately. Or, the line could be extended with a very small slope for these few notes. But if the range is to be extended to a half or a full octave higher, another anchor point should be established for the end of the series and another radius drawn. Similarly, the measurements of this same treble bell will furnish the anchor points for the lines of deviation for Small Diameter, Height, Thickness.

The Thickness Line really gives only an <u>indication</u> of this dimension. Although the outside proportions can be fully determined, the thickness measurement can be firmly established only by actual experience -- with, of course, the necessary thickness added to allow for the removal of metal in bringing it down to pitch, in tuning. Excessive thickness is

wasteful, for it requires extensive machining, but a bell cast too thin is even more so, for it has to be recast. The Thickness Line for the series can fairly well be determined by casting and tuning every fifth bell, charting the resulting thicknesses and then allowing enough additional metal for tuning.

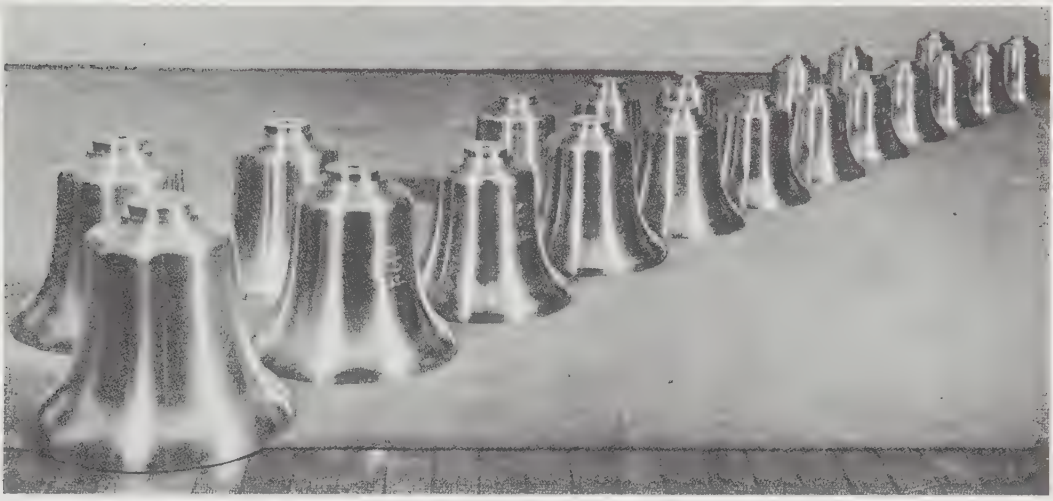

Plate 3. A Series of Bells Cast to the Established Line - F_5 to C_7

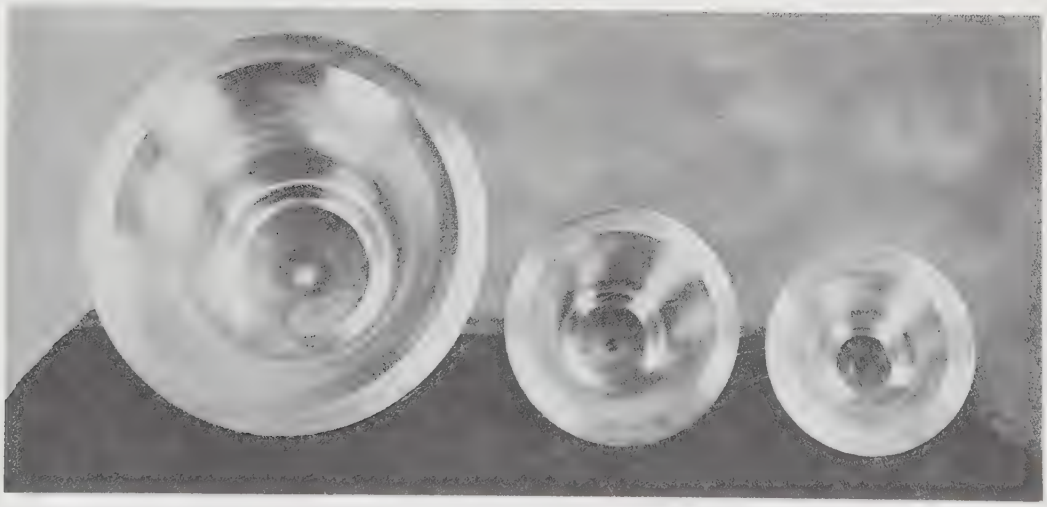

Plate 4. F_5, F_6, and C_7 Bells showing how the F_6 Bell, an Octave of the F_5, is much more than one-half its size; and how the C_7 Bell is hardly any smaller than the F_6 several tones below it.

It has often been remarked here that the up-scaling of the treble bells is one means of increasing their strength, necessary to achieve an intensity comparable to the bass register, but that this in itself is not enough to assure complete tonal balance. Another phenomenon must be considered: the performance of the individual bell within itself. As intensity is due not only to the total mass, but to the total amount of decibels that all partials contribute, and as clarity and exact pitch perception are due to the contributions of the Octave and especially the Prime, it may be reasoned that a form of bell possessing these partials in adequate strength is mandatory for adequate intensity, clarity, and tonal balance. Tonal balance comes not from over-all intensity alone. It is also the result of the contribution of the partials in their respective intensities and how all this together affects the ear.

To design a series of trebles with these desirable attributes -- intensity, clarity, exact (or at least adequate) pitch definition -- was the purpose of the extensive experimentation undertaken. The curves of the bell, the relationship of interior to exterior profile, and especially the tuning, also had to be studied before a result at all satisfactory was realized.

The result of this experimentation follows.

THE PROFILE

Once the over-all proportions have been fixed, the actual curves given to the bell can vary greatly. The bell may be somewhat "pot-formed" with straight, diverging lines extending downward from the shoulder. It may be "cylindrical" with lines dropping almost vertically from the shoulders to the waist. It may be deeply "vased" with its waist much concaved above the sound-bow. Or it may be "full-waisted" with only a shallow curve between shoulders and sound-bow. Its sound-bow may be large or small, with an almost vertical, a very sloped, or a curved lip. Within limits, the same bell, with the same diameters and height, can have many forms. But if it be too pot-formed, too vased, have too deep or too shallow a curve at the waist, with too small or too full a lip, it is probable that the position of some of the overtones will be disturbed.

Every change in the form changes the series of overtones. Within certain limits, many different forms of the same sized bell can possess that series of overtones required for the most musical usage of bells. It is the relationship of the inner to the outer profile which compensates for these differences in form and thus achieves this same series of overtones. But the timbre of all these bells will be different, even though the series of overtones is correct. As the walls differ in shape and

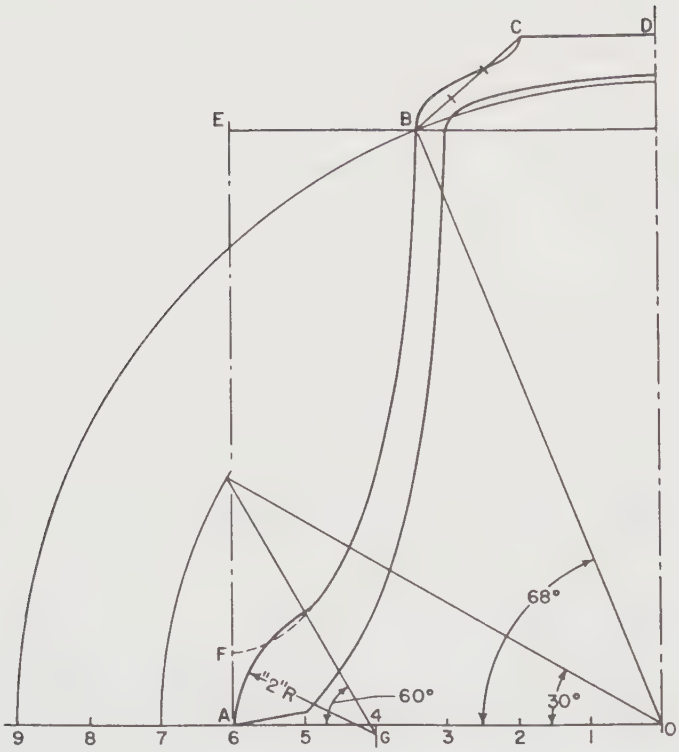

Figure 41. Establishing the Profiles of the Bell

thickness and in their relationship to the sound-bow, the strength of the partials will vary. In our experiments, we sought that shape of bell which gave a strong Hum-Tone, a much greater Prime, and as much Octave as possible, so that the bell would gain in performance.

In determining the profiles of the bells in this series, the lowest bell of the upper two octaves of the 4-octave range, a C_5, will be designed first. This will incorporate the several results obtained experimentally. As the series progresses upward, the proportions will change and hence the profiles as well. The gradual change can be followed on a chart showing the entire series of profiles, based on the layout lines determined from the lines of proportion.

Consider now the tracing of the outer profile of the C_5 bell.

1. The diameter, 16 1/8", is taken from the chart of the Lines of Proportion. On a horizontal line, with Point O (for origin) at the right end, one half this diameter, 8 1/16", is laid off, to Point A. The resulting line AO is divided into six parts, numbered 1 through 6, with 6

at A. The line is extended and numbered to 9. This is termed the "base line". The distance between two of these numbers is the unit of measure used here.

2. From Point O a vertical centerline is drawn.

3. The outer lip of the bell, the sound-bow, is an arc of radius "2". Its center is found by swinging an arc of radius "7", from O as a center, until it meets a line from O making 30° with the horizontal. From this intersection of arc and line, a line is drawn with the horizontal at 60° until it cuts a vertical dropped from 4. This is Point G, the center of the arc describing the outer lip. The limits of the arc are the base line and the 60° line through G.

The position of Point E will vary as the bells reach into the treble. As a lip of greater proportion is called for, neither Point G nor the radius "2" will be constant.

4. The remainder of the curve is an ellipse with one-half its major axis EF on the vertical line AE, with F a distance of "1" above A. The minor axis is EB.

The line OB and the base line OA are the foundation lines upon which the entire series is based. The point B is very important, as it determines the <u>slant height</u> of the bell (here always termed Height) and the Small Diameter. Point B indicates that the Height is .74 of the Great Diameter, and that the Small Diameter is .56 of the Great Diameter.

5. From B, draw a horizontal line to the centerline. The vertical height of the bell is found by adding, above the horizontal through B, twice the distance between this horizontal and the point where radius 9 intersects the centerline. The full vertical height is thus OD.

6. The distance CD is not quite "2". Divide BC into 3 parts and draw the counter-curve as shown with the lower curve tangent to the profile line.

If all the bells of this upper series were of the same proportion, lines parallel to AB could be drawn through the half-diameters laid off on the bass line AO, and their intersection with line OB would determine the location of the shoulder of each bell. This is the system used to draw the bells in that part of the series which follows the AMP line, giving the same proportion and profile to every bell.

In the upper series, from around C_5, where the scale is increased, line OB is also of utmost importance. Rather than taking it as a straight line to O, a correction can be made from the charts of proportions, and the line used in establishing the profiles of the trebles. Lines from A to B will not be quite parallel in this register, as the Heights decrease and the Small Diameters increase.

The inner profile of the bell is less complicated to establish than the outer. It is, in general, the same curve as the outer profile but is carried down to the base line AB without a counter-curve. Its nature is elliptical.

The Thickness, that at the sound-bow, is determined from the proportion chart, with enough addendum to assure that the pitch is high rather than low. Founders do not expect to "hit the note" in the higher bells, although in the larger ones some have developed their skill to such a degree that only a small amount of machining is necessary to tune to the closest tolerances.

It will be noted that as the bells reach higher frequencies, the Thickness increases rapidly in proportion to the other measurements. From A, a line of slope of 10° is drawn until it intersects the inner profile line. About "1" in length, this line determines the inner lip. Above B, the inner profile is taken roughly parallel to the arc from 9 until it intersects the centerline. As the bell vibrates very little above B, this is of less importance.

The way the bell is tuned is of the greatest importance. One partial may be treated either in one part of the bell or in another. It may be influenced to a greater degree in machining one part of the bell rather than another. It must be treated where it will contribute most to the result. This requires special skill, for, while treating one partial, others are affected. Thus the bell is treated according to the desired strengths of the partials tuned. A bell may have its overtones tuned down to their correct pitches, but where and how they are tuned make a considerable difference.

We have noted several times that the bell must be cast thicker than necessary to allow for lowering the partials to pitch. However, if some of the partials come out too close to tone, and tuning would thus bring them below their pitch, the bell might still be made to serve -- but as a note a half-tone lower. This may be done if the inner profile is still thick enough to allow this operation. But lowering the pitch of this bell will relegate it to another series; even if it could be given the correct partials and a pleasing tone would result, its strength would not be the same as what it would have been had it remained closer to its original weight and sounded the higher note.

Between the two profiles is what is termed the "neutral line". This line, half-way between the other two at all times, should present a constant curve with no fractures, and of course no counter-curve. If this neutral line were to display any marked breaks, it would signify a physical handicap and show that the bell's inner proportions were at variance with an orderly production of sound. This would cause undesirable effects in the over-all tonal picture.

18D

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PART V
THE COMPLETELY BALANCED CARILLON IN ALL ITS ACOUSTICAL AND PHYSICAL ASPECTS

A. L. Bigelow

From the studies in the first part of the book, we concluded that the carillon could become an acceptable musical instrument only if it could be made to perform in an acceptable musical manner. It must first be in tune, so that it is capable of playing melody and harmony with pleasing effect; all registers of the instrument must have equal intensity, so that no register will be particularly loud -- the bass -- and none too soft and tinkly -- the treble. If these two requirements are met, the instrument may well keep the singular tuning of its bells, with their Minor Thirds, and the peculiar blending from the protracted decay of the notes; for tuning and ring will be accepted as attributes which give the carillon its distinctive character. Indeed, it is "natural" for a carillon bell to have a ring and possess a Minor Third.

But faulty tuning and tonal imbalance are inexcusable in any musical instrument. The tuning must be pure and the notes homogeneous, all of the same family and of the same character, if a collection of bells called a carillon is to be an instrument on which music may be expressed. The bells must have the same intensity, that is, the ear must judge them to be of equal intensity; their ring, or rsonance, must not sound too differently in one part of the instrument than in another. Equal quality, equal performance are necessary, for without them it is difficult to express musical thought; the following of a passage is interrupted by imperfect technicalities, unevennesses, in the tone; the esthetic sense is disturbed by out-of-tune notes, by harshness, tinniness, inadequate tonal response from the several parts of the carillon. And because of this the mind, perhaps quite unconsciously, tends to notice the imperfections, the unmusicalness, of the instrument rather than to follow the musical thought that the performer is interpreting. It is not the fault of the listener; it is more often the fault of the carillon.

As we have said before, many times it is just this which leads

the musician, trained to expect correct performance in an instrument if he is to play it or listen to music played upon it, to reject the carillon as a folkloristic curiosity; perhaps it sometimes adds to local color, but it is, he judges, far from a developed instrument in the sense of other developed instruments.

Thus in seeking a way to overcome the most obvious drawback to musical performance -- tonal imbalance between the upper and lower registers -- we learned that upscaling the trebles would go a long way to accomplish this. But we also implied that the internal tonal structure of the individual bell must also contribute. The increased intensity due to increased decibel output, plus clarity and sonority, would help to give the treble the tonal importance of the basses.

This was indeed the greatest step in realizing the goal. But there are others, and without them perfect tonal balance could still not be realized. This chapter will attempt to show how the carillon achieves its matched series of notes, each occupying a definite place in the range of the instrument, all belonging to one and the same family, for the production of cultured musical expression.

How can the trebles, even with their increased scale and decibel output, ever match the strength of the basses? The mass of the basses is hundreds of times that of the trebles; hence their power is many times greater.

Here, an acoustical phenomenon enters the picture: a low tone does not sound as "loud" to the ear as does a high tone. In a choir of mixed voices, for instance, if absolute balance is desired, it is necessary to have more basses than sopranos. A single trumpet can make more "noise" than two or three tubas and be heard above them. So it is with bells. The presence of the higher bells, even small-scaled, can be remarked above the basses. The tone may be inadequate; there is no resonance and they are quite overridden by the larger bells. But they do make sound and one does know they are there. We know now that if this sound were stepped up by bigger trebles of greater mass to disturb more air and thus give a greater impression of loudness, tonal balance would then be much nearer.

Figure 42 shows the results of the several steps taken to make an ideal set of bells into an ideal carillon. We begin with the curve of the decibel output for the series (1). Just above this line is another (2), showing what our ears construct from the decibels when our senses react to them. The bass is softened while the treble becomes louder. Thanks to this phenomenon, many carillons whose trebles are light and

tinkly have been termed pleasing instruments when not too much tone was expected from them. But we know that their use is limited, that the treble register is of lesser value and should be only sparingly employed.

When the scale of trebles is increased, the decibel output is changed accordingly. The next line (3) in the chart compares the new decibel output of this register to the original output, and the line just above it (4) the effect contributed by the higher-the-louder phenomenon.

> NOTE: This pheomenon is limited to a few thousand cycles. Notes of frequencies higher than ten thousand cycles per second will again become "softer" as the limits of our hearing, and especially pitch perception, are approached. In working this phenomenon with respect to bells, however, this point is still some distance away.

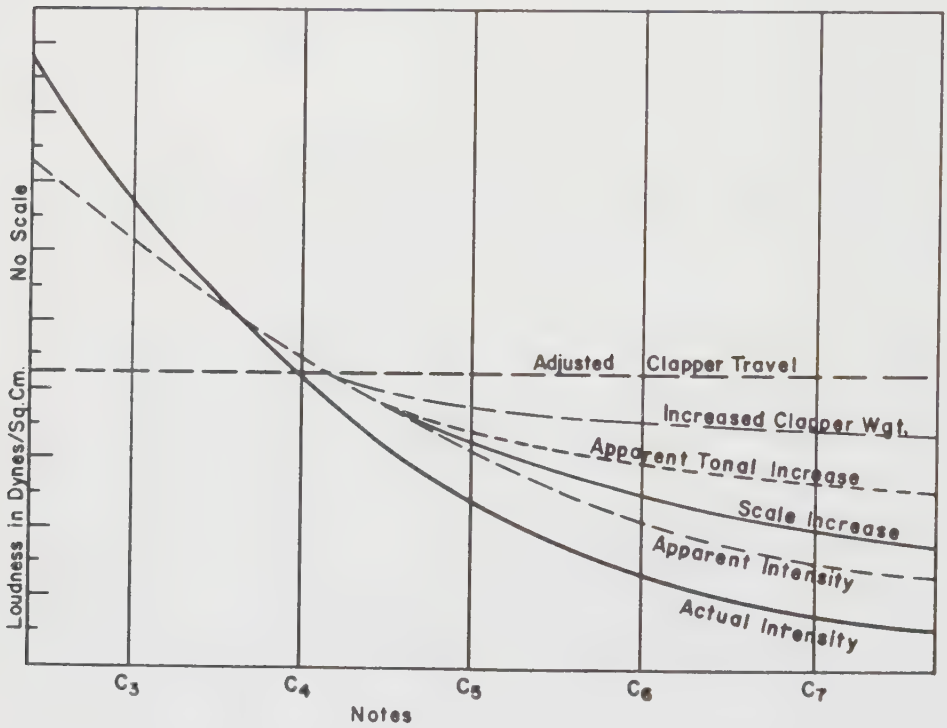

Figure 42. Achieving Tonal Balance

There is still some ground to cover before adequate "loudness" in the treble is achieved. Now that their proportions have been increased, the trebles may be struck with heavier clappers - heavier not only because

the new mass of the bell is greater, but even heavier than their proportional weight, as the upper bells are reached.

The next line (5) on the chart shows the increase in loudness realized by the heavier clappers.

We have now stepped up considerably the tonal yield of the trebles, though the impression we receive from the bass has remained the same. One more step is necessary, one which will increase still further the tone of the trebles and reduce the tone of the basses, so that the line representing these differences is flattened out to the same level. When this is accomplished, tonal balance will have been realized. This last step is effected by mechanical means: increasing the proportional travel of the clappers of the treble bells and decreasing the proportional travel of the basses.

In spite of the much greater output of the trebles after upscaling, if the basses were struck with blows proportional to those given the trebles, they would again overshadow the little ones. If it is normal to give the treble clappers a travel of about 1 3/4", at this rate the clapper of the C_3 bell would travel 15" and produce far too loud a tone. Further, when a bell is hung "dead" and struck by a clapper or a clock hammer with a travel of an inch or two, and when the same bell is pealed, where the clapper travel can be measured in feet, rather than in inches, there is no question of which is the louder.

Another fact to consider, one which makes it desirable to strike the basses with a reduced blow, is the physical strength of the performer.

The great weight of the clappers of the bass bells, as compared to the light weight of the treble clappers, precludes from the outset that a bellmaster be able to cause a clapper to travel a distance of more than a very few inches. The carillon is played through a mechanical action built so that the performer can move the heavy clappers with a certain ease, but not very far. Through the leverage of the keyboard and transmission bars, the travel of the clapper is reduced, thus reducing the weight and force required to sound the bell. This allows control, very necessary for the realization of touch. When the travel is greater, control is not possible.

It is thus quite acceptable that the blow of the basses be reduced, if only for physical reasons. For tonal balance, it is imperative. Actual response from the bell is due to the mass (weight) of the clapper times the speed with which the clapper strikes. With greater travel, more speed can be realized; and the greater the speed, the louder the tone -- not a desirable condition in the bass register.

Mass × velocity = energy (here considered as tonal output).

All bass bells must be struck with the proper weight of clapper; they cannot be reduced in weight to give the required softer tone. Clappers of lesser weight would not have enough energy to bring out the desirable partials with the needed intensity, but would, instead, awaken many undesirable upper partials inordinately, causing the bell to sound tinny. So the weight of the clapper must be retained; it is the blow that must be reduced. This is realized in one way by the limitations of the keyboard. Keys, and pedals too, usually have a travel of about 2". The clappers can be made to travel this distance by direct bell-crank connection, or, lengthening or shortening the arms of the crank will increase or decrease the distance. In the medium and especially the lower registers, it is quite necessary to decrease the travel. If the same travel is maintained, the touch suffers, for the effort expended to play the bells will mount greatly, note by note, as the bass register is descended. There will be little control and therefore little touch. Controlling the excessive tone of the basses, on the one hand, and the touch, on the other, fortunately go hand in hand.

Throughout their history, bells have most often been swung, and the clapper is therefore suspended from a pivot in the head (Figure 43). This assures that the clapper will hit the inside of the sound-bow perpendicularly, tending to bounce away from the bell immediately upon striking, by the elasticity of the metal. This gives the best tonal result. A clapper weighing 140 pounds, suspended from a pivot in the head of a C_3 bell and whose ball is 2" from the bell, will exert a force of 47 pounds on the pull wire. Thus a force somewhat greater than 47 pounds will be needed to overcome the inertia and cause the clapper to move to the bell. Just how much more force is given the clapper depends upon the amount of tone desired. If the ball's travel is started from a distance of 1" from the bell, the necessary force will be over 49.6 pounds, which is the force exerted on the pull wire by the clapper at rest in that position. Comparing this to the little force necessary to move the 2 1/2 pound clapper of the C_7 treble, we readily understand that a means must be sought to lessen this great difference, to render more nearly equal the physical effort required to play the instrument in the bass and treble.

It is, however, neither desirable nor necessary to make the touch equal throughout. The basses are capable of more tone than the trebles and it is quite natural to expect a greater exertion to get it. The bells are bigger and the clappers heavier, so psychologically it is not surprising to have to use more energy to sound a deep note than a high one. As the bass is controlled by the feet, capable of greater force than the arms, there will be no particular hardship if the clappers offer more

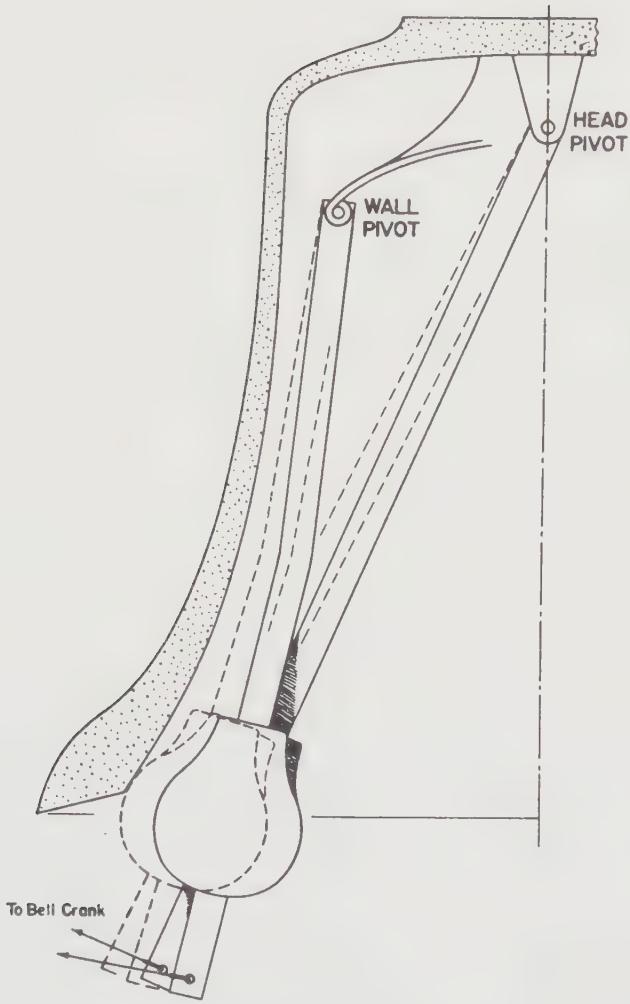

Figure 43. The Clapper Action

resistance. Lesser resistance aids faster playing but it is not often necessary to play fast in the bass. The longer decay period of the larger bells limits the number that can be sounded in a short space of time, for fear of "campanological mush" -- so much deep tone in the belfry that understanding of the music is impaired. One does not use as many basses as mediums and trebles.

Also, a certain amount of resistance restrains the performer from letting his pedal work "run away with him"; too light a touch is uncomfortable. When there is no resistance in the pedal, an effort of restraint is necessary. And as touch is due to the resistance of the clapper as well as to the ease by which it is moved to the bell, without resistance there

can be no adequate control. The clapper would take the force given it by the performer and, once the inertia from its position at rest is overcome, it would move to the bell without discipline, the player being unable to exercise but little further control upon it.

The proper resistance also returns the clapper to its initial position, after the note is struck. With the correct amount of reduction in the force to move it, and with an adequate amount of resistance remaining, a properly balanced key will "follow" the performer. When he moves his hand up or down, the key will remain in contact with it at all times, even when there is fairly rapid repetition. It can only leave the contact of the hand when it is "flipped" down by a short, swift blow, and the momentum given the clapper is sufficient to carry it to the bell. This also holds true for the pedal.

Even so, the difference in force required at the two ends of the keyboard is too great, and must be reduced. This will be done by lightening the action in the bass and by heavying the action in the treble.

The bass is our greatest concern. We cannot reduce the force required by reducing the weight of the clapper: as explained, a bell must have a clapper in proportion to its weight if it is to sound properly. We must reduce the pull and not the clapper. A solution has been sought by moving the point of suspension closer to the wall of the bell and shortening the stem a bit. Even if it helps physically, this does not solve the problem acoustically, because the bell no longer receives a direct blow from the clapper. When it does not, the clapper tends to scrape, in the measure of the deviation of the stroke from the perpendicular (Figure 44). Still, this method of clapper suspension reduces measurably the force required to sound the bell. At 2" from the bell, the 140 pound C_3 clapper exerts a pull of 19 pounds on the control wire, or only 40% of the pull on the wire when the clapper is suspended from the head.

When a clapper starts its travel from a point near the bell, more force is required than when it starts its travel from a point farther away -- but in this last instance the performer must apply force longer. If the pedal -- or key -- is now slowly depressed, the resistance increases as the clapper nears the striking point. While a clapper suspended from a point just under the head takes more initial force to move it than one suspended from a point nearer to the wall, this force increases only slightly as the clapper nears the bell. To the contrary, while a clapper pivoted near the wall takes less initial force to move it, the force increases at a greater rate.

But it is not enough to consider the pull of the clapper on the control wire when the clapper is at rest or moving slowly to the bell.

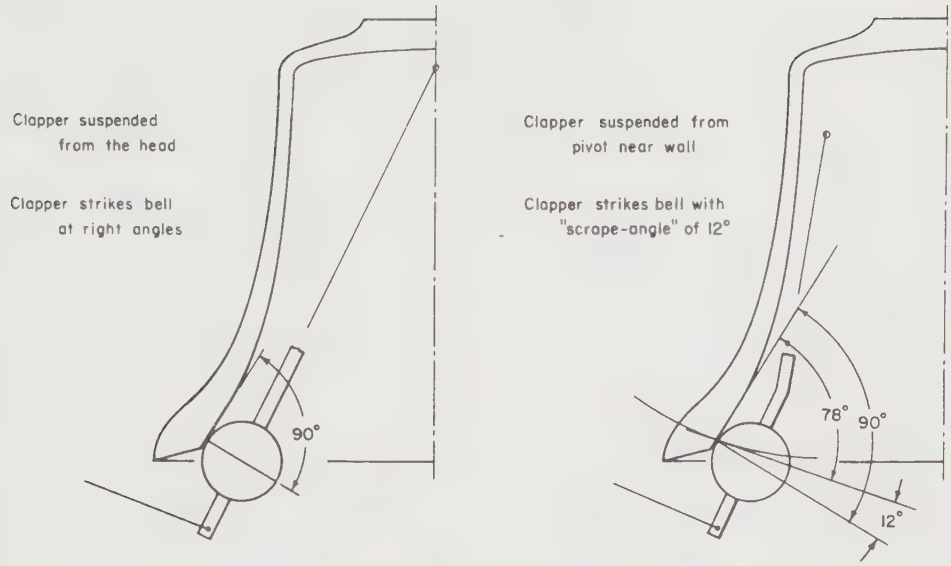

Figure 44

Clappers have no meaning unless they are in motion, and in relatively fast motion. When the performer strikes the key, he expects to get the result at once. When the key is struck, the clapper is given momentum and once the clapper has been started on its course by the initial blow this momentum will carry the clapper to the bell. Thus the increase in resistance is evened out.

The system of suspending a clapper from a point near the wall has much to offer but its one notable deficiency is the tone itself. Because the bell is not struck perpendicularly, the blow tends to produce friction, scraping, of clapper upon bell, robbing the bell of some of its tone.

Figure 44 shows the clapper hanging from a pivotal point quite close to the wall. Draw a tangent to the bell at the point of contact of the clapper. From the pivotal point as center, and with a radius the length of the distance from the pivot to the striking point, swing an arc through the striking point. At the striking point, establish a tangent to the arc. The two tangents, one to the bell and the other to the arc, will determine the "scrape angle". It measures 12°, which means that the path of the clapper, at the point of contact, deviates from the perpendicular by 12°.

Another way to reduce the force required to strike the bell is to shorten the arm of the clapper. As the direction of travel will be

retained, the bell will always be struck with a perpendicular blow. The locus of the points of suspension for clappers of any length is a line from the striking point to its point of suspension in the head. A clapper suspended from any point on this line, if its ball hits the sound-bow at the proper place, will strike the bell correctly. Figure 45 shows the results of shortening the clapper in this fashion. This system does not cause too great a reduction of the force needed to start the clapper on its travel, hence it is by itself no solution. Yet it does assure in every instance correct tonal production.

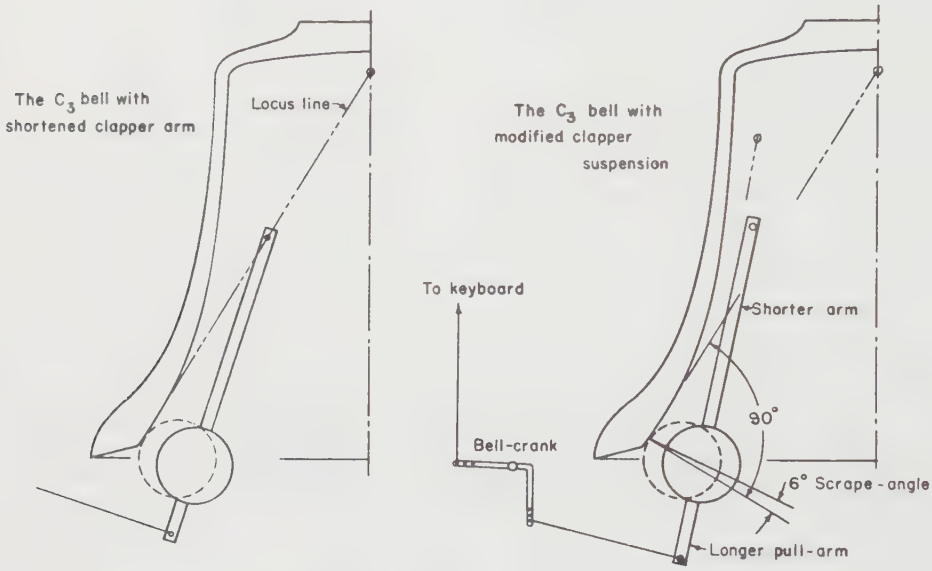

Figure 45

It will therefore be imperative to return to the method of hanging the clapper from a point nearer the wall of the bell. The question is, how far may this be done without impairing the tone? It is evident that there is going to be some friction, if the clapper does not strike perpendicularly.

The amount of scrape decreases fast with the reduction of the angle. If the scrape angle is now reduced by half, the scrape will be measurably reduced and the tone of the bell will gain. Of course, the force required to move the clapper will be increased, for the pivotal point is moved farther from the wall.

To reduce as much as possible the increase in the pull effected by moving the pivotal point farther from the wall, the length of the

clapper can be reduced. If the clapper is shortened by 20", the pull is brought down to 25 1/2 pounds when the clapper is at 2" from the sound-bow, comparing favorably with the 19-pound pull when the clapper is pivoted close to the wall. And the tonal result has gained. The pull can be reduced further by lengthening the clapper below the ball, thus increasing the leverage (Figure 45).

We have made our comparisons with the clappers located 2" from the striking point. Bass bells ordinarily require a travel of 1", or even less. Clappers starting their travel from a point closer to the bell require, of course, a greater force to move them. By adjusting the bell crank, shortening the arm of the pull crank, or lengthening the arm of the horizontal crank, both the force and the travel distance will be reduced.

Some manufacturers of carillon action have sought to lessen the force required to play the basses by adding counterweights to the bell-cranks. Although this practice does help when pressure is applied slowly, when speed is necessary for a loud result the counterweight is a hindrance. The force necessary to overcome the inertia of both clapper and counterweight is far greater than that needed to move the clapper alone. And when the clapper-counterweight system is in motion and the clapper strikes the bell, the counterweight tends to continue its travel, effectively keeping the clapper on the bell until the counterweight's travel has been expended. Thus the bell is hindered from vibrating freely, normally, once the clapper has hit, since the clapper is held against it for a time. This greatly impairs the tone. In many cases where counterweights were originally used, their removal brought improvement to action and tone.

Consider now the mediums and trebles. When the clappers become too light to return to their initial positions, the force of gravity on the returning clapper unable to pull up the weight of the wire and key, springs are installed behind the clappers. Nor is it enough that the key be returned to its up-position on the keyboard: it must also return with the speed necessary for a repetition of the stroke. Springs are in some measure necessary in almost the entire upper two-thirds of the range. The springs are adjustable, their tension being lessened or increased to compensate for the weight of the clappers.

Once the instrument is balanced, with an even tone from bass to treble, and with an even action (though slightly heavier in the bass than in the treble), just how the bellmaster controls it is purely a matter of touch, comparable in its way to touch on keyboards of other percussive instruments.

There are a multitude of ways by which a key can be struck: the

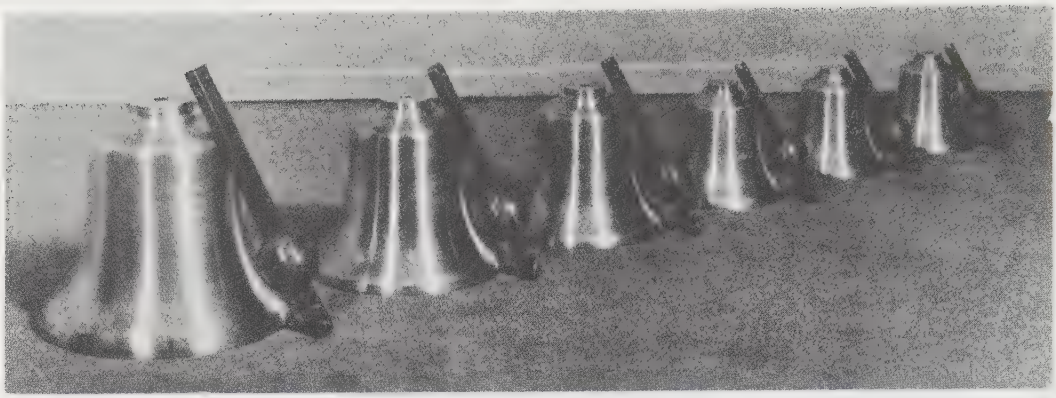

Plate 5. A Study of the Clappers and the Bells They Ring. The Weights are Proportionally Increased as the Series Reaches its End.

"flip", the "carry-through", and so on, all necessary to musical expression. The accomplished bellmaster knows and uses all of them. The instrument must be able, mechanically as well as acoustically, to follow the moods of the musician at its keyboard. The instrument must first be a fitting and proper medium, and what the bellmaster does with it after that depends on an experience and understanding that long association with bells has given him, an understanding of its possibilities and its limitations, too. The unique character of its tones, the ring -- different in the treble from the bass - the great number of pleasing tonal combinations, the typically "carillonistic effects" possible only on this instrument because of the character of its tones -- all these must become second nature to the bellmaster, so that he may forget himself and his medium, as every real musician does, and live only with the thought he wishes to express and the beauty he sends out from his tower.

When the carillon has achieved this state, it will have become a fully developed musical instrument. Perhaps some of the considerations explained in this book will help to make it so.

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GROUP THEORY AND THE BELL

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It is shown that the modes of vibration of the bell can be classified into families according to their behaviour under the symmetry operations of the undistorted bell. These families are characterized by the number of nodal meridians rather than the number of nodal circles used in the usual *ad hoc* classification scheme. The phenomenon of bell warble is shown to be a direct consequence of the approximate nature of the symmetries of real bells and a new practical method for removing warble is suggested. It is argued that certain misshapen medieval bells were eccentrically cast not because of poor technique but deliberately in order to reduce warble.

1. INTRODUCTION

The acoustically significant modes of vibration of bells have been investigated by a number of workers. Most of the references can be traced from the paper of Grützmacher *et al.* [1] and the dissertation of van Heuven [2]. The nodal patterns for the radial motion of the bell, which is the acoustically significant part, always consist of n circles with $n \geqslant 0$ and $2m$ meridians with $m \geqslant 0$. The circles lie in planes parallel to and at various distances from the base while the meridians lie in planes through the axis of symmetry at equally spaced azimuths. A mode is usually specified by quoting the values of n and $2m$ although there is some ambiguity in this procedure. Grützmacher *et al.* [1] regard the modes with a fixed number of circles and varying numbers of meridians as forming a family but this is a completely *ad hoc* scheme. In section 6 it will be shown that a physically meaningful scheme emerges by classifying the modes according to their behaviour under the symmetry operations of the bell and that this is determined by the value of m.

The phenomenon of bell "warble" is well known to campanologists and is generally considered to be undesirable. Bell founders go to considerable lengths to reduce this effect which arises because most of the modes occur in nearly degenerate pairs or "doublets" [1], the nodal meridians of one coinciding with the antinodes of the other. The components of each of these doublets beat together producing the warble. Much of the art of bell founding is contained in the removal of these beats. The process is difficult because the act of reducing the beating of one doublet, which is achieved by cutting the bell in a suitable manner, may increase the beating of others. A detailed knowledge of the nodal patterns of all the important modes is required and their precise locations have to be found empirically for each new bell.

No realistic attempt at solving the problem of the bell theoretically has ever been made and the reasons underlying the occurrence of many of the modes in nearly degenerate pairs have not hitherto been fully understood. The difficulties involved in setting up the eigenvalue equation for the system in detail are severe but it is shown in sections 5 and 6 that these details are not required in order to understand the occurrence of the doublets. They are a direct consequence of the approximate symmetries of real bells. The only knowledge of the eigen-

value equation which is required in order to prove this is that it exists. Having established the origin of the doublet structure, a simple practical method for eliminating the warble is suggested in section 7.

2. THE SYMMETRY GROUP OF THE BELL

By a symmetry of a mechanical system is meant some operation, which need not necessarily be mechanically feasible, which leaves it in a state indistinguishable from the original. It is easy to show in any given case that the collection of all symmetry operations of the system forms a mathematical group. This is the "symmetry group" of the system. It is necessary to establish the nature of this group for the bell for later use.

If a church bell is stood on a horizontal plane and a vertical cross-section is taken through the apex, or crown, then the result is of the type shown in Figure 1. The entire bell shape can be generated by taking, say, the right-hand half of the cross-section ABC and rotating it about the vertical line AD through the apex A. In other words the bell has, geometrically at least, axial symmetry about AD. For a carillon bell the shape of ABC may be different but the same axial symmetry is present.

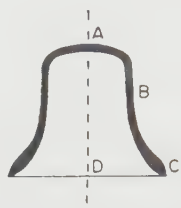

Figure 1. Vertical cross-section through the crown of a typical church bell.

If one assumes the density and elastic properties of the bell metal to be uniform then the symmetry operations depend upon geometry alone. Clearly a rotation through any angle, ϕ, about the axis AD is a symmetry operation. Let this be called $C(\phi)$. Its inverse is $C(-\phi)$ and the case of $\phi = 0$ is the identity operation, E. Another symmetry operation, usually denoted by σ_v, is to take the mirror image of the system in any plane containing the axis AD. There are infinitely many of these σ_v operations. The infinite group composed of E, $C(\phi)$ and σ_v is well known in molecular vibration theory and is denoted as $C_{\infty v}$ in the Schoenflies system. The character table is well known [3, see appendix L] and is reproduced in Table 1. There are infinitely many classes and irreducible representations. The number of members of each class is denoted in the table by the number preceding the typical element. The notation for the irreducible representations is borrowed directly from molecular vibration theory. A well-known theorem of group theory [3, see section 14] states that the dimensionality of a representation is equal to the character of the identity element in that representation. Hence the $\Sigma^{\pm}$ are one dimensional and all other cases are two dimensional.

TABLE 1

Character table for the group $C_{\infty v}$

	Irreducible representations						
Class	Σ^+	Σ^-	Π	Δ	Φ	Γ	...
E	1	1	2	2	2	2	...
$2C(\phi)$	1	1	$2\cos\phi$	$2\cos 2\phi$	$2\cos 3\phi$	$2\cos 4\phi$	...
$\infty\sigma_v$	1	−1	0	0	0	0	...

3. SYMMETRY OPERATIONS ON THE DISTORTED BELL

If an arbitrary displacement of the bell from its equilibrium configuration is made then it can be described by means of generalized displacements $\eta_1, \eta_2, \ldots$, some of which might be, for example, Fourier expansion coefficients. The convention of collecting these into a column matrix $\boldsymbol{\eta}$ is adopted in the present work. They can be regarded as the components of a state vector, $\vec{\eta}$, relative to some set of linearly independent basis vectors, $\hat{e}_1, \hat{e}_2, \ldots$, spanning a linear vector space. These can be collected into a row matrix, $\mathbf{e}$, so that $\vec{\eta} = \mathbf{e}\boldsymbol{\eta}$. If the deformed bell is subjected to a symmetry operation, S, of the undeformed bell then a new configuration is obtained. This new configuration could equally well have been achieved by simply applying a new set of displacements, $\eta_1', \eta_2', \ldots$, to the original system. This new set is related to the old one by a linear transformation, $\boldsymbol{\eta}' = \mathbf{S}\boldsymbol{\eta}$, where the matrix $\mathbf{S}$ depends upon the precise nature of the operation S [4, see section 2.6]. In other words the symmetry operation, S, corresponds to a rotation in the vector space. Thus each of the symmetry operations of the bell can be regarded as an operation on the displacements alone and this is the view adopted in the remainder of the present paper. It is straightforward to show that the matrices $\mathbf{S}$ form a representation of the group of symmetry operations of the bell [4, see section 2.7].

4. THE EIGENVALUE EQUATION

The authors have made a survey of the scientific literature on bells covering some 50 papers published over the last 80 years. In none of these papers has any evidence of non-linear vibrations been reported. It is therefore reasonable to assume that all the modes of vibrations of church and carillon bells behave as though the systems were linear, free of applied forces and subject to some "equivalent viscous" damping. Rigid body degrees of freedom are no problem because in practice at least one point of the bell is held fixed. The only applied force comes from the clapper which can be regarded as supplying a set of initial conditions on the generalized displacements and velocities, the subsequent motion being force free. In practice the displacements are never large so it is not surprising that non-linear effects are unimportant. The nature of the damping is complicated [1]. It consists of structural, viscous and acoustic components, the relative importance of each of these varying from mode to mode.

The free vibrations of the bell will be determined by some eigenvalue equation of the form

$$\mathbf{W}\mathbf{y}_n = (1/p_n)\,\mathbf{y}_n,$$

where $\mathbf{W}$ is the usual dynamical matrix [5]. p_n is a complex eigenvalue whose imaginary part is equal to the angular frequency of the nth mode and whose real part is the damping factor of this mode. $\mathbf{y}$ is related to the generalized displacements, η_j, by

$$\mathbf{y} = \begin{pmatrix} \dot{\boldsymbol{\eta}} \\ \boldsymbol{\eta} \end{pmatrix}.$$

5. GENERAL CONSIDERATIONS OF SYMMETRY AND DEGENERACY

The dynamical matrix, $\mathbf{W}$, is a characteristic of the system and is therefore independent of the generalized displacements. It was shown in section 3 that a symmetry operation can be regarded as a transformation of the displacements only, so $\mathbf{W}$ must be invariant under the

operations of the symmetry group of the system. Under a symmetry operation, R, the matrix $\boldsymbol{\eta}$ is transformed into $\boldsymbol{\eta}' = \mathbf{R}\boldsymbol{\eta}$ so that

$$\mathbf{y} \to \mathbf{y}' = \mathbf{r}\mathbf{y}, \quad \text{where} \quad \mathbf{r} = \begin{pmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{pmatrix}.$$

Thus the eigenvalue equation transforms into

$$\mathbf{W}(\mathbf{r}\mathbf{y}_n) = (1/p_n)(\mathbf{r}\mathbf{y}_n).$$

In other words if $\mathbf{y}_n$ is an eigenvector of eigenvalue $1/p_n$ then any symmetry operation, R, of the system transforms it into another eigenvector, $\mathbf{r}\mathbf{y}_n$, with the same eigenvalue. Consequently $\mathbf{r}\mathbf{y}_n$ must be expressible as a linear combination of only those linearly independent eigenvectors, $\mathbf{y}_1, \ldots, \mathbf{y}_g$, say, which have the same eigenvalue, $1/p_n$. This is true for any symmetry operation of the system and any eigenvector in the degenerate set. Hence all the eigenvectors belonging to a given eigenvalue must themselves form a vector space which is invariant under the symmetry group of the system. The linearly independent eigenvectors, $\mathbf{y}_1, \ldots, \mathbf{y}_g$, therefore transform according to a g-dimensional representation of the symmetry group. The vector space spanned by $\mathbf{y}_1, \ldots, \mathbf{y}_g$ is either irreducible or can be reduced into subspaces each of which transforms according to an irreducible representation of the symmetry group. It thus never happens that eigenvectors belonging to the same irreducible vector space belong to different eigenvalues.

The question must now be posed as to whether eigenvectors transforming according to different irreducible representations necessarily have different eigenvalues. The general answer is clearly that they do not, but that any degeneracy which does occur between different irreducible representations can have nothing to do with the symmetries of the system. This is true provided that one has indeed included all the symmetries of the system in the symmetry group employed and has not in fact used some subgroup of the true symmetry group. Two sets of eigenvectors which are not mixed under the symmetry operations of the system but which do happen, due to dynamical peculiarities, to have the same eigenvalue are known in molecular vibration theory as "accidentally degenerate" and this terminology will be taken over into the present work. Such an occurrence is rare in molecular vibrations [4, see chapter 6] and there is no reason to suppose that it will be any more common with continuous systems. The results of this section can be summarized in the following theorem.

Theorem. Provided there are no accidental degeneracies each set of degenerate eigenvectors of the dynamical matrix spans a vector space which transforms according to an irreducible representation of the group of symmetry operations of the system.

It follows from this theorem that the eigenvectors, and hence the modes of vibration, can be classified according to their behaviour under the symmetry operations of the undistorted system. It also follows, if the theorem quoted at the end of section 2 is borne in mind, that the degeneracy structure of the vibration spectrum can be obtained by inspection of the character table of the symmetry group.

6. THE DEGENERACY AND CLASSIFICATION OF BELL MODES

If one assumes that there are no accidental degeneracies then the possible multiplicities in the bell's vibration spectrum are equal to the dimensions of the irreducible representations of $C_{\infty v}$. Examination of the characters of the identity operation as listed in Table 1 thus shows that any modes transforming like $\Sigma^{\pm}$ are singlets while all others are doublets. The operations

contained in $C_{\infty v}$ are such that their effects upon the distorted bell can be ascertained by looking at a cross-section perpendicular to the axis of symmetry. Hence the classification of the modes in terms of their behaviour under the elements of $C_{\infty v}$ will depend upon the pattern of nodal meridians. Different numbers of nodal circles will be possessed by different members of the same symmetry family.

The singlet modes are easily identified since, as can be seen from the character table, they are either symmetric or antisymmetric under the elements of $C_{\infty v}$. Since this group includes rotations through any angle ϕ, $0 \leqslant \phi < 2\pi$, about the symmetry axis, the only possibilities are modes with no nodal meridians: i.e., with $m = 0$. It follows that all modes with $m \geqslant 1$ are doublets, and this is in agreement with experiment. The singlet modes should divide into two families, Σ^+ and Σ^-, depending on whether they are symmetric or antisymmetric under σ_v. The only known bell modes with no nodal meridians are the "breathing" modes where every cross-section of the bell normal to the symmetry axis remains circular and centred on the symmetry axis but with periodically varying radius. These modes are all symmetric under σ_v and so belong to a Σ^+ family. The behaviour of a typical cross-section is shown schematically in Figure 2 together with the first few of the other symmetry families. Although none of the well-established bell modes are of the Σ^- type these certainly exist in the form of torsional vibrations of the bell about the symmetry axis. Such modes have never been reported experimentally but this is certainly because, being of no acoustical significance, they have never been searched for. A search for modes of this type is currently being undertaken by the authors.

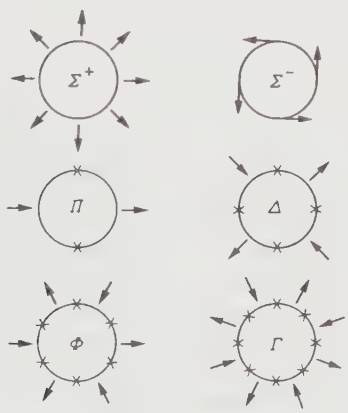

Figure 2. Schematic illustrations of the motions in a horizontal plane of bell modes of the first few symmetry types.

It has been established that all modes with $m \geqslant 1$ are doublets but it remains to complete their classification in terms of the irreducible representations of $C_{\infty v}$ according to which they transform. This can be done by considering their behaviour under the operation $C(\phi)$ for any ϕ in the range $0 \leqslant \phi < 2\pi$. The matrices corresponding to $C(\phi)$ for the two-dimensional irreducible representations of $C_{\infty v}$ can be written in the form

$$\begin{pmatrix} \cos m\phi & -\sin m\phi \\ \sin m\phi & \cos m\phi \end{pmatrix}$$

where $m = 1, 2, \ldots$ correspond to Π, Δ, ... The degenerate pairs of modes which are transformed under $C(\phi)$ in the manner described by these matrices have $2m$ nodal meridians, each nodal meridian of one member of the pair coinciding with an antinodal meridian of

the other. Hence modes with 2, 4, 6, 8, . . . nodal meridians belong to the Π, Δ, Φ, Γ, . . . symmetry families. These are shown schematically in Figure 2 where the positions of nodal meridians are marked with crosses. All the known bell modes can be classified into families in this way and those which are most important acoustically are listed in Table 2 with their chief properties. It should be noted that none of these modes belong to the $\Sigma^{\pm}$ families and so they are all doublets. It is interesting that this symmetry-based classification scheme which has been introduced here divides the modes into families with common numbers of nodal meridians. This is precisely the opposite of the *ad hoc* scheme of Grützmacher *et al.* There is some confusion in the literature as to how the number of nodal circles should be counted [1, 2, 6]. This difficulty does not affect the arguments used in the present work but for convenience the system of Grützmacher *et al.* is adopted [1].

TABLE 2
Classification of the most important bell modes

Frequency ratio [1]	Name	Number of nodal circles, n	Number of nodal meridians, $2m$	Irreducible representation
1	Hum	0	4	Δ
2	Fundamental	1	4	Δ
2·4	Minor 3rd	1	6	Φ
3	Quint	0	6	Φ
4	Nominal	1	8	Γ

At the beginning of this section the assumption was made that no accidental degeneracies would be present in the bell's vibration spectrum. It followed that the modes would be either singlets or doublets. If the assumption were false then some of the modes would be mutually degenerate resulting in multiplets of order greater than two and causing a partial break down in our classification scheme. There is no experimental evidence for such higher order multiplets ever arising in bells. This justifies the assumption for practical purposes but does not mean that accidental degeneracies can never occur in bells: it simply means that they are rare events.

7. THE ORIGIN AND REMOVAL OF WARBLE

It was shown in the previous section that two properties of the acoustically important modes of vibration of ideal bells arise as a direct consequence of the symmetry group being $C_{\infty v}$. First, they occur in degenerate doublets. Second, the nodal meridians of one member of each doublet coincide with the antinodal meridians of the other. Because all azimuths are dynamically equivalent for an ideal bell the locations of the nodal meridians are indeterminate and depend upon the initial conditions supplied by the clapper.

Suppose a perturbation which has some but not all of the symmetries in $C_{\infty v}$ is applied to the ideal bell. The theorem of section 5 holds for the composite system whose symmetry group must be some subgroup of $C_{\infty v}$. In general it will possess fewer degeneracies than the unperturbed system so that some, but not necessarily all, of the doublets will be split into pairs of non-degenerate singlets. In the extreme case the perturbation contains none of the original symmetries, apart from the identity operation, and the composite symmetry group is the trivial C_1 whose only irreducible representation is the one-dimensional unit matrix. Consequently all the doublets are then split.

If $C_{\infty v}$ is to be the symmetry group of a real bell then it will have to be perfectly cast and have complete uniformity of physical properties of the bell metal. In practice there will always be deviations from this ideal and the nature of these deviations will vary from one bell to another. In addition, the bell founder often deliberately introduces small breakings of the symmetries by casting the names of the donors of the bell into the structure. The real bell can be regarded as an ideal bell subject to a set of small perturbations. It is highly unlikely that the net perturbation will have any symmetries and so the doublets will nearly always be split into pairs of singlets. The indeterminacy of the nodal meridians will be removed because not all azimuths are now dynamically equivalent. The locations of these meridians now depend upon the details of the perturbation rather than upon the operation of the clapper and they will have to be located empirically if required. For bells cast by the usual careful techniques these "intrinsic" perturbations should always be small. The frequency difference between the members of a split doublet will therefore be small and so they will produce an audible beating or, in bell jargon, warble.

The usual technique for reducing warble depends upon locating a nodal–antinodal meridian of the offending pair and grinding off some metal along it. This influences one member of the pair much more than the other and so changes their frequency difference. The basic idea is to try to compensate for the intrinsic perturbations by introducing a further but controlled perturbation. The practical problems of performing such an operation are obviously considerable.

It is the authors' view that a much simpler and probably superior approach to the problem of removing warble is as follows. Introduce a single and azimuthly highly localized perturbation which is large enough to swamp the effects of the small intrinsic perturbations but not so large as to distort the bell modes significantly. Every doublet will now have a nodal–antinodal meridian very close to this location. If the clapper is hung so that it strikes the bell along this common meridian then nearly all the energy will go into those members of the split doublets with their antinodes located there. Thus only one member of each doublet will be excited and so no warble can be produced. The most obvious way in which such a perturbation can be achieved is to cast a rib into the bell which runs vertically up its whole length on one side only. It could be on the inside or outside of the shell and the clapper would be arranged to strike the bell at this rib's location.

Figure 3. Horizontal cross-section through an eccentrically cast bell.

A more sophisticated version of the authors' solution of the warble problem is of some interest. This is for historical reasons rather than because it is likely to be any more effective. The variation is to make the applied perturbation not highly localized but still of such a form that it will lead to every split doublet having a nodal–antinodal meridian at the same easily predicted location. One way of doing this is to displace the inside of the bell with respect to the outside so that their axes of symmetry are still parallel but no longer coincide. A horizontal cross-section through such a bell is shown schematically in Figure 3. The resulting system has a plane of symmetry which is defined by the axes of symmetry of the inside and outside surfaces. This plane cuts the inside surface of the cross-section shown in Figure 3 in the points A and B. The symmetry group of the system is C_{1v} which has only one-

dimensional irreducible representations so that all the doublets are split. Every split doublet has a nodal antinodal meridian at A and another at B. Thus there will be no warble provided the clapper strikes along one of these easily predictable meridians. It is interesting that this displacement technique was employed in some surviving medieval bells whose sound is indeed largely free of warble [7]. It has usually been supposed that this lack of warble was fortuitous and that the bells were eccentrically cast because of primitive technique. It does not therefore seem unreasonable to claim that the authors have rediscovered a lost craft secret by means of group theory!

8. CONCLUSIONS

It has been demonstrated that, although the details of the eigenvalue equation are not known, some insight into bell vibrations can be obtained by applying the techniques of group theory to the problem. The information obtained is all of a qualitative nature but nevertheless leads to two definite advances in the subject. First, it gives a physically meaningful scheme for classifying the vibration modes. Second, it leads to an understanding of the root cause of the warble phenomenon and so to various ideas for eliminating the effect in practice.

ACKNOWLEDGMENTS

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Part VI

CHIMES AND TUBULAR BELLS

Editor's Comments on Papers 20 and 21

20 ROSSING
Excerpt from *Acoustics of Percussion Instruments, Part I*

21 HUEBER
Simulation of Bell Tones with the Help of Pipe-Bells and Piano Sounds

Chimes or tubular bells have a bell-like quality that is widely used in compositions for band and orchestra. The bending modes of a tube have frequencies resembling those of a tuned bell, and these can be brought into closer proximity by selecting a tube of optimum diameter, attaching a mass to one or both ends, or tapering the tube.

Paper 20, excerpted from the editor's paper on the acoustics of percussion instruments, is a brief discussion of chime acoustics. More extensive studies of the modes of vibration of pipes are described in Paper 21. Included in this study are the modes with axial nodes that occur in pipes of large diameter.

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ACOUSTICS OF PERCUSSION INSTRUMENTS, PART I

Thomas D. Rossing

[*Editor's Note:* In the original, material precedes and follows this excerpt.]

Pitch of complex tones

Before discussing the acoustics of chimes, I want to call attention to an important subject in psychoacoustics: the determination of the pitch of complex sounds. When the ear is presented with a tone composed of harmonically related partials, it is easy to predict what pitch will be heard. It is simply the lowest common denominator of these frequencies, which is the fundamental. The ear identifies the pitch of the fundamental even if the fundamental is very weak or missing altogether. For example, if presented with a tone having partials with frequencies of 600, 800,1000, and 1200 Hz, the pitch will nearly always be identified as that of a 200 Hz tone, the "missing fundamental." This well-known characteristic of the ear makes it possible for the undersized loudspeaker of a table radio to produce bass tones and also forms the acoustical basis for certain mixture stops on a pipe organ.

When the ear is confronted with an inharmonic series of partial tones, however, the determination of the pitch is more subtle. Various theories of pitch perception predict similar but not identical behavior. One of the easiest theories to understand is the "residue theory" formulated by Schouten and colleagues[15] about 20 years ago. According to residue theory, the ear picks out a series of nearly harmonic partials somewhere near the center of the

audible range, and determines the pitch to be the largest near-common factor in the series. For example, if frequencies of 630, 830, and 1030 were presented, the perceived pitch would correspond to approximately 208 Hz (since 630/3 = 210, 830/4 = 208 and 1030/5 = 206), rather than 200 Hz, which is the "difference" tone. Of course, the exact pitch heard depends upon the loudness of the partial tones. For a discussion of modern theories of pitch perception, a recent article by Wightman and Green[16] is recommended.

TABLE III
Principal modes of a C# chime

mode no.	frequency	note	decay time(60dB)
1	62 Hz	B_1 +	64 sec
2	173	F_3 -	89 sec
3	338	E_4/F_4	98 sec
4	555	C_5#	87 sec
5	818.5	G_5#	19 sec
6	1126	C_6# +	10.3 sec
7	1472	F_6#	6.8 sec
8	1851	A_6# -	2.2 sec
9	2253	C_7# +	1.4 sec
10	2675	E_7 +	
11	3155	G_7 +	
12	3598	A_7/A_7#	

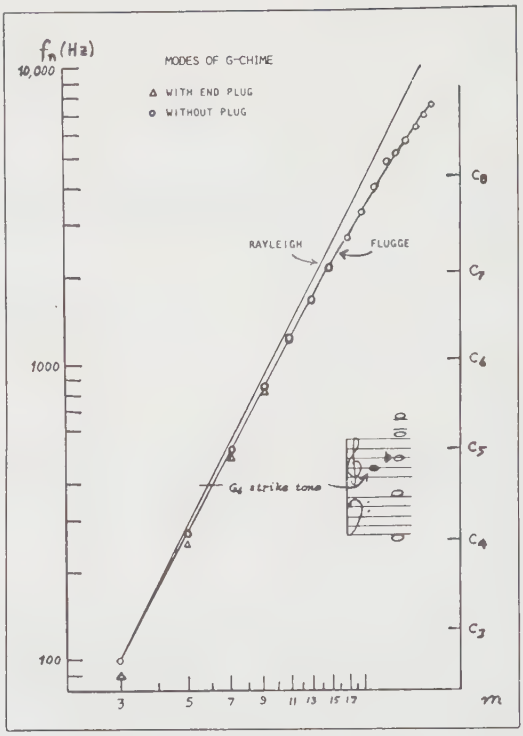

Fig. 12. Frequencies of a G-chime as a function of mode number m, along with those predicted by thin-bar theory (Rayleigh) and thick-bar theory (Flugge). The first five modes are shown on musical staffs, along with the subjective *strike tone.*

Chimes

One of the interesting characteristics of chimes is that there is no mode of vibration with a frequency at, or even near, the pitch of the strike tone one hears. The frequencies excited when a chime is struck are very nearly those of a free bar described earlier. Modes 4,5, and 6 appear to determine the strike tone. This can be understood by noting that these modes for a free bar have frequencies nearly in the ratio $9^2 : 11^2 : 13^2$ or 81 : 121 : 169, which are close enough to the ratio 2 : 3 : 4 for the ear to consider them nearly harmonic, and use them as a basis for establishing a pitch. The largest "near-common factor" in the numbers 81, 121, and 169 is about 41.

In Fig. 12 is a plot of the frequencies of the G chime as a function of *m*, along with those predicted by the thin bar theory of Lord Rayleigh and also the more detailed theory for a thick bar.[17] Also shown are the frequencies of vibration with the end plug removed. Note that the end plug lowers the frequencies of the first few modes, but has little effect on the higher modes. The strike tone, which lies one octave below the fourth mode, is indicated.

The ratios of the modal frequencies of a chime tube with and without a load at one end are given in Table IV. Also given are the ratios considered desirable for a tuned carillon bell. Note the similarity between the partials of a chime and those of a carillon bell. Adding a load to one end of a chime lowers the frequencies of the lower modes more than the higher ones (see also Fig. 12), and thus "stretches" the modes into a more favorable ratio. The end plug also adds to the durability of the chime, and helps to damp out the very high modes.

The bell-like quality of chimes is well-known, of course, and has been utilized in many compositions for band and orchestra (Tchaikovsky's "1812 Overture," for example). This bell-like timbre can be maximized by selecting the optimum size of end plug for each chime. For the G-chime in Table IV, the 193-g plug, with which the chime was originally fitted, is pretty near the optimum. Most chime makers use the same size plug throughout the entire set of chimes, however, so the timbre changes up and down the scale, typically being optimum near the center. We are of the opinion that a set of chimes "scaled" to have the same timbre throughout would have some advantages.

A well-tuned chime not only has its overtones tuned to resemble those of a carillon bell, but it is also free of "beats" between modes with nearly, but not quite, the same frequencies of vibration. These beating modes occur when the chime tube is not perfectly round or its wall thickness is not perfectly uniform. As a result, the transverse vibrations will have slightly different frequencies in two different transverse directions, resulting in beats when both modes are excited. These undesired beats can be

TABLE IV
Ratios of mode frequencies for loaded and unloaded chime tube (compared to strike tone)

n	Thin Rod	Tube	Loaded with 193g	435g	666g	tuned bell
1	0.22	0.24	0.23	0.22	?	0.5 or 0.6
2	0.61	0.64	0.63	0.62	0.61	1
3	1.21	1.23	1.22	1.22	1.22	1.2 (1.5)
4	2	2	2	2	2	2 (2.5)
5	2.99	2.91	2.93	2.95	2.94	3
6	4.17	3.96	4.01	4.04	4.03	4
7	5.56	5.12	5.21	5.21	5.18	5.33
8	7.14	6.37	6.50	6.43	6.37	6.67
strike tone		416 Hz	393 Hz	383 Hz	381 Hz	

(Diameter of tube is 1.37% of length; strike tone is G_4 with $f = 393$ Hz. All modes are compared to nominal strike tone.)

eliminated by squeezing the chime in a vise or thinning the wall slightly on one side to bring the modes into tune.

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SIMULATION OF BELL TONES WITH THE HELP OF PIPE-BELLS AND PIANO SOUNDS

K. A. Hueber

This article was translated by H. Leighton and T. D. Rossing from "Nachbilding des Glockenklanges mit Hilfe von Rohrenglocken und Klavierklangen," in Acustica ***26:****334–343 (1972)*

SUMMARY

In the present work the natural frequencies of the flexural vibrations of a pipe-bell were measured and calculated and an investigation made of the vibrations which are of definite significance to obtain the required sound impression. It was found further under which conditions the bell sound could be simulated by the keyboard sound. The tones of the bell-chord struck on the piano corresponded closely with the stepped-up amplitude tones of the pipe-bell spectrum. Finally it was shown in which way the sound spectrum of church bells corresponds with that of pipe-bells and piano-bell chords.

1. INTRODUCTION

The following specifications are most essential for the physical description of bell tones: (1) frequency of the partial tones, (2) amplitude relationships of the partial tones, and (3) damping of the partial tones.

The frequencies of the partial tones in pipe-bells and the frequency relationships of these sound-determining partial tones were investigated. Since bending vibrations of circular cylindrical shells are not harmonic by nature, it is notable that the sound impression from pipe-bells results in a nearly harmonious musical sensation. A characteristic group of bending vibrations of the same type as rod bending vibrations is responsible for the bell tones. This "bell formant" with its almost harmonic frequency relationships, which also forms the core of the sound spectrum of church bells, makes it possible to simulate bell tones with the help of piano sounds.

2. FREQUENCY SPECTRA OF PIPE-BELLS

The analysis of the vibrations of pipe-bells was made in two ways: (1) analysis of the vibrations according to their vibration form (measuring the nodal lines); (2) analysis of the sound field.

The following measurements were made on three brass pipe-bells:

1. Pipe E (e^0 or E_3): outside diameter d = 37.5 to 38.5 mm (partially out of round), wall thickness h = 1.5 mm, length l = 2.095 m;
2. Pipe G (g^0 or G_3): d = 39.5 mm, h = 1.5 mm, l = 1.923 m;
3. Pipe B (b^0 or B_3): d = 39.0 mm, h = 1.5 mm, l = 1.787 m.

The pipes were pierced diametrically by two holes approximately 4.2 cm from one end. Through these holes was drawn a thin string for hanging. The G and B pipes were closed on one end by a cap with a hole approximately 3 cm in diameter.

2.1 Measuring procedure

The pipes were first hung normally and struck with a soft hammer. The resulting sound was recorded on tape in order to determine the approximate frequencies of the partial tones and their intensity relationship as they died out.

2.2 Analysis of the vibrations according to their vibration form

In order to measure the nodal lines, two automatic measuring procedures were developed: (a) To determine the nodal rings, the gear of a Brüel and Kjaer chart recorder was furnished with a chain mechanism by which a microphone could be pulled along the vibrating pipe over intervals of approximately 2 cm. The resonance amplitudes could be examined with the same chart recorder. Bending vibrations were excited by an electrodynamic system with a coil fastened to a wood piece glued on the pipe approximately 6 cm from the end. The pole of a strong magnet was inserted into the coil, so that it vibrated in the same manner as the coil of a loudspeaker. Through the coil was passed a sinusoidal current from a generator, which was adjustable to 0.1 Hz, and whose output was also applied to an electronic counter for exact measurement of frequency. The applied frequencies, determined from the recordings of the struck pipes, were adjusted until the microphone located at an antinode of the driven pipe showed a maximum amplitude. With the excitation frequency determined this way, the microphone was pulled along the pipes.

(b) To determine the nodal meridians, a microphone was mounted on a stand placed on a Brüel and Kjaer revolving table centered under the pipe. The position of the microphone, as it rotated approximately 3 cm from the pipe, and the measured amplitude were recorded directly on polar graph paper with a chart recorder. Figure 1 shows the nodal meridians of pipe B. The measured results are compiled in Tables 1, 2, and 3.

2.3 Analysis of the sound fields

A Brüel and Kjaer frequency analyzer was used to measure the frequencies of the individual partial tones. The frequency was measured with an electronic counter each time the partial tone reaches its maxi-

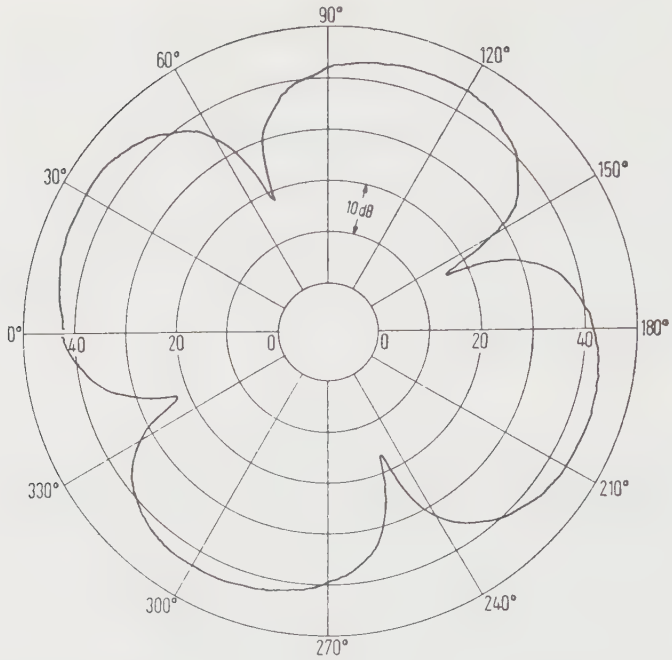

Figure 1. Sound field of pipe B at 2281.2 Hz.

mum amplitude, usually one second after the stroke. The diagrams in Figures 2, 3, and 4 show the sound fields of the three pipe-bells E, G, and B. One recognizes very well the amplitude ratios of the individual partial tones; these amplitudes are of great importance for the sound phenomenon. The damping of the various partial tones is different, as can be seen from their resonance curves. The wider the curve, the stronger the damping. If the resonance is strongly damped, the determination of the frequencies is, of course, less exact than in the case of less damped tones. One can read the relative dB-levels of the partial tones. The longitudinal half-wave numbers *i* for the corresponding bending vibrations of a hollow rod are recorded above the individual partial tones.

2.4 The strike tone

By listening, the following strike tones on pipes were established for pipes E, G, and B: e^0, g^0, and b^0 (E_3, G_3, and B_3). However, these notes, with the approximate frequencies of $e^0 = 164.8$ Hz, $g^0 = 196.0$ Hz, and $b^0 = 233.1$ Hz on the standard scale of frequencies, do not exist. In order to explain the strike tone, which is not a real note but rather is formed in the ear, the reader is referred to a publication of Grützmacher [1].

f Hz	z_K	z_M	Anmerkungen
37,2 (?)	—	—	wegen zu geringer Intensität nicht meßbar
102,2	3	2	
199,4	4	2	
326,3	5	2	
482,1	6	2	
665,8	7	2	
874,2	8	2	
1105,9	9	2	
1358,0	10	2	
1630,3	11	2	
1916,5	12	2	
2163,7	—	4	2 M deutlich 2 M undeutlich
2189,1	13	4	
2241,7	13	2	
2328,1	14	4	
2406,7	16	4	
2487,6	17	4	
2588,2	18	4	2 M deutlich, 2 M undeutlich
2692,9	19	4	
2875,5	14	2	1 M deutlich, 1 M undeutlich
3124,1	21	4	
3270,6	22	4	
3424,1	23	4	
3592,5	24	4	4 M nicht deutlich
3763,6	25	4	
3944,2	26	4	

Table 1. Frequency f, number of nodal rings and meridians z_M of pipe E.

3. BENDING VIBRATIONS OF PIPE-BELLS ACCORDING TO THE VIBRATION FORMS

Pipe-bells are circular cylindrical shells with a constant wall thickness of approximately 1.5 mm and an average diameter of 37 mm. The theory of vibrations of such cylindrical shells has been treated many times; the literature is cited by Grützmacher, Kallenbach, and Nellessen [2]. Basically, there are three kinds of bending vibrations: (a) longitudinal bending vibrations of the rod type; (b) peripheral bending vibrations; (c) bending vibrations with peripheral and longitudinal waves.

3.1 Longitudinal bending vibrations of the rod-type ($n = 1$)

These vibrations have the peripheral wave number $n = 1$; they have two shell nodal lines. The longitudinal half-wave numbers i go through the values $i = 1, 2, 3, 4. . .$ The number of nodal rings K is given by the

f Hz	z_K	z_M	Anmerkungen
45,4	2	—	M wegen zu geringer Intensität nicht meßbar
124,5	3	2	
241,6	4	2	
394,4	5	2	
581,4	6	2	
800,0	7	2	
1047,1	8	2	
1320,2	9	2	
1615,4	10	2	
1930,9	11	2	2 M undeutlich
2242,5	13	4	
2666,1	17	4	
2805,6	18	4	
2979,3	14	2	2 M undeutlich
3115,5	20	4	
3285,7	21	4	
3353,5	15	2	2 M undeutlich
3466,8	22	4	
3656,5	23	4	
3854,7	24	4	
4060,2	25	4	
4127,1	17	2	2 M undeutlich
4272,8	26	4	
4522,3	18	2	2 M undeutlich
4716,5	28	4	
4927,4	19	2	2 M undeutlich

Table 2. Frequency f, number of nodal rings z_K, and meridians z_M of pipe G.

f Hz	z_K	z_M	Anmerkungen
52,5	2	—	M wegen zu geringer Intensität nicht meßbar
143,2	3	2	
277,5	4	2	
453,9	5	2	
673,0	6	2	
913,9	7	2	1 M undeutlich
1205,4	8	2	
1515,6	9	2	
1849,4	10	2	
2222,4	12	4	
2281,2	—	4	
2313,0	13	4	
2545,7	15	—	keine deutliche M
2834,7	17	4	
3002,5	18	4	
3175,1	19	4	
3359,9	20	4	
3559,6	21	4	
3989,8	22	4	

Table 3. Frequency f, number of nodal rings z_K and meridians z_M of pipe B.

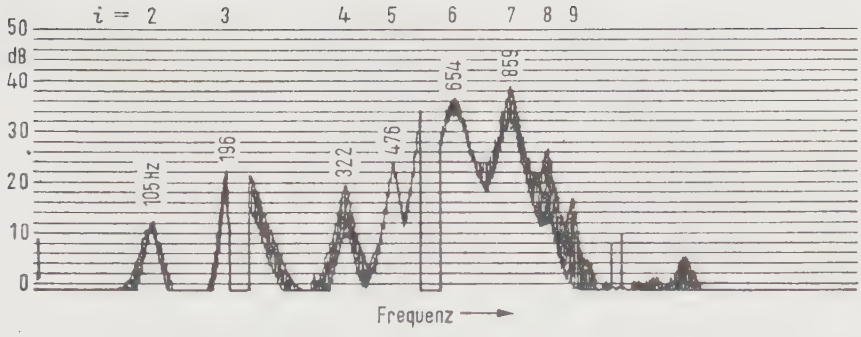

Figure 2. Eigenfrequencies of pipe-bell E. Over the peak of each partial are the frequency and the longitudinal half-wave numbers i of the corresponding longitudinal vibrations.

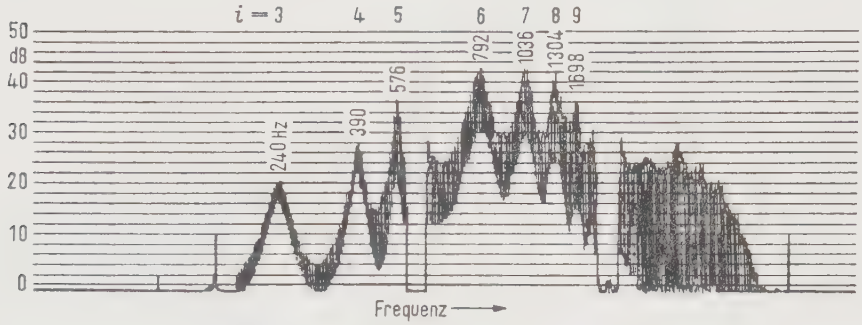

Figure 3. Eigenfrequencies of pipe-bell G.

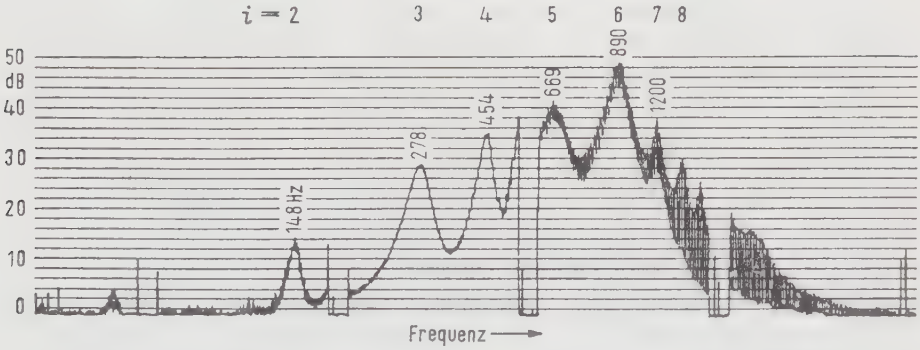

Figure 4. Eigenfrequencies of pipe-bell B.

relationship $K = i + 1$, since pipe-bells are hung freely movable, and thus fulfill the end conditions of free rods. The radially stiff plates that close one end of the pipes G and B have no influence on the end conditions of the longitudinal bending vibrations; that is, the rim requirements of free rods are also true in this case.

Longitudinal bending vibrations $n = 1$ are best calculated through the formula for cylindrical shell bending vibrations set up by Flügge [2, p. 83]:

$$f = \frac{1}{2\pi} \sqrt{\frac{E\,\Delta}{\varrho\, a^2 (1-\nu^2)}} \tag{1}$$

with

$$\Delta = \frac{(1-\nu^2)\,\lambda^4 + \beta\,\{(\lambda^2+n^2)^4 - 2[\nu\,\lambda^6 + 3\,\lambda^4 n^2 + (4-\nu)\,\lambda^2 n^4 + n^6] + 2(2-\nu)\,\lambda^2 n^2 + n^4\}}{(\lambda^2+n^2)^2 + (3+2\,\nu)\,\lambda^2 + n^2 + \beta\,\dfrac{3-\nu}{1-\nu}\,(\lambda^2+n^2)^3}.$$

f = frequency
a = average radius of shell
h = wall thickness of shell
l = cylinder length
i = longitudinal half wave number
m_i = eigenvalue
$\lambda = \dfrac{m_i a}{l}$ = longitudinal wave factor
n = peripheral wave number
$\beta = h^2/12a^2$
E = elastic modulus
ν = Poisson constant
ρ = density

The first eight natural frequencies of pipe-bell E were calculated according to Equation (1). The natural values m_i, which depend on the rim requirements in the hollow rod, are determined in pipe-bells by the roots m_i in the free-rod equation: $1 - \cos m \cosh m = 0$ [3]. The density of the material was determined to be $\rho = 8.4$ g/cm^3. The elasticity constants were taken from tables: elasticity module $E = 11.7 \times 10^5$ kg/cm^2, Poisson constant $\nu = 0.35$.

Thus, there results for the first intrinsic frequency ($i = 1$) a value of 38.6 Hz. If we reduce the difference from the measured value of 37.2 Hz to the uncertainty due to the unmeasured modulus E, and if we use the measured value of the first natural frequency f_1 (37.2 Hz) to determine exactly the modulus E, there result the calculated values f_{ber} for the first eight natural frequencies of the pipe-bells that are quoted in Table 4. The agreement with the measured frequencies can be considered to be very good.

The natural bending frequencies, calculated according to Equation (2) for the vibrations frequencies of transverse or bending vibrations of rods and hollow rods show, as is known, increasingly larger positive deviations from the measured values with growing longitudinal wave factor. The error is greater than + 5% when $\lambda > 0.2$. In the case of pipe-bell E,

f	f_{ber} Hz	f_{gem} Hz
f_1	37,2	37,2
f_2	102,1	102,2
f_3	198,9	199,4
f_4	326,1	326,3
f_5	482,2	482,1
f_6	665,4	665,8
f_7	873,9	874,2
f_8	1105,5	1105,9

Table 4. Calculated frequencies f_{ber}, and measured frequencies f_{gem} of pipe-bell E.

which has the calculated value f_6 according to Equation (2), the error compared to the measured value already amounts to + 3.6%.

$$f_i = \frac{m_i^2 \varkappa}{2 \pi l^2} \sqrt{\frac{E}{\varrho}} \tag{2}$$

f = frequency
l = length of the rod
i = longitudinal half-wave number
m = eigenvalue
K = radius of gyration
E = elasticity modulus
ρ = density

As can be deduced from Figures 2, 3, and 4, the longitudinal bending vibrations ($n = 1$, for the values $i = 3, 4, 5, 6, 7, 8, 9$), because of their amplitudes, basically determine the sound impression. The lowest longitudinal bending vibration ($i = 1$) cannot be found in the sound spectra of pipe-bells E and G. The next vibration ($i = 2$) is of only minor importance for the sound because of its small amplitude. The amplitudes of the vibrations $i = 4$ and up increase fast, reach their maximum with the values $i = 6$ and $i = 7$ (ca. 4 times the dB-level of the vibration $i = 2$), and then more or less quickly decrease. The frequencies of the transverse vibrations of the rod, calculated according to Equation (2), behave like the squares of the eigenvalues of the free rods. The frequency ratios of the longitudinal bending vibrations for the values $i = 3, 4, 5, 6, 7, 8, 9$ of pipe-bell E, which are decisive for the sound impression, are somewhat smaller than the corresponding ratios of the squares of the eigenvalues of free rods. These frequency ratios are compared with the corresponding pure intervals in Tables 5 and 6.

With increasing ratio a/l in the pipe-bell, the frequency relationships of the longitudinal bending vibrations $n = 1$ become smaller. In the case

Verhältnisse der Quadrate der Eigenwerte m	Reines Intervall
$m_4^2 : m_3^2 = 1{,}653$	$1{,}667 = 5:3$ große Sext
$m_5^2 : m_4^2 = 1{,}494$	$1{,}50 = 3:2$ reine Quint
$m_6^2 : m_5^2 = 1{,}397$	$1{,}389 = \frac{4}{3}\cdot\frac{25}{24}$ übermäßige Quart
$m_7^2 : m_6^2 = 1{,}331$	$1{,}333 = 4:3$ reine Quart
$m_8^2 : m_7^2 = 1{,}284$	$1{,}25 = 5:4$ große Terz
$m_9^2 : m_8^2 = 1{,}249$	$1{,}25 = 5:4$ große Terz

Table 5. Comparison of the ratios of the squares of eigenvalues m of free rods with the corresponding pure intervals.

Verhältnisse der gemessenen Frequenzen f	Reines Intervall
$f_4 : f_3 = 1{,}64$	1,67 große Sext
$f_5 : f_4 = 1{,}48$	1,50 reine Quint
$f_6 : f_5 = 1{,}38$	1,39 übermäßige Quart
$f_7 : f_6 = 1{,}31$	1,33 reine Quart
$f_8 : f_7 = 1{,}26$	1,25 große Terz
$f_9 : f_8 = 1{,}23$	1,25 große Terz

Table 6. Comparison of the ratios of measured frequencies f of longitudinal bending vibrations ($n = 1$) of pipe-bell D with the corresponding pure intervals.

of the longitudinal bending vibrations of pipe-bells, the frequency ratio $f_4 : f_3$, which is so characteristic for bell sound, creates an interval of a sixth. The size of this interval depends on the ratio a/l and is a decimal number between the values 1.64 and 1.60. When $a/l \geqq 0.019$, the interval of the sixth falls short of the value 1.60, and the bell sound phenomenon decreases.

Figure 5 shows the frequencies of the vibration modes, ordered according to their vibration form, for an open tube with the ratio of $a/l = 0.023$. The measurements of the radius and of the wall thickness differ a bit from the usual values in pipe-bells. As in pipe-bells, no more than 4 meridians were measured (compare 3.2). However, the frequencies of the longitudinal bending vibrations $n = 1$ (2 meridians) of the tube in Figure 5 deviate substantially, in contrast to the pipe-bells, from the frequencies of the transverse vibrations of the rod calculated from Equation (2).

Table 7 compares the frequencies F of the longitudinal bending vibrations ($n = 1$) of the pipe in Figure 5 with those frequencies f of the rod calculated according to Equation (2), and also compares the characteristic frequency ratios $f_4 : f_3$ and $f_5 : f_4$, which form intervals of a major sixth and

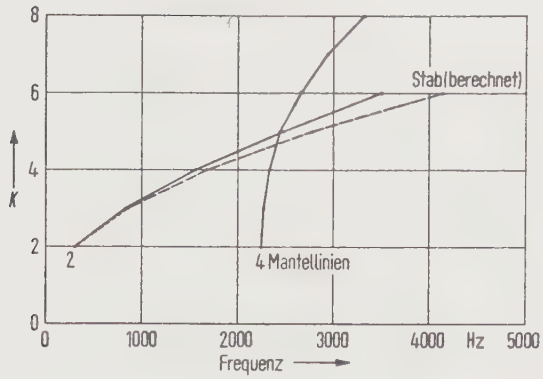

Figure 5. Eigenfrequencies of an open pipe classified according to vibration form. $a = 1.45$ cm, $l = 64$ cm, $h = 0.1$cm. K is number of nodal rings. Longitudinal vibrations, $n = 1$ (2 meridians) and cylindrical shell vibrations, $n = 2$ (4 meridians). Transverse vibrations of a rod calculated from Equation (2).

Längsbiegeschwingungen, $n = 1$			Transversalschwingungen des Stabes		
i	K	F Hz	i	K	f Hz
1	2	310	1	2	310
2	3	830	2	3	850
3	4	1560	3	4	1670
4	5	2470	4	5	2770
5	6	3500	5	6	4130

Frequenzverhältnisse F	Reines Intervall	Frequenzverhältnisse f	Reines Intervall
$F_4 : F_3 = 1{,}58$	$1{,}60 = 8:5$ kleine Sext	$f_4 : f_3 = 1{,}65$	$1{,}67 = 5:3$ große Sext
$F_5 : F_4 = 1{,}42$	1,39 übermäßige Quart	$f_5 : f_4 = 1{,}49$	$1{,}50 = 3:2$ reine Quint

Table 7. Comparison of the frequencies F of the longitudinal bending vibrations ($n = 1$) of the pipes of Fig. 5 with the frequencies f calculated from Equation (2) for transverse rod vibrations. Longitudinal half-wave number $= i$, K is number of nodal rings.

a pure fifth, respectively, with the corresponding frequency ratios $F_4 : F_3$ and $F_5 : F_4$.

3.2. Peripheral bending vibrations and bending vibrations with peripheral and longitudinal waves

Bending vibrations with peripheral and longitudinal waves were measured only for a peripheral wave number of $n = 2$ (4 shell nodal lines). The natural frequencies of the vibrations for the values $i = 1$ to approximately $i = 12$ are not recorded in the tables. As the calculation according to Equation (1) shows, these frequencies lie very close together. The natural frequency of the first bending vibration with peripheral and longitudinal waves ($n = 2$, $i = 1$) of pipe-bell E is 2105 Hz, that of the peripheral bending vibration $n = 2$, $i = 0$ lies at 2104 Hz, close below. As follows from the diagrams of the sound fields of pipe-bells E, G, and B (compare Fig. 2, 3, and 4), these vibrations are of secondary importance for the sound impression because of their minimum amplitudes.

In contrast to pipe-bells, in which only the bending vibrations with peripheral and longitudinal waves with peripheral wave number $n = 2$ (4 shell lines) were measured, we can observe bending vibrations of the peripheral kind and bending vibrations with peripheral and longitudinal waves up to a value of $n = 6$ (12 shell lines) in the case of an open tube (Fig. 6), where the eigenfrequencies have been arranged according to their vibration form. The frequency spectrum is completely inharmonic.

4. PIANO BELL TONES

Observations that certain note combinations or chord structures, when struck on a piano in a sharply differentiated frequency area and with open dampers, evoked bell-like tones served as the starting point of this study. Six chord structures, in which the bell tone emerges in a most curious way, will be presented here. Figure 7 shows the piano bell chords of types 1a, 1b, 1c and 2a, 2b, 2c. It is characteristic of type 1a to show the combination of major sixth, pure fifth, diminished fifth (or augmented fourth), perfect fourth, and major third. In place of the perfect fourth and

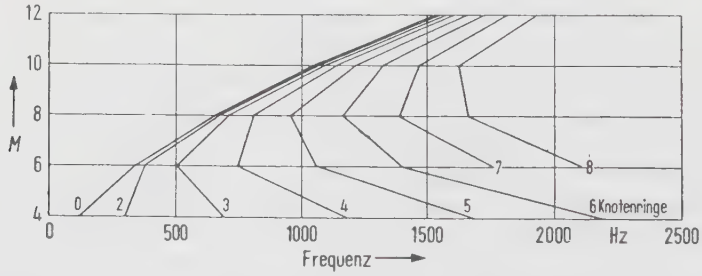

Figure 6. Eigenfrequencies of an open tube classified by their vibration form. $a = 9.875$ cm, $l = 100.5$ cm, $h = 0.25$ cm, M = number of meridians.

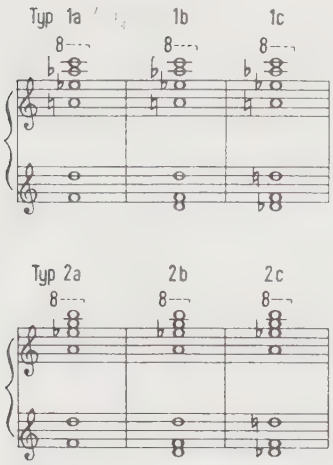

Figure 7. Piano bell chords, types 1a, 1b, 1c, 2a, 2b, and 2c.

major third, in type 2a we have two major thirds. The b- and c-variants of types 1 and 2 are different from the a-form only in that they add one more note below the lowest note of the a-type, which note forms the interval of a minor third in the case of types 1b and 2b and the interval of a major third in the case of types 1c and 2c.

Figure 8 shows a chord structure in which the bell tone in the low frequency range is less apparent.

The bell tone disappears very quickly in the above-mentioned chords when the lowest note lies below c^1 (C_4) in types 1a and 2a, below a^0 (A_3) in types 1b and 2b, and below as^0 (A_3^b) in types 1c and 2c. Figure 9 gives a synopsis of the range encompassed in the piano bell chords.

How can these note combinations or chords that evoke bell-tone effects be explained? If one compares the tone spectrum of pipe-bells with that of the piano bell chords, the noteworthy fact emerges that the notes struck on the piano correspond with the notes that are most prominent in the pipe-bell spectrum As has been shown, these are the ones that belong to the partials for the longitudinal vibrations with $i = 3, 4, 5, 6, 7, 8$. In the case of the piano bell chords, the transcendental frequency ratios of the longitudinal bending vibrations of the hollow rod for $i = 3, 4, 5, 6, 7, 8$ are transposed in the tempered pitch of the piano. Figure 10 shows which partial notes of the pipe-bells correspond to the individual notes

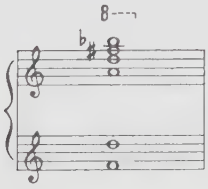

Figure 8. Chord structure shown by a few particular bell sounds in the low-frequency range.

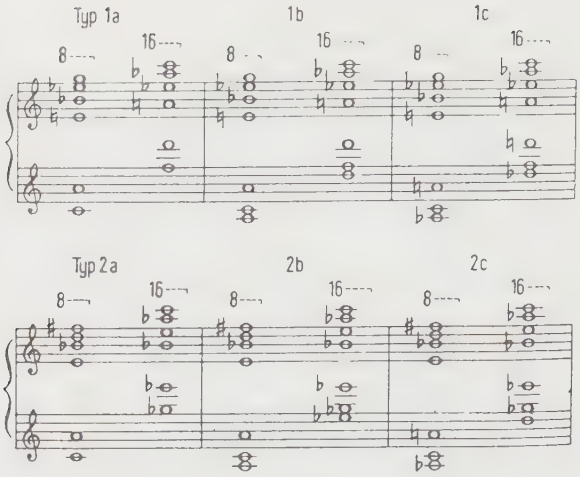

Figure 9. Range of piano bell chords of types 1a, 1b, 1c, 2a, 2b, and 2c within which bell sounds are simulated.

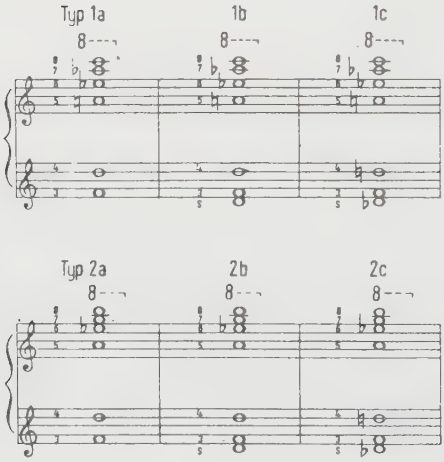

Figure 10. Piano bell chords of types 1a, 1b, 1c, 2a, 2b, and 2c. The numbers by the notes designate the numbers *i* of longitudinal half waves in the corresponding longitudinal bending vibrations of a hollow rod. S = strike note.

of the piano bell chords. The figure in front of each note in the chord designates the longitudinal half-wave number *i*, which is the order number of the corresponding longitudinal bending vibration of the hollow rod. *S* indicates the strike note. The strike note, which can be heard clearly in pipe-bells—it is the note which we sense as the "basic note" in striking the pipe-bell—cannot be observed in the sound spectrum as a

real note. In church bells, we find a real note, the prime (1 nodal ring, 4 meridians), in the same range as the strike note. If we strike a note in the piano bell chords (types 1b, 1c, 2b, 2c) that corresponds to the strike note (that is, it rings as a real note like the prime of a church bell), then the sound impression becomes fuller and approximates more nearly that of a church bell. If, in the case of types 1a, 1b, 2a, 2b, the notes $i = 3$ and $i = 4$ form the interval of a minor third instead of a major sixth, the bell-sound effects can be achieved only in the high frequency range.

The fact that certain note combinations struck on the piano result in bell-sound effects not only depends on the structure of the chords, but is probably also caused by the great similarity of the transients in the striking of pipe-bells and piano chords.

4.1. Sound spectrum of the piano bell chords

The sound spectra of piano bell chords were investigated at the Physical-Technical Institute in Braunschweig. From the recordings, tapes were prepared for analysis with a transient analyzer from Wandel and Golterman. The frequency of the partial notes was established by tuning an audio generator to find each maximum. The corresponding frequency was measured to 1 Hz with a Beckman electronic counter.

Figure 11 shows the sound spectrum of the piano bell chord d^1 - f^1 - d^2 - a^2 - dis^3 - g^3 - h^3 (D_4 - F_4 - D_5 - A_5 - $D_6^\sharp$ - G_6 - B_6). The frequency is recorded to the right, the amplitudes of the partial tones are recorded in dB at the top. In Table 8 the frequencies are again compiled, and the note designations with the deviations in cents from the equally tempered pitch (a^1 = 440 Hz) are given.

It is obvious from Figure 11 and Table 8 that not only the notes struck on the piano appear in the sound spectrum of the bell chords, but also those that were made to sound through resonance. These are the partials

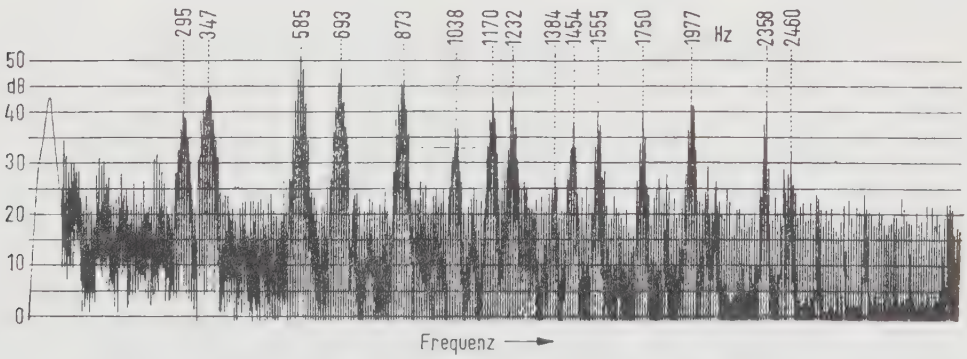

Figure 11. Sound spectrum of bell chords d^1, f^1, d^2, a^2, dis^3, g^3, h^3 (D_4, F_4, D_5, A_5, $D_6^\sharp$, G_6, B_6).

f Hz	MT	A cent
295	d^1	+ 8
347	f^1	−11
585	d^2	− 7
693	f^2	−14
873	a^2	−14
1038	c^3	−14
1170	d^3	− 7
1232	dis^3	−30
1384	f^3	−16
1454	fis^3	−31
1555	g^3	−14
1750	a^3	−10
1977	h^3	+ 1
2358	d^4	+ 6
2460	dis^4	−20

Table 8. Frequencies f in the sound spectrum of piano bell chord d^1, f^1, d^2, a^2, dis^3, g^3, h^3 (D_4, F_4, D_5, A_5, $D_6^\#$, G_6, B_6). MT is the musical designation, A is deviation in cents from the scale of equal temperament with a^1 = 440 Hz.

of the harmonic series built on the two lowest sounding notes d^1 and f^1 (D_4 and F_4):

Series I: d^1, d^2, a^2, d^3, fis^3, a^3 (D_4, D_5, A_5, D_6, $F_5^\#$, A_6);
Series II: f^1, f^2, c^3, f^3, a^3, c^4 (F_4, F_5, C_6, F_6, A_6, C_7),

The resonance notes show a somewhat smaller amplitude than those notes struck on the piano, and they have as their effect that the sound spectrum of the piano bell chords contains partial notes that are not present in the sound spectrum of the pipe-bells but can be found in church bells.

5. COMPARISON OF THE SOUND SPECTRA OF CHURCH BELLS AND PIPE-BELLS

The series of vibrations with one node circle above the stroke ring and increasing meridian number [4], which is so important in the whole sound spectrum of a church bell, corresponds exactly with the series of partial notes of the longitudinal bending vibrations in pipe-bells with the values i = 3, 4, 5, 6. . . . In the partial-note series of church bells with 1 nodal circle and 4, 6, 8, 10, 12 nodal meridians, the meridian numbers correspond to twice the longitudinal half-wave numbers i of the pipe-bells. Only the partial note of the prime in church bells (vibration form: 1 nodal circle, 4 nodal meridians) has no equivalent in the pipe-bell spectrum. One hears the prime (the "fundamental") only as a strike note, which, as already

indicated, is not a real note. The third, which is so important in the sound spectrum of church bells (the "characteristic note," vibration form: 1 nodal circle, 6 meridians), has as its opposite in the pipe-bell spectrum the partial note that corresponds with the longitudinal bending vibration $i = 3$. This note is not immediately recognizable as a third, since the prime is missing; however, it proves to be the third if we start with a strike note lying in the range of the prime. Thus, the series of partial notes so characteristic of the pipe-bell spectrum, proves to be also the nucleus of the sound spectrum of church bells.

Table 9 compares the partial note series of the longitudinal bending vibrations (for the values i = 3, 4, 5, 6. . .) of pipe-bells with the corresponding series (1 nodal circle, 4, 6, 8, 10, 12 meridians) of church bells. The note designation of the relevant partial note, which is customary in church bells, is given in the third column. The sound spectrum of pipe-bells differs substantially from that of church bells because in it there are missing the following important partial notes of the church-bell spectrum:

Lower octave (vibration form: 0 nodal circles, 4 nodal meridians),
Prime (vibration form: 1 nodal circle, 4 meridians),
Fifth (vibration form: 0 nodal circles, 6 meridians),
Tenth (vibration form: 2 nodal circles, 6 meridians).

Of course, there are also some partial tones in the mixture range that are missing; they are not of very great importance for the bell sound, however.

The mathematical proof that in the case of the church bells the series: 1 nodal circle, 4, 6, 8, 10, 12 meridians of the partial notes corresponds with the longitudinal bending vibrations of pipe-bells for $i = 3, 4, 5, 6, \ldots$ cannot be given, even approximately, since the frequency equations have not yet been set up due to the extremely complicated shape of the bell.

Röhren-glocken i	Kirchen-glocken 1 Knoten-kreis M	Tonstufe
—	4	Prim
3	6	Terz
4	8	Oberoktave
5	10	Duodezime
6	12	Doppeloktave
7	14	Quart über der Doppeloktave
8	16	Sext über der Doppeloktave
9	18	Tripeloktave

Table 9. Comparison of the partials from bending vibrations of pipe-bells with the corresponding partials from church bells.

i = number of longitudinal half-waves
M = number of meridians.

6. COMPARISON OF THE SOUND SPECTRA OF CHURCH BELLS AND PIANO BELL CHORDS

Piano bell chords, especially those of types 1b, 1c, 2b, 2c, show, as has already been stated, partial notes which are the result of the resonance notes in their sound spectra, and which cannot be found in the pipe-bell spectrum but are present in the spectrum of church bells. In Table 10 we have the most important partial notes of a d^1 (D_4) church bell according to their vibration form. It deals with the transposing of the sound spectrum of an as^1 ($A_4^\sharp$) bell which is quoted in the work "Acoustical Investigations on Church Bells" by Grützmacher, Kallenbach, and Nellessen [4] (Paper 7). The transposition at d^1 (D_4) was undertaken in order to simplify the comparison with the sound spectrum of the keyboard bell chord: d^1 - f^1 - d^2 - a^2 - dis^3 - g^3 - h^3 (D_4 - F_4 - D_5 - A_5 - $D_6^\sharp$ - G_6 - B_6, compare Figure 7 and Table 8). The comparison of Tables 8 and 10 shows that in the sound spectrum of the piano bell chords there can be heard partial tones of

K	*M*	MT	Tonstufe
0	4	d^0	Unteroktave (Unterton
0	6	a^1	Quint
0	8	g^2	
0	10	d^3	Doppeloktave
0	12	$+ g^3$	
0	14	$+ h^3$	
1	4	d^1	Prim
1	6	$+ f^1$	Terz
1	8	d^2	Oberoktave
1	10	$- a^2$	Duodezime
1	12	$+ d^3$	Doppeloktave
1	14	$- g^3$	Oberquart über der Doppeloktave
1	16	$- h^3$	Obersext über der Doppeloktave
1	18	d^4	Tripeloktave
1	20	f^4	
2	4	$+ g^2$	
2	6	$+ f^2$	Dezime
2	8	b^2	
2	10	$- e^3$	
2	12	$+ a^3$	
3	4	$- dis^3$	
3	6	d^3	Doppeloktave
3	8	e^3	
3	10	$+ g^3$	
3	12	$+ h^3$	
4	4	a^3	
4	6	gis^3	
4	8	$+ a^3$	
4	10	c^4	

Table 10. Partials of a d^1 church bell according to its vibration form.

K = number of nodal rings, *M* = number of meridians, *MT* is the musical tone designation.

church bells that result from resonance. These are: the tenth (vibration form: 2 nodal circles, 6 meridians); the double octave (vibration form: 0 nodal circles, 10 meridians and 3 nodal circles, 6 meridians), which lies a bit below the partial note: 1 nodal circle, 12 meridians; and also the partial notes that lie in the range of a fifth above the double octave (vibration forms: 2 nodal circles, 12 meridians; 4 nodal circles, 4 meridians; 4 nodal circles, 8 meridians).

Permit me to express special thanks to all the individuals and organizations that encouraged this work: The Dr. Adolf-Scharf Foundation for Furthering of Science, in Vienna; Professor Dr. Werner Lottermoser, who supported my work in every way; his esteemed colleague Engineer E. Jenkner, who conducted the analyses at the PTB in Branschweig; and Professors Dr. W. Kallenbach and Dr. Herbert Bednarczyk, TH, Vienna, who gave me valuable stimulation.

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Part VII

ELECTRONIC CARILLONS

Editor's Comments on Papers 22 and 23

22 CURTISS and GIANNINI
Amplification of Small Bells

23 SLAYMAKER
Bells, Electronic Carillons, and Chimes

Because of the high cost of cast bells, electronic carillons have become popular. Most electronic carillons use some type of tone generator, usually tuned rods struck by hammers, along with amplifiers and loudspeakers to radiate bell-like tones. The quality of the sound depends upon the quality of the loudspeakers but especially on the ability of the tone generators to generate the important partials of bell sound in the right proportion and with appropriate attack and decay times. One alternative to the use of electroacoustic tone generators is to amplify the sound of small bells electronically. Another alternative is to synthesize the sounds of the bells electronically.

Early experiments with amplified bells are described in Paper 22. Amplification allows the bell founder to design each bell for optimum tone quality without concern for loudness. This would probably result in high-frequency bells with thicker waists than used in most carillons, according to the authors.

The design of tone generators is discussed in Paper 23. Rectangular rods with two independent pickups can be tuned to give a more satisfactory bell sound than simpler generators.

Electronic carillons are played from a keyboard that resembles that of an organ console. They usually allow the carillonneur to select different bell sounds, given such names as Flemish bells, English bells, chimes, harp bells, and so forth. (See, for example, Klein, 1961.)

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Amplification of Small Bells

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and
G. M. Giannini, *Curtis Institute of Music*

(Received November 9, 1933)

THE rapid progress of electrical amplification of sound has brought with it many possibilities in the field of music. One such advantageous possibility is the substitution of small bells with suitable means for amplification to take the place of the large and heavy bells in a carillon.

There are several objections to the use of large bells in a carillon. They require a heavy tower construction which is costly, the bells themselves are expensive and the bells are not of the same tone quality throughout the musical range of a carillon. The change of tone quality of the bells is due to a change in the design of the bells of the upper register from that of the lower bells in order to get sufficient volume from these small bells. Also the playing action of the large bells as generally used today is large, bulky, and rather strenuous to play.

Several advantages of electrical amplification are less weight, less space required in tower; in fact only sufficient space for proper size of loudspeakers is necessary. The rest of the equipment such as the amplifiers, the small bells with their pickup device could be placed in some out of the way place. The small bells could be struck electrically or mechanically but either way the striking action and clavier would be much smaller and easier to operate than the present clavier for large bells and lastly the output of the small bells may be amplified to practically any required sound level.

Measurements have been made on the intensity of sound of an actual set of bells of a carillon in order to determine the amount of audio power necessary from a set of loudspeakers to cover adequately a given territory with the same efficiency as a set of bells. By basing the required amount of volume on these measurements and by considering the noise level existing at the time of these tests and their effect on the ability to hear clearly any passage of music the bells are interpreting, a definite relation was established on the audio power required to cover a certain locality with given noise levels. This is an important advantage of audio amplification over large bells because the noise level may be determined for any location and an audio power output specified which will give good reception for bell music on that location.

From previous tests on large bells, it has been noted that the general shape of a bell has a great deal to do with its harmonic content and pleasing tone. Taking these facts into consideration a set of small bells (14) was made by the John Taylor Company of Loughborough, England. These small bells are listed in Table I. All except Bell No. 15 were designed with a thin waist and no sound bow. Bell No. 1 is a tenor bell about

Table I.

Bell	Note	Frequency (cycles)	Weight (g)	Time to reach 45 db (sec.)	Type of clapper
1	*C*	129.3	1065	9	Soft
2	*G*	387.5	600	8	Medium
3	*A*	435.0		7	"
4	*A*♯	460.8		6	"
5	*B*	488.2		5	"
6	*C*	517.3		5	"
7	*C*♯	548.0		5	"
8	*D*	580.6	460	6	"
9	*D*♯	615.1		6	"
10	*E*	651.7		5	"
11	*F*	690.5		5	"
12	*F*♯	731.5		6	"
13	*G*	775.0		5	"
14	*G*♯	821.1	275	4	"
15	*G*♯ (Thick)	821.1	1150	10	"
16	*C*	4138.4	30	1½	Wood

The frequencies as specified in this table to which most bells are tuned does not cover these small bells. Being experimental bells, these bells were not exactly pitched as can be noted in the following table.

fifteen inches high and about ten inches across the top. The smallest bell No. 16 is about three inches high and one and three-quarters across the lip. Bell No. 15 is of a design similar to the heavy bells of a carillon and has a thick waist and sound bow.

A preliminary test of the small bells was made with amplification on a two-hundred watt amplifier and speaker. A velocity microphone was placed about four feet from the bells on a line with their effective sound bow. Comments were solicited from a number of persons and they all seemed to agree that the thick waisted bell was the most pleasing and sounded the closest to the tones of heavy bells. These small bells were not perfect in tune and had beats which were disconcerting when the bells were played rapidly. The thin waisted bells sounded rather harsh because of the relation of the upper harmonics of these bells to their strike note. These harmonics almost covered the strike note. This result brings up the question of designing amplifiers which attenuate the higher frequencies to a level of good tone balance in these small bells. Such conditions have been observed with other percussion instruments which have overtones of the higher order, the same as these small bells. The overtones are of varying intensity and attenuate rapidly while the frequency response curve of a good amplifier is flat over the frequency band in question which makes the faithful reproduction of bells and other percussion instruments difficult. The intensity of the overtones creates an impression of harshness and because of the rapid variations, the ear cannot become accustomed to the correct relation that exists. This necessitates an amplifier of such a frequency response that it attenuates the higher partials of the small bells without limiting the range of such a set of bells. Several such amplifiers may be necessary because of the range of bells from 120 cycles to 16,500 cycles.

In making a study of these small bells, measurements were made with a calibrated oscillator on the strike note of each bell of the time required for each bell to attenuate to 45 db sound level from the same force of strike, and in several cases, of the weight of the bells. The data are recorded in Table I. In this test the velocity microphone was placed three feet from the bell under test. The initial intensity was 60 db.

From the original group of bells, several bells were chosen to make an analysis of the placement of their partials. A calibrated oscillator was used again together with a microphone placed about three feet from the bell and on the same level as the sound bow. The calibrated oscillator was coupled into the same amplifier as the microphone, so that the source of sound of oscillator and bell was the same. Location of the partials was found by the beat note method. The tone analyses of Bells No. 1 and No. 6 are shown in Table II. The analyses of Bells No. 14 and No. 15 are shown in Table III and will be spoken of later. Referring to No. 1 Bell in Table II, it may be seen the harmonic content does not follow the numerical relation advanced for larger bells of 1, 2, 2.4, 3, 4, 5, and 6. The relation of this bell appears as 1, 2.03, 2.49, 3, 3.97, 5.3 and 6. This, however, seems to follow the results of tests made on large bells as given in a recent paper[1]

TABLE II. *Frequency content of bells.*

Bell No. 1	Pitch in cycles	Bell No. 6	Pitch in cycles
Hum note	65	Hum note	250
Strike note	132	Strike note	500
Third	162	Third	625
Fifth	195	Fifth	750
Nominal	258	Nominal	1005
Tenth	345		
Twelfth	389		

Bell No. 14 given as thin bell in Table IV.

TABLE III. *Radiation characteristic of bell (thick bell).*

Degrees	Sound level (db)	Energy flux density (micro-watts/cm²)	Degrees	Sound level (db)	Energy flux density (micro-watts/cm²)
0	77	0.0335	100	87	0.3150
10	75.5	.0240	110	85.5	.2250
20	74.5	.0190	120	82.5	.1100
30	77	.0335	130	79	.0550
40	82	.1000	140	77	.0335
50	84	.1550	150	75.5	.0240
60	85	.2000	160	72	.0105
70	86.5	.2750	170	71	.0084
80	87	.3150	180	70	.0066
90	88	.3900			

[1] Curtiss and Giannini, *Some Notes on the Character of Bell Tones*, J. Acous. Soc. Am. 5, 159 (1933).

by the authors. Bell No. 6 follows the relation of 1, 2, 2.5, 3 and 4.2. The higher partials of this bell were abundant but were closely bunched and of about the same intensity.

In this set of small bells under test there were two bells tuned to approximately the same pitch. One of these bells (14) was thin waisted with no sound bow. The other bell (No. 15) of Fig. 1 was

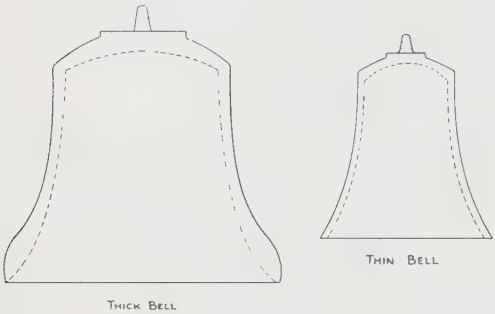

FIG. 1.

shaped the same as large bells having a thick waist and a sound bow. A close analysis was made of both bells which is recorded in Table IV. A number of persons were questioned as to which of these two bells was the most pleasing in tone and character. The general opinion was that the thick waisted bell was the most pleasing when amplified. About the only difference in these bells is that the thick bell did not attenuate as rapidly, possessed several inflection points and the hum note was much weaker than the hum note of the thin bell. Of these differences the most prominent was the decrement of attenuation and inflection points. These inflection points are peaks or humps in the attenuation curve of a bell after strike. These points are caused by a partial reaching maximum volume after strike. Any note which contains a number of inharmonic partials that appear at various times after strike should have sufficient body to last long enough to take away any harshness or jumble. If such a note is rapidly attenuated as was the case of the smaller bells, the note leaves the impression of overbalance with higher harmonics. The inflection points are rather important too. Each one of these points represents one of the harmonics building up in the note after strike. When the attenuation of the bell is not rapid and these important harmonics are allowed to build up slowly and then attenuate, the complete tone of the bell is rounded and pleasing. With the thin bells no such inflection points were found which means all the harmonics were present at strike and attenuated at the same rate. This does not follow the pattern of the heavy bells of a carillon.

TABLE IV. *Comparison of thick and thin bells.*

	Thin	Thick
Sound level in db		
Struck with medium clapper	60 db	75 db
Time in seconds to reach 45 db level	4 sec.	10 sec.
Weight of each bell	275 g	1150 g
Decrement (time in sec.) 70 db (strike)		
65 db	0.8 sec.	0.6 sec.
60 db	2.0 sec.	2.0 sec.
50 db	4.4 sec.	8.0 sec.
Time appearance of curve inflections points after strike	None	1.2 1.8 2.4 sec.
Harmonic frequencies strength	Pitch in cycles	Pitch in cycles
Hum note	394 (fair)	390 (weak)
Strike note	789 (strong)	780 (strong)
Third	1057 (fair)	1059 (fair)
Fifth	1190 (weak)	1167 (weak)
Nominal	1576 (fair)	1560 (fair)

While making tests on these small bells it was thought of general interest to obtain a sound radiation characteristic of a bell. Because the small bell (No. 15) which was patterned after large bells corresponded to large bells in tone analysis and attenuation, it was assumed it

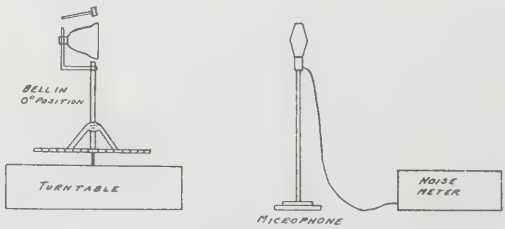

FIG. 2. Sound radiation characteristic of a bell.

would give the representative radiation characteristic of large bells and of all bells generally. For this purpose Bell No. 15 was mounted in a horizontal position with means for rotating through various angles before a velocity microphone and the calibrated noise meter, as shown in Fig. 2.

The radiation characteristic as found shows

that most of the energy comes from the sound bow at an angle of 90° from the axis of the bell. Very little radiation occurs either at the mouth of the bell or the top support. The radiation characteristic is shown in Fig. 3.

The bells used in this test were of two entirely different designs. From the results of each pattern as may be deduced Tables II and IV, the bell with the thick waist and sound bow is much more desirable for tone. The only disadvantage of this type is the weight incurred for the lower range bells. A compromise in design for the lower bells will no doubt have to be made in order to keep these bells within the limit of small bells. This arrangement of design would probably only be necessary for the lower eight notes, considering the *C* below Middle *C* as the lowest.

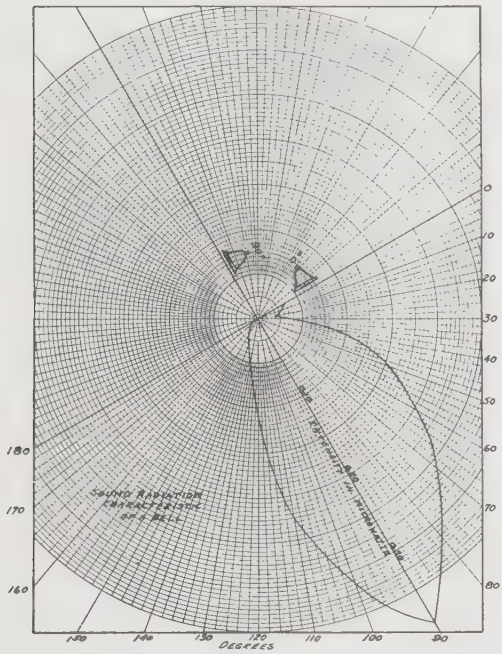

Fig. 3.

Along with a redesign of the small bells, some thought will have to be given to amplifiers and pickup devices which will correctly attenuate certain undesirable higher harmonics. All percussion instruments seem to offer this problem in some manner. To attenuate the higher harmonics does not mean the elimination of the upper partials. The partials of a bell as listed in previous paragraphs are essential for the full tone of a bell. In fact for good response of bell tones it would be desirable to carry the frequency range to 15,000 cycles. The problem is the attenuation of a quantity of certain harmonics which bunch up immediately after strike and tend to give a harsh tone to the bell. Producing small bells with enough mass to create inflection points in the attenuation curve and careful design of amplifiers should give pleasing and good competitive results to the large cumbersome bells.

The radiation characteristic of a bell as shown in Fig. 3 shows a striking comparison to a semi-directional loudspeaker taken in one plane. The sound bow in this case is the voice coil and the

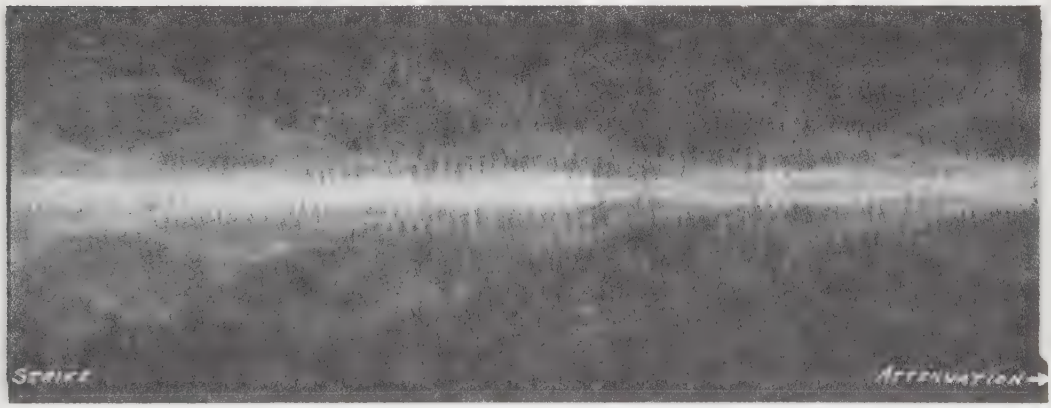

Fig. 4.

sides of the bell the horn. The mouth of the bell which would seem to be the horn of the mechanical actuator is almost a dead spot. Greatest radiation takes place at the sound bow and occurs mostly along the outside surface of the bell.

Fig. 3, indicates that a stationary bell gives just as much sound output from a tower as a pealing bell. The only advantages to a swinging bell is that it tends to break up any standing waves that may exist in a tower and to distribute the sound a little more evenly. But it is reasonable to say that our present system of mounting bells stationary is just as efficient for sound radiation as a set of swinging bells.

An oscillogram was made of the thick walled bell in one test and is added to show the character of attenuation. Six rotations were made by the drum holding the film at the speed of ten revolutions per second. This picture gives a good idea of the tone content of a bell. See Fig. 4.

Conclusions

Small bells with proper amplification seem to be much more desirable for use in carillons than the large heavy bells that have been in use for centuries. Heavy bells require large and heavy towers which are costly to build and maintain in this age. A great many towers of today, whether church or public building, are not large or strong enough to take a set of large bells. Many of the owners of these towers are anxious to obtain a carillon without added expense of rebuilding their tower. Small bells with amplification offer one solution.

The authors of this paper wish to express their appreciation to the John Taylor Company of Loughborough, England for specially casting and supplying the several types of small bells used in the tests described in this paper.

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Bells, Electronic Carillons, and Chimes*

F. H. SLAYMAKER†

Summary—**Bells and chimes, unlike the more familiar string and wind instruments, produce tones in which the overtone structure cannot be expressed as a series of harmonics. The accuracy of tuning of the various overtones varies widely but the better cast bell carillons, electronic carillons, and tubular chimes do have very accurately tuned overtones. This paper describes the results of measurements on cast bells, electronic carillons, and tubular chimes. Data will be given on relative amplitude and decay rates of the various overtones. The reaction of "out-of-tuneness" will be discussed and explained. A new type of tone source for electronic carillons will be described. With this new tone source, a rod of carefully controlled rectangular cross section is used. This rod can give in a single unified structure essentially the same overtone array as that of an accurately tuned cast carillon bell.**

Background

MOST PEOPLE will recognize the cup-shaped cast bronze bell as a bell when they see one, whether they are experts or laymen. Also they will recognize the polished brass tubes, such as used with pipe organs or in the percussion section of a symphony orchestra, as chimes. The so-called electronic carillon, however, is less familiar in its appearance although it is becoming more and more familiar as a sound. The most obvious external characteristic of an electronic carillon is the sound of bell-like tones coming from a set of loudspeakers in a church tower. In a little room tucked away in the church someplace there is likely to be a case containing a set of tuned rods. These rods, about an eighth of an inch in diameter and a foot or two in length are struck with little hammers. The vibrations of the rods are picked up with magnetic or electrostatic pickups, amplified, and radiated from loudspeakers. Once the sound is out in the air, however, the easy and certain choice of identifying names vanishes and the use of the words bell, chime, and carillon becomes confusingly intermixed. It might be presumed by most engineers and physicists that the actual tone can be described in quantitative terms as a fundamental and a series of harmonics; *i.e.*, a Fourier series. But even in the scientific analysis of the tone of a bell, chime, or electronic carillon, preconceived ideas about tone description must be forgotten. Fig. 1 shows the sound spectrum of the ideal cast carillon bell, the tubular chime, the simple minor-tuned clamped-end rod as used in electronic carillons and the more familiar harmonic series. More overtones, higher in frequency, are present than those shown in Fig. 1, but the higher overtones become less and less strong. The sequence of musical notes marking the position of each component in the complex sound spectrum of the bell, chime, and the clamped-end rod do not follow a Fourier series at all. By far the majority of electronic carillons made in this country have a tonal spectrum that can be expressed by the set of components given for the minor-tuned clamped-end rod or for the tubular chime. It is the attempt to identify the electronic carillon tone with the tone of a cast bell or tubular chime that causes so much confusion in the use of the words carillon, bell, and chime. To some people the tone is bell-like if it is deep and rich in contrast to the dainty etherealness of the tubular chime. To others, perhaps those who have heard poorly tuned bells, the word bell suggests something harsh and "clangy," while the word chime suggests something pleasant and sweet. The Guild of Carillonneurs defines a carillon, in part, as an instrument made up of twenty-three or more chromatically tuned cup-shaped bells, while an instrument having less than twenty-three bells would be called a chime. To other people it is the minor interval (the *E* flat in Fig. 1) that is characteristic of the true bell tone while the major interval (*E* natural in Fig. 1) is characteristic of chimes. And so it goes. For the purposes of this paper, a chime is a tubular chime; a bell is the cast campaniform bell; and a carillon is the traditional carillon as defined by the Guild of Carillonneurs. The electronic carillon is a new instrument, as yet undefined.

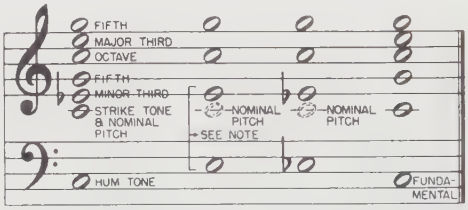

Fig. 1—Sound spectra of the ideal cast carillon bell tuned to middle "C"; the tubular chime tuned to a nominal middle "C"; the simple minor-tuned clamped-end rod tuned to a nominal middle "C"; and the harmonic series fundamental one octave below middle "C." (Note: Weak in the tubular chime.)

Sooner or later in any discussion of bells, chimes, and electronic carillons, the question comes up, "Why do they sound out of tune?"

The most obvious answer is that some of the instruments *are* out of tune. The better grades of bells, chimes, and electronic carillons, however, are so accurately in tune that the tuning discrepancies are too small to be noticeable. Even so, some notes do *sound* out of tune. Let us examine the spectrum in Fig. 1 to see what clues appear. All of the complex tones represented are supposed to sound like a *C* of some sort. Yet only in the harmonic series, such as approximated by a string or wind instrument, and in the spectrum for the cast carillon bell do we find the lower two components falling on a *C*. In the simple rod and the tubular chime the lower two components fall on some other note. From a look at the spectrum it is a little surprising that the rod and

* Presented at the National Electronics Conference, October 3, 1955; Manuscript received by the PGA October 3, 1955.
† Stromberg-Carlson Co., Rochester, N. Y.

the tubular chime sound like C at all since we have two E's or E flats and only one C. Also, no component is present physically at the nominal pitch; *i.e.*, there is no actual middle C present in the tone. I don't know a really good explanation for the apparent pitch but here are a couple of theories.

The C component an octave above middle C is likely to be the most prominent component in the entire spectrum. The pitch may sound an octave lower because the presence of the two lower frequency components influences the listener's judgment as to the octave. Another theory is that the C and G components, because they are separated by an amount equal to the frequency of middle C, combine in the ear to produce the impression of a pitch of middle C. Unfortunately the casual listener hasn't heard of these theories and he often hears an E or E flat or a G instead of a C and becomes very much confused. "Somebody sure hit a wrong note," he will say, or, "That thing sounds out of tune." The chances of such misunderstandings occurring with the string or cast bell tone are much more remote since in a space of two octaves there are three C's, whereas for the rod and chime there is only one.

What can be done to alleviate the out-of-tuneness of the simple rod or chime? A compromise is necessary, a compromise between (a) a deep sounding tone in which the lower E or E flat components are very strong but which is likely to sound more like an E of some sort than a C, and (b) a thin light tone in which the lower E's or E flats are suppressed to make the C more prominent. The light thin tone, of course, is suitable for use indoors with the support of an organ or a symphony orchestra but is not very appropriate for use in a bell tower. The outdoor tone needs the body and bigness.

The amplified vibration of the clamped-end rod are often compared to the cast bell tone because of the strength that it is possible to give to the lower components. The lower components in the tone of the tubular chime, however, are very weak. The tubular chime is too small physically to be a good acoustical radiator at the long wavelengths of the lower components. The tubular chime tone, then, is dainty and light but sounds better in tune than the deeper and more bell-like tone of the simple clamped-end rod.

Since tower music should sound rich and deep to be emotionally satisfying, how can one avoid the necessity of making a compromise between depth of tone and the impression of out-of-tuneness? To combine the feeling of bigness with accurate pitch, the lower two components in the tone must be C's for a C bell. When this condition exists, it is possible to make the tone very deep, and still have the tune sound like C, by increasing the relative acoustical power in lower components. The lower components in the harmonic series, of course, meet this requirement as do the lower components in the spectrum of the ideal cast bell. The spectrum of the simple rod or the tubular chime does not fulfill this condition. A tone with a spectrum coinciding with the harmonic series, however, will sound like a harp, a banjo, or some string instrument rather than like a bell.

Although ideal for tower music, cast bells themselves are heavy, expensive, and unwieldy. The electronic carillon is much more feasible in this modern economy than the cast bell carillon, and it would be very desirable to have an electronic carillon in which the tone fulfilled the condition for both bigness of tone and accuracy of pitch. It would be possible, of course, to create tones synthetically by mixing the tones of several rods. With a careful selection of the rods a great variety of effects could be obtained. It is a problem to strike several rods at once, however, and maintain the effect of a single bell being struck. A single structure producing the desired spectrum as an inherent result of its shape would be an ideal tone source.

Technical Section

The final structure, surprisingly enough, turned out to be a rather simple one, even though it was the result of a rather lengthy research program. The answer is a rod of rectangular cross section in which the ratio of the sides is the same as the ratio of the frequencies in a minor third; *i.e.*, 1.2 to 1 in the just intonation, or 1.189 to 1 in the equally tempered scale. The allowed frequencies of a clamped-end bar are:[1]

$$f_n = \pi/2l^2\sqrt{Qk^2/p}B_n^2 \qquad (1)$$

where

f = frequency, cycles per second
$n = 1, 2, 3, 4, \cdots$
l = length of bar in centimeters
Q = Young's modulus in dynes per square centimeter
p = density in grams per cubic centimeter
k = radius of gyration of the cross section in centimeters
$B_1 = 0.597$, $B_2 = 1.494$, $B_3 = 2.500$
$B_n = n - \frac{1}{2}$ for $n > 2$.

The ratio of successive allowed frequencies for a clamped-end rod are given in Table I. For a rectangle, the radius of gyration, about a center line perpendicular to h, the height of the rectangle, is $h/\sqrt{12}$.

TABLE I

Ratios of Successive Allowed Frequencies for a Clamped-End Rod and the Departures from the Tuning Shown in Fig. 1

Allowed Frequencies for A Clamped-End Rod	Departure from the Spectrum of Fig. 1, Assuming f_5 is Tuned to C
(1) $f_1 =$ —	—
(2) $f_2 = 6.236\, f_1$	—
(3) $f_3 = 2.801\, f_2$	$E^b + 65$ cents*
(4) $f_4 = 1.960\, f_3$	$E^b + 30$ cents
(5) $f_5 = 1.653\, f_4$	$C \pm 0$ cents
(6) $f_6 = 1.494\, f_5$	$G - 5$ cents

* A cent is 1/100 semi-tone, or the 1200th root of 2.

When the rectangle is turned through 90 degrees the other side becomes the "h" and a new value of the radius of gyration is obtained corresponding to a new set of

[1] P. M. Morse, "Vibration and Sound," McGraw-Hill Book Company, Inc., New York, N. Y., 2nd ed., p. 158; 1948.

allowed frequencies of vibration. Striking the rod on the corner excites both sets of allowed frequencies.

The allowed frequencies of vibration for a simple clamped-end rod, however, do not fit very well into the musical scale as shown in Fig. 1. They come too close together (See Table I). If the *C* component in the rod spectrum of Fig. 1 is tuned accurately the *G* above will be flat and the *E* flat below will be sharp. The lowest *E* flat will be even sharper. If, on the other hand, we use a rod that is pivoted at one end[2] instead of being rigidly clamped, the allowed frequencies are too far apart. A flexible support consisting of a notch, or necked portion of the rod, at the clamped-end can be made to give the proper spread to the allowed frequencies. The components of the spectrum shown in Fig. 1 correspond to the third, fourth, fifth, and sixth allowed frequencies or normal modes of vibration. The first two allowed frequencies are so far apart that they are of little use and they usually are too low in frequency to be passed by the reproducing system.

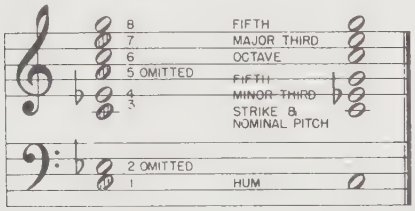

Fig. 2—The sound spectrum of the rectangular rod tuned to middle "C" compared with the spectrum of an ideal cast bell.

Fig. 2 shows two sets of components as obtained from the two directions of motion of a rectangular rod. The odd numbered set is obtained from the vibration of the rod perpendicular to its broad face and the even numbered set is obtained from the vibration perpendicular to the narrow face. Notice that the even numbered set corresponds exactly to the clamped rod spectrum in Fig. 1 for a tone that sounds like *C*. The odd numbered set, then, would sound like *A* if the *A* component (No. 5) were left in the reproduced tone. But the *A* can be removed by using two pickups as shown in Fig. 3. Pickup No. 2 is mounted opposite the broad face to pick up the odd numbered set of components. It is located at a point along the rod that is opposite a node for the normal mode corresponding to the *A* component. In the absence of the A component, there is no chance of the resulting tone sounding like both a *C* and an *A* sounding at once. The lower *E* flat in the even numbered set can be removed by suitable location of the pickup or by filtering it out of the electric circuit. The resultant tone contains a low *C* and one at middle *C* corresponding to the hum and strike tone of the bell, a minor third, an octave above the strike tone, a major third and a perfect fifth. Only one component is missing in the one to one correspondence between the components of a bell and the tone produced by the rectangular rod. It is the first perfect fifth. It is entirely a matter of coincidence that this particular component in a bell tone is weak and dies out rapidly,[3] so that the fact that it does not appear in tone of rectangular rod works out very well.

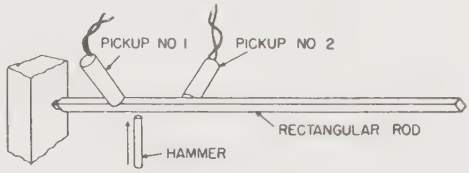

Fig. 3—Orientation of pickups and hammer used with rectangular rod.

As a preliminary to the research which resulted in this rectangular rod tone generator, we analyzed the tone of a large number of cast carillon bells.[3] Table II shows a comparison between the tuning of a well-tuned carillon bell and a well-tuned rectangular rod. The equally tempered scale is taken as the normal standard though in some instruments the overtone structure fits the just intonation better.

TABLE II

COMPARISON OF TUNING OF A WELL-TUNED RECTANGULAR ROD AND A WELL-TUNED CAST CARILLON BELL

	Rectangular Rod Deviation from Equally-Tempered Scale	Cast Carillon Bell Deviation from Equally-Tempered Scale
Hum Tone	0 cents	+ 5 cents
Strike Tone	0 cents	0 cents
Minor Third	+ 2 cents	− 7 cents
Perfect Fifth	—	+18 cents
Octave	− 1 cent	− 1 cent
Major Third	+ 8 cents	− 8 cents
Perfect Fifth	+10 cents	−12 cents

The extremely high accuracy of the tuning of the octaves in the rectangular rod is due to a very great extent to extremely precise control of the cross sectional shape of the rectangular rod. The other components differ from the tempered scale frequencies by an amount so small that it is less than the difference between a third on the tempered scale and a third figured on the just intonation; *i.e.*, if the minor third were one derived from the harmonic series it would be about 16 cents sharp compared to the tempered scale, while a major third figured on the just intonation would be 14 cents flat compared to the tempered scale.

So, with a carefully designed rectangular rod as a tone source, an electronic carillon can be built in which the components in the tonal spectrum fulfill the requirements for accurate pitch and the production of a deep, rich, and satisfying bell tone.

[2] L. Raleigh, "The Theory of Sound," Dover Publications, New York, N. Y., vol. 1, 2nd ed., p. 286; 1945.

[3] F. H. Slaymaker and W. F. Meeker, "Measurements of the tonal characteristics of carillon bells," *J. Acoust. Soc.*, vol. 26, pp. 515–522; July, 1954.

Part VIII

HANDBELLS

Editor's Comments on Papers 24, 25, and 26

24 PRICE
Handbells from Earliest Times

25 ROSSING and SATHOFF
Modes of Vibration and Sound Radiation from Tuned Handbells

26 ROSSING
Tuned Handbells, Church Bells and Carillon Bells

Although handbells have been traced back at least to the third millenium before Christ, the modern type of tuned handbell was developed in England in the eighteenth century, mainly for practice by tower bell ringers. More recently handbells have come into their own as musical instruments, and handbell choirs have become very popular in schools and churches.

The early history of handbells is the subject of Paper 24 by Percival Price. The acoustical characteristics of handbells are discussed in Papers 25 and 26. The sound spectrum of handbells is quite different from that of church bells. There are usually only two tuned partials, the strike note being determined by the lower one. Handbells are much lighter than church bells of the same pitch, and they have no sound bow. A comparison of vibration modes in handbells and church bells is made in Paper 26.

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HANDBELLS FROM EARLIEST TIMES

Percival Price

Handbells, the instrument you play collectively to produce a music sweet, charming, and like no other, have a longer, more dignified, and more mysterious history than you may imagine. Moreover, the use of handbells for music has not been limited to Europe and America, but has long occurred in Asia and Africa as well.

We are now near the end of the second millenium after Christ. In the third millenium before Christ the Chinese emperor, Huan T'i, ordered twelve bells to be made as pitch standards. These are not extant; but we know that they looked like grain scoops, so they must have been held in the hand. The oldest existing bells with handles date from about 1600 B.C., and are found in China. Some of them had clappers on the inside, and others were struck with hammers.

Illustration 1

The Chinese wrote about the tone qualities of their bells, and ascribed marvelous powers to their sound. This was not just blind superstition, but was based on their theories of sound as a transcendental power.

A thousand years later, in 600 B.C., we find a slightly different shape of handbell in China: longer, and with a more developed handle.

Illustration 2

That these bells differed in pitch was probably not so important to their hearers as that they differed in tone color, which is an older aesthetic attraction in music than melody.

Tone color is indeed a more outstanding attribute of most bells' sounds than is pitch, which may be rather indefinite. The Chinese described the sounds of their bells by terms such as 'Yellow Bell', 'Forest Bell', and 'Echoing Bell'; and these terms probably first referred to tone colors, and later to pitches.

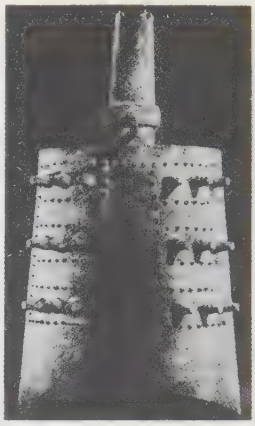


Illustration 3

A further development of the Chinese handbell was to cast it with spikes or 'nipples' on the outer surface. These were not designed to improve the tone in a physical sense, but to help it give off *liu*, the spiritual essence of sound, a force which, they believed, travelled far beyond the audible part of the sound, as do radio waves. An item more important than the 'nipples' for our observation is the loop at the base of the handle, so that the bell can be hung in a row and a melody played on them by one man. It brought in the concept of the bell as an an article fixed in one place, rather than portable, and from this, of bells larger than a man could hold.

It took the advent of Confucianism in the 5th century B. C. to take the religious use of the handbell and develop it musically. Sets of 9 to 16 bells attached to a frame and struck by hammers in the hands of one man were added to the Confucian orchestra. The set was called a *pien-chung*. It is the oldest known bell instrument on which music of more than a 3- or 4-note range could be played.

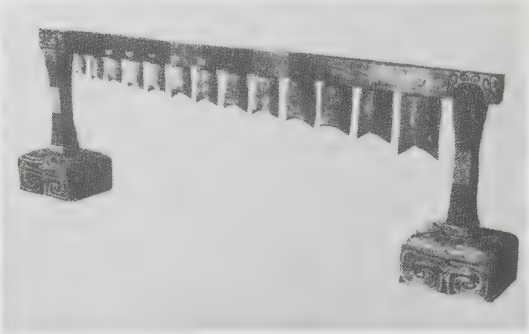


Illustration 4

The *pien-chung*
in use
over 2,000 years

The *pien-chung* continued in use for over two thousand years. Around A.D. 500 we find the bells more circular in form: this was a Chinese compromise between their ancient grain-scoop or 'fish-mouth' shape and the concentric and flared shape introduced with Buddhism from India.

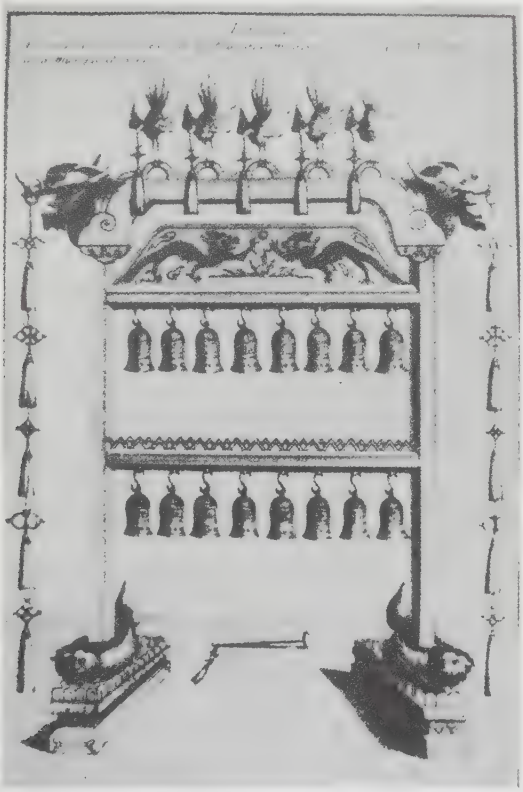


Illustration 5

In some instruments the handle was replaced by a loop; in others it was retained, as we see in an illustration of a *pien-chung* in the Korean imperial court orchestra around 1600.

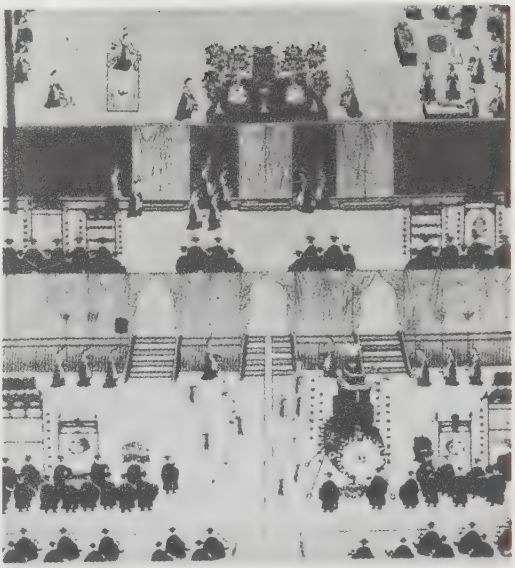


Illustration 6

In this scene of a performance before the emperor there are two *pien-chung*, both to the right: one is on the center terrace with a large bell to its right; the other is on the lowest terrace with drums to its left. There are other percussion instruments and winds; and they do not all play together as in our orchestras, but answer each other with their different tone-colors. And just as among percussion instruments, a sound on a drum, a musical stone, or a wood-block ceases almost immediately after it is struck, while a sound on a bell lingers as if a spirit inside were crying out, so it was perhaps this attribute of resonance, more than any other, that caused men to regard the bell as a medium of divine potency.

The Hindus carried the attribute of divine potency in the sound of the handbell so far as to have them used by some of their gods, as we see in a rock-carving of Parvati, consort of Siva.

Illustration 7

Such bells were so highly venerated that they were sometimes prayed to directly. A prayer to Parvati's ran:

> OM to the bell striking terror to our enemies by thy world-wide sound!
> Drive out from us all our iniquities.
> Defend us and bless us, O Lord.

We spoke of the circular form of the Buddhist handbell. It came from the much older form of the Hindu handbell, seen best in a bell dedicated to the monkey god, Hanuman, whose form is given to the handle. In gripping such a bell the ringer receives the double benefit of contact with the god by the touch of his image and by the sound of 'his' bell. The flared contour of the body of the bell is not copied from our Western handbells, as some people have supposed, but is much older. It is a stylization of the lotus, and symbolized the world-lotus, out of which issue hosts of the created world, just as sound issues out of a bell. Of course the flared shape also allows for a wide toss of the clapper, as in our handbells.

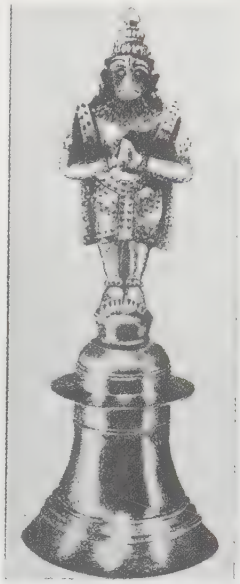


Illustration 8

The Buddhist handbell emerges from the Hindu handbell

Buddhism adopted the Hindu handbell with few changes. It slightly modified its shape, and eradicated all images of gods. By the 8th century of our era the ringer no longer gripped the anthropomorphized form of a deity, but a shaft with claws, the abstract symbol of The Power of the Thunderbolt. With this properly used the ringer could compel the gods; improperly used it could destroy the ringer. Its sound was no longer only the accompaniment to prayer. In places it was prayer.

Illustration 9

This form of handbell was born in the heights of the Himalayas, in tiny foundries in dimly lit cells off the courts of temples. It is still so made, by only a man and his wife. But it is not cast as trivia. First the man must purify himself, and next, make the mold. Then, when the stars are all in their proper places according to astrological divination he fills it, praying wildly as he pours the metal. When it has had time to cool it is the wife, perhaps with a child in her arms as in our illustration who breaks open the womb-like mold and sets the bell forth as a new creation.

Illustration 10

Illustration 11

Such bells were carried by holy men across central Asia, down into China, and out into eastern Siberia, in their work of Buddhist evangelization (see illustration). One or two have even been found in Alaska. In Japan this bell was not only adopted for temple worship, but was the origin of a variant, the *ain*, in which the bell was smaller, the handle longer, and a colored tassel and a swastika sun-symbol added. This bell finds its place in *goeka* music, which is roughly the Buddhist counterpart of plainsong, rendered by group singers including the laity.

Illustration 12

Each singer has an *ain* and a *shoko,* a small metal disk struck with a light hammer; and as they sing they sound them alternately to give a 'sparkle' to a unison vocal line.

This use of handbells is rhythmic, not melodic, and the bells are all of an indefinite high pitch, like our smallest orchestral triangles. *Goeka* music is very old, and originally was performed only twice a year, but now it is rendered, sung by men and women together on many occasions. Today it is highly organized, there being several thousand *goeka* societies in Japan; and these, like the A.G.E.H.R., hold regular regional and national conventions. *Goeka* music, however, has always retained its religious nature, both in regard to the repertory performed and to the care for the bells. They are made in conformity with religious rites, and are never sold on the open market.

The handbell is not only remarkable for the antiquity of its origin, but for the universality of its use. When European explorers of a few centuries ago found it in Africa they thought at first that it had been brought from Europe, and was a sign that other explorers had recently preceded them. Now we know better. The handbell is indigenous to many regions there, both for magic and for music. We have long known of the high development of percussion music by Africans; but it is only recently that we have appreciated the African's melodic use of the handbell. African handbells are of various shapes and sizes. The commonest form is rather flat, recalling the very early Chinese shape, but without the concave 'fishmouth' rim.

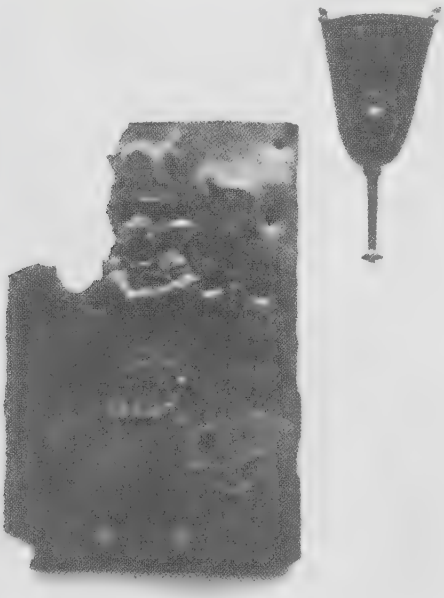


Illustration 13

Some African handbells are in clusters of two to six or more, so that different notes can be tapped out on them. In some places they are used in orchestras, and accompany songs and dances.

In Egypt, the corner of Africa where influences from Asia, Africa, and Europe meet, the handbell was rung in temple rites in the 8th century B.C., or three centuries before Chinese Confucianism adopted it. Ancient Egyptian handbells were small, but good castings. And just as Hinduism chose to form the handle into a statuette of the god whose aid the bell invoked, so it was in some Egyptian cults. Then, as new religious influences spread, the old symbols were dropped as distasteful. However, the bell was seldom left plain. Gods and animals gave way to flowers, flowers to lines, and eventually, in Coptic bells, to the Christian cross.

Illustration 14

Tradition tells us that the bell was introduced into the Christian religion in Africa, and that this was in the form of a handbell. Just as wandering Buddhist teachers carried handbells, so did the early Christian desert fathers. One of these fathers was St. Anthony, who lived in Egypt from A.D. 250 to 350, and founded Christian asceticism. Dwelling as a hermit, he was constantly tempted and tormented by demons; but the power in the sound of his handbell as an instrument dedicated to serve Christ was strong enough to ward them off.

Illustration 15

The picture of St. Anthony, painted a thousand years after he lived, shows his association with the handbell. The Christian missionaries of western Europe rang handbells wherever they went. They were the first church bells long before there were tower bells on churches. Several of these very early missionaries' handbells still exist. One, which can be traced back to St. Patrick, is in the National Museum of Ireland, in Dublin.

The association of the handbell with St. Anthony was not missed by composers of music. We find it, for example, in a motet to St. Anthony by the 15th century Flemish composer, Busnois. He introduces it for touches of realism, and he indicates just where it is to be rung by placing Latin words around a handbell with a Tau or Egyptian cross for a handle drawn on his score.

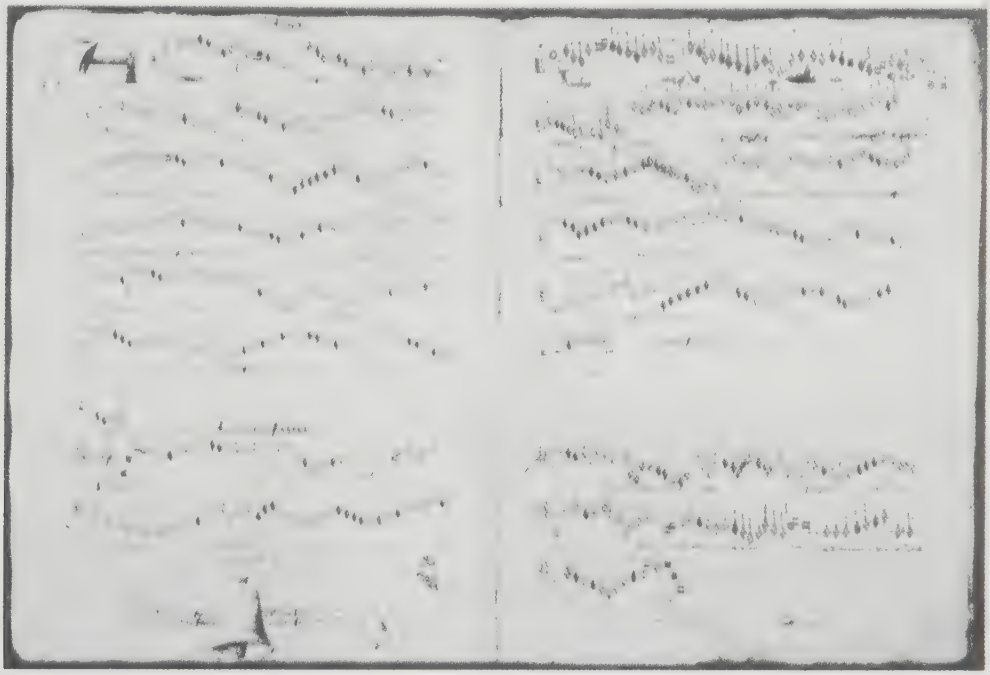


Illustration 16

We have no doubt that handbells were in use in Busnois' time. They are illustrated in manuscripts made both then and during several centuries previous to him. We find them in the border decorations of both sacred and secular works: decorations which were not required to have any closer relationship to the text than is the pattern of a wallpaper to events that occur in a room. Thus in a 13th century Psalm Book we find a girl in hobby-horse costume ringing two handbells while a boy plays a fife and a drum.

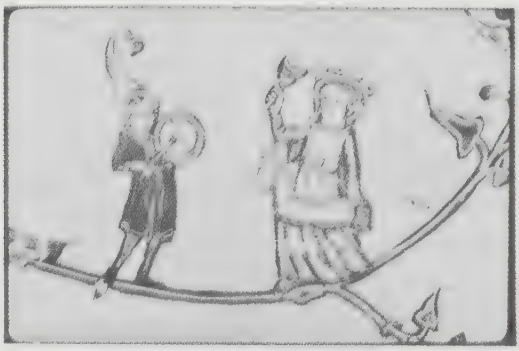


Illustration 17

In a 14th century manuscript of *The Romance of Alexander* the use is much more fanciful. Two hybrid creatures, half human, half dragonfly, face each other; and while the female one is pouring something into a vessel the male one is sounding a trumpet and ringing a handbell.

Illustration 18

A 14th century Psalm Book is decorated with a grotesque showing us the wrist movement for ringing two handbells, one in each hand.

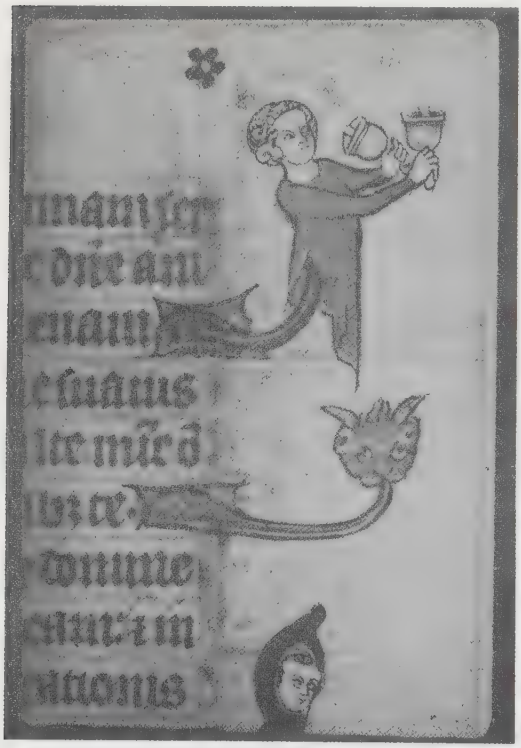


Illustration 19

Of course the handbell had many practical uses in the Middle Ages. Town criers used it for calling people to hear news. Thus it is not inappropriate for a book on sermonizing to show a man running and ringing two handbells as if to call people to hear the 'good news', while two perched owls, symbols of wisdom, suggest the prudence of stopping to listen.

Illustration 20

Another use was to ring when a person was dying, so that people would pray for the departing soul as it was entering the world beyond. This, called the 'passing bell', was sometimes rung on handbells. A 15th century Book of Hours from France illustrates it by showing a sad-faced priest walking through the streets with two handbells jiggling at his sides. We can imagine their sound to be much more subdued than that of the bells in the previous illustration.

Illustration 21

Ringing in the presence of death had a far more ancient origin than just to call for prayers. The belief that the sound of a bell had metaphysical power, which we found in ancient China, has been universal, and is not entirely discarded to this day even in some Christian circles. In earlier times handbells were rung beside a coffin as it was taken for burial, so as to keep evil spirits from entering the body. Chaucer refers to this in 'The Pardoner's Tale' (lines 336-7) in Canterbury Tales:

> And as they satte they herde a belle clinke
> Biforn a cors, was caried to his grave.

We see it depicted in the Bayeux Tapestry, where two boys, walking under (more likely it was on either side of) the bier of Edward the Confessor king of the English, ring handbells as his body is being transported to Westminster Abbey for burial in 1066.

BELLS

rung for sorrow

rung for joy

In contrast to these lugubrious ringings, the handbell's notes signalled occasions for general rejoicing. This gave it a special significance at Christmas, and a use which later, after carol singing developed, encouraged the exploration of purely musical uses of handbells. Their association with Christmas is seen in the border around a picture of the shepherds in the field in an early 15th century Book of Hours. Half lost in an amazing proliferation of flowers, a grotesque—a man growing out of a flower—rings three handbells. (see illustration, upper right corner).

This depiction is significant not only as an early illustration of the ringing of two bells with one hand, but as the earliest one I know showing handles resembling our modern loop strap-handles.

Illustration 23

Illustration 24

The association of handbells with the Nativity was carried a step farther when the 15th century Flemish artist, Geertgeen of St. Jans, depicted them in the hands of the infant Christ. In his oil painting, The Adoration of the Virgin, he placed the Mother and Child, ringed by angels, in a burst of golden light against an almost black background; and in each of the infant's hands he painted a small handbell. These were of a type used in ancient nunneries and seldom seen now, with prongs to catch the clapper should it work loose in overzealous ringing. At the bottom center, in the obscurity, Geertgeen also painted a hovering angel ringing two handbells downwards, as if to alert earth below to the happy event.

The desire to ring more bells than the hands could hold when accompanying one's own singing, reciting, or dancing, led to putting them on a portable frame where they could be jingled softly or shaken loudly in effects of rhythm and tone color. We see such a frame held by a troper or singer of vocal embellishments in a 12th century score of liturgical music.

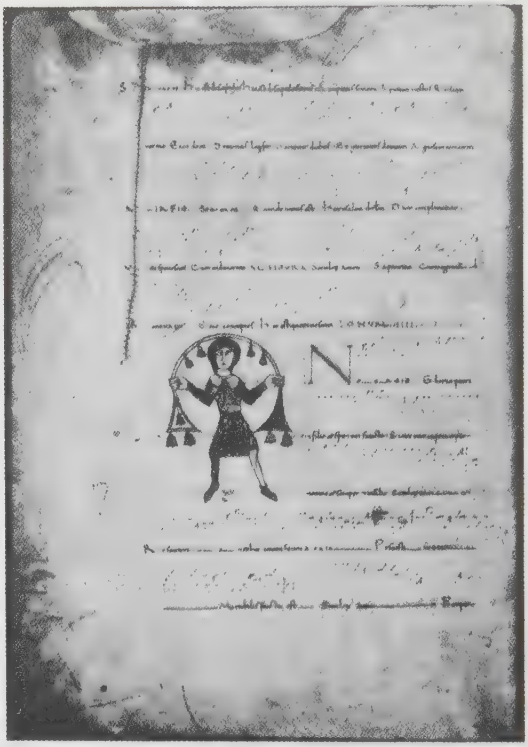


Illustration 25

Such bells need not form a melodic scale; and so for the origin of sets of small bells arranged in melodic series — the forerunners of our four- or five-octave handbell sets of today — we must look elsewhere. We do not find it in liturgical uses, or in the bell's ability to ward off demons or stimulate praise to highest heaven, but in its usefulness in demonstrating the musical theories of the ancient Greeks.

Pythagoras, who lived in the 6th century B.C., noted the dependence of musical intervals on arithmetical ratios of lengths of string at the same tension, such as 2:1 for octaves, and 3:2 for fifths. The handbell was the most reliable medium to check this with because it was very durable, it retained its pitch permanently, and it could be carried.

Illustration 26

We do not need to believe that Pythagoras had handbells like ours just because he is portrayed as a vigorous old man holding two handbells in one hand, and two hammers for striking them in the other, in a 14th century copy of the works of Cassiodorus, a scholar of the 6th century A.D. who referred to him.

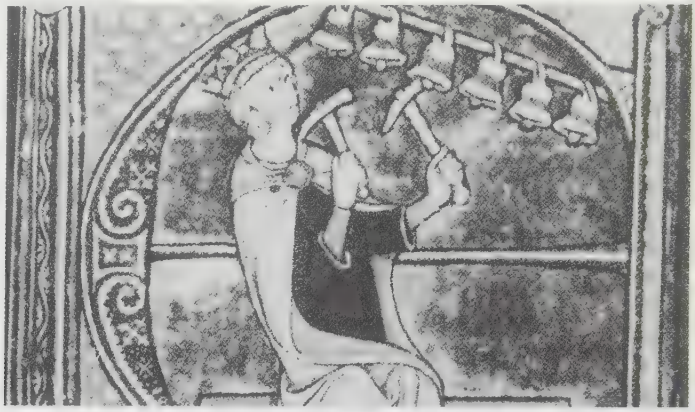
It is appropriate that alongside this there is shown a woman seated at a wheel of twelve bells, with a psaltry on her lap and an organ nearby, as if all three instruments had a common reason for being there. The reason is that out of Pythagoras' relationship of fifths was built up the medieval scale of twelve notes to the octave; and the twelve bells on the wheel, if tuned to these notes, could be used as pitch standards for tuning both stringed and wind instruments.

Illustration 27

The bells which a troper used might be attached to a portable frame, but those which gave the correct pitch to a choir before it started its procession through the church were best kept in a side room, fastened permanently on a horizontal bar. In a 15th century secular work we see pictured what might be a choirmaster using these to rehearse a trio in a vestry while a churchwarden tiptoes through with scales for weighing the collection. Here there are only three bells on the bar; usually there are more: six, eight, or twelve. In medieval texts the instrument is referred to as a *cymbala*, simply the Latin word for 'bells'.

Since bells were considered the best instruments for expressing praise, and because King David was a musician who frequently praised the Lord, it was concluded that David played not only the harp but also the *cymbala*. This is repeatedly shown in medieval manuscripts at the initial letter E of "Exutate Deo", the Latin for "Praise the Lord", the beginning of the 80th Psalm in the Vulgate Bible and the 81st in the King James version.

In a 13th century Franciscan manuscript David is shown playing on six bells with heavy mallets. As in most of the medieval depictions the bells appear to be handbells with their handles removed, for they also have clappers inside.



We mentioned that about 1700 years before the *cymbala* appeared in Europe, the Chinese had an instrument called the *pien-chung* which evolved from hanging handbells stationary and striking them with mallets. In Europe there was also the counterpart of the Chinese and Korean musical ensemble with bells. We find evidence of this as early as the 11th century in an illustration in a Burgundian Bible. To the right an organ is being played, in the center a viol and a wind instrument are being sounded, and to the left a set of bells is being struck. There is no doubt that the musicians are playing together.

Illustration 29

A 12th-century bishop of Augsburg has given us in a few words a description of the musical effect of such an ensemble along with voices:

In this direction a choir of men's deep voices,
And in the other, one of boys' high voices, praise God.
The organ sounds together with flutes, cithars, and lutes,
And the set of bells, on being struck, rings out.

The illustration in the Burgundian Bible shows stick-like keys on the organ, with their letter names above them. Once keys had been applied to the row of whistles which constituted the primitive organ, it was natural to apply them to the row of handbells constituting the *cymbala*. The keys or levers were pushed down by the full hand, as seen in a 13th century Spanish illustration.

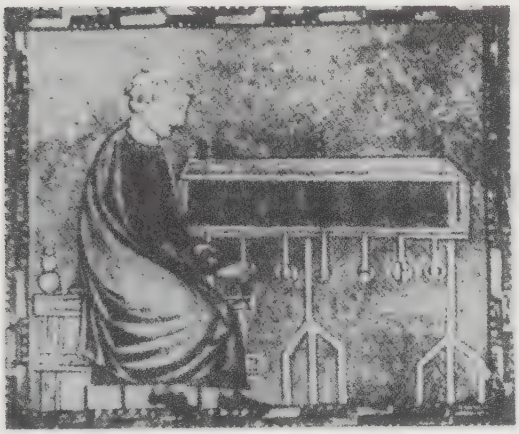


Illustration 30

This type of ringing required a different technique than either swinging the bell with the hand or tapping it with a mallet. Eventually it brought into use larger and heavier bells than could be put on a light frame; and this meant transferring the instrument to the tower, where in England, with the passage of time, the Latin word *cymbala* became corrupted to 'chimes'. In the Low Countries, the arts of both bellfounder and instrument-maker were so developed by the 17th century that the *Cymbala's* range was extended to two or three octaves, complete with semitones; and these 25 to 35 tower bells were made playable from a more refined design of key, although one necessarily for full-hand touch because of the heavy clappers. By a seeming contradiction this instrument acquired the Latin name for only four bells: *quadrillo,* modified in French to 'carillon'.

The handbell also gave rise to a third tower instrument, the swinging peal. We have already spoken of the use of handbells by such early Christianizers as St. Patrick in order to attract listeners. When permanent congregations were formed and small churches and religious houses erected, the handbell was first rung at a doorway or a window opening to call the faithful in to services. Later it was fastened either in a window opening or over a door. Although this made it a fixture, in Western churches it was still swung like a portable handbell. In the window it was pivoted at the center and twirled; over the door it was fastened at one end of a pivoted stick or rod and rocked by pulling a rope, as we see in an amusing border decoration on a page in a 12th century Psalm Book.

Illustration 31

When cities grew up around churches, and towers were extended above the rooftops, they were filled with enormous bells that could be heard for miles, not only to call to service but "to sound thunderous praise unto the Lord." Nevertheless, the concept of ringing them like swinging a handbell was still retained in western Europe.

Large bells

are mounted in towers

Illustration 32

In Spain and Portugal, and later in Latin America, bells of several tons fastened high in bellchamber openings, were rotated in the same manner as the little bells in ground floor windows had been.

In more northern towers large bells were swung to and fro in the way that handbells on sticks had been rung previously, except that wheels gradually replaced the leverage arms, and steel frames the great oak ones that were necessary to withstand the sidethrust in this type of ringing.

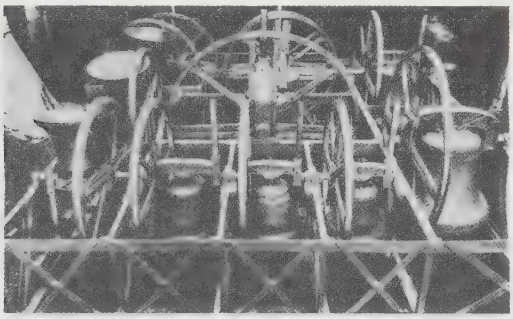


Illustration 33

We should add that this is not the traditional way of ringing in Eastern churches, and that Orthodox people look upon our method with dismay. Why move the bell to hit the clapper, they ask, when it is so much easier to move the clapper to hit the bell? They leave their bells stationary.

In England the Puritan regime of 1649-1660 had a profound effect on both tower-bell and handbell ringing. It forbade tower-bell ringing except one bell to call to church on Sunday. It threw all handbells out of the church except one in the pulpit for the preacher to ring if anyone fell asleep during his sermon. It allowed the town crier's and the night watchman's handbells. The poet Milton, writing at the time, must have found the sound of the latter agreeable in the small hours of the night, for he refers to it in Il Penseroso:

. . . the bellman's drowsy charm
To bless the doors from nightly harm.

Handel set these words to music with a suggestion of handbell sounds in the accompaniment.

After the Puritan regime ended, the limitations on bell ringing in England were lifted. One could ring as many bells in the tower as one liked, when one liked, and, within reason, for as long as one liked. Formerly there had scarcely been more than five bells in a tower, and these were not necessarily all musically related, for some had been installed at different times than others, and for different purposes. Now there was a movement to increase their number to six, eight, or even ten in the larger churches, to have them sound a diatonic scale, and to ring them all together. On the continent this was done by allowing each bell to swing at its natural pendular speed, so that little bells (short pendulums) rang fast, and large bells (long pendulums) rang slowly. In England a device to hold the bell after it rang once was installed on each bell, so that each could be made to wait until all the others had sounded before it was sounded again. And then, the order of sounding need not be the same as before.

This ringing in changing orders of notes, which on six or more bells can be kept up for a long time without repeating an order, is called 'change ringing', and standard practices of changing the orders are called 'methods'. Change ringing became a popular pastime in England in the latter part of the seventeenth century, bringing men of different social stations together to pull ropes in a secluded room in a tower as a team heard but not seen over town or countryside.

Illustration 34

As change ringing spread, the ringers of one tower challenged those of another to the perfect completion of a method or two that took hours to perform. Since the necessary practice for this on tower bells was sometimes a little trying on even the most loyal neighborhoods, it was often partly done on handbells.

Frederick Sharpe, the celebrated English campanologist who was guest speaker at the A.G.E.H.R. convention in Detroit in 1963, told me that the handbells of that time had wooden handles, and no springs to hold the clapper off the bell and no leather pegs in the ball of the handle. Instead of this last, they had wooden pegs in the clapper, and these had been introduced around 1660 by William and Robert Cor, two brothers who made handbells at that time. He showed me two of their bells in his possession.

Some of this handbell practice took place in homes, and when Handel came to reside in England in 1714 he found it in some of the first homes he was invited into. This as well as ringing in towers caused him to dub Britain, "the ringing isle." We don't know whether he meant it as a compliment or not.

When the handbells were not in use they were usually kept locked in the church tower. Nevertheless, they were so enticing an instrument that people of the neighborhood, not necessarily tower-bell ringers, tried other successions of notes on them. The first pieces were such hymn and folk-song melodies as were possible on the limited number of bells. Many of the favorite tunes required more notes, and so bells were added — to some sets a few, to others many — both to extend the range and to provide the most essential chromatic notes.

Eventually virtually every village in England had its band of handbell ringers. Throughout much of the year the town might not be very aware of them, for they played most of the time without an audience, simply, like an old-fashioned chamber trio, for the musico-social pleasure of ringing together. Then, as if handbells had been invented for Christmas, they would come out in public at this season and ring carols in the street and along country roads. Their reward was invitations into homes for mulled beer and mince pies. Apart from this there would be occasional playing at parties in the great houses, and at May Day festivities, which were revived right after the Puritan regime. In addition, they gave a kind of esoteric performance that linked them with the primeval use of handbells: ringing over the grave of a deceased ringer.

Many of the ringers were untutored musically, and could not read staff notation. But they knew the numbers by which they called their bells for changes: '1' for the highest pitched, '2' for the next, and so on down in the reverse order of the letter names for notes. Consequently they adopted this system with some additional indications for notating their handbell music apart from changes, and it has prevailed in Great Britain to this day.

Unfortunately for the development of handbell music in England there were some influential tower-bell ringers who considered the use of handbells other than for change ringing a waste of time. These were men with that gift which enabled them to hear and delight in sounds coming out of a tower not as music, but as progressions of mathematical combinations and permutations. They felt that the people on the continent of Europe knew very little about bells because they rang their church bells "any old way;" and that bells — either in the tower or in the hand — were worthy only to ring changes, God having intended other instruments for music.

This attitude would have kept handbell music in Britain from developing as an independent art if there had not also been men with the opposite point of view. They saw to it that, athough their teams used their handbells to learn their notes on the tower instrument, they also spent some time learning music for handbells on them. This permitted more complicated music, and so from handling one to four bells per ringer held in the hands, they learned to manage eight or ten per ringer laid out on a long table before all of them. This was developed particularly in the north of England, where local audiences responded with delight to hearing music other than changes in the rich golden tones of handbells.

Encouraged by this, they started giving performances farther and farther from home, especially after travel was facilitated by the advent of railways. Bands which received high acclaim on tours studied repertory which would attract a larger public for future ones. They became recognized as concert artists, which meant that they were material for the booking agent. Many bands of merit were ignored, and others calculated to be of box office value were exploited.

You know the history from here on. Scott Parry in his excellent book, *The History of Handbells,* tells how the art became planted in North America. From about 1840 bands of such ringers, some of them immigrant families, toured up and down the eastern United States, entertaining chiefly at churches, dining halls, and fairs. Alfred Storey, Director of Extension Services of The University of Michigan and the descendant of such a family, has told me of their experiences. They were English, and travelled throughout Pennsylvania and New York from about 1885 to 1915.

Illustration 35

While they did not have to pretend to be Swiss like the ringers Barnum imported, or even dress in exotic costumes, they did have to present society with music in an unfamiliar medium, and therefore take the risk of its being rejected because there were no accepted standards for criticism. They guarded against this by always playing some changes, which no critic could find fault with, and also vocal numbers, music on other instruments, and combinations of these with handbells.

No matter how excellent the musicians, such itinerant careers gradually ended with the closing of Chautauqua circuits on the one hand and vaudeville houses on the other. Yet even while this was happening, handbell music received an impetus which eventually brought it to its present popularity

and high musical standard. At the turn of the century Dr. Arthur Nichols of Boston and his daughter Margaret, later to be known as Margaret Shurcliff, made trips to England in connection with restoring the bells of the Old North Church to their original use. There they learned change ringing, and returning as enthusiasts made up teams among family and friends to practice it on handbells in their homes. Out of this grew the New England Guild of English Handbell Ringers, established in 1937, and which was to become the first of the twelve areas in our American Guild of English Handbell Ringers, incorporated in 1954. In 1967, handbell ringers in Great Britain founded their association, known as the Handbell Ringers of Great Britain.

These two associations are unique, and I feel honored to be asked to write music for their art. Not being satisfied with the explanation that Barnum's 'Swiss Bellringers' were a myth, I have inquired high and low to find a parallel cult of handbell music in the many countries I have visited on bell research. If there is a rare isolated pocket in Scandinavia, or Japan, it is a colony of the English handbell-ringing art. It all stems from the unique English type of handbell, and for this credit must be given first to the Cor brothers, whose little wooden pegs in the clappers started men thinking of developing handbell ringing for more musical uses than driving evil spirits away with their tone color or praising God with their din.

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Modes of vibration and sound radiation from tuned handbells

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INTRODUCTION

Bells have belonged to every culture. Bronze bells were known to have been cast in China from at least 1000 yrs BC. The art of tuning carillon bells was perfected during the 17th century by Dutch bell founders, especially the Hemony brothers. Carillon bells since then have generally had from five to eight harmonically tuned modes of vibration (see, for example, Lehr[1]; also a bibliography of over 50 papers on bell acoustics is available from the authors).

Handbells also date back at least several centuries BC.,[2] although tuned handbells of the present-day type were developed in England in the 18th century. One early use of handbells was to provide tower bellringers with a convenient means to practice change ringing. In more recent years, handbell choirs have become very popular in schools and churches; some 2000 of them are reported in the USA alone. In spite of their widespread use as musical instruments, however, handbells do not appear to have been the object of much acoustical study.

Handbell choirs customarily employ two to five octaves of chromatically tuned bells. The pitch of each bell is determined by the fundamental, supported by one or more harmonic partials. These plus many inharmonic partials determine the timbre of handbell sound. The dominance of the first two harmonic partials of the radiated sound makes it feasible to play several bells together in chords without a clash between the upper partials, as occurs in carillons.

The purpose of this paper is to describe the modes of vibration of tuned handbells, the way in which they radiate sound, and to discuss the significance of these in determining the timbre or tonal quality. As yet, no calculations of the modal frequencies or of the radiated sound field have been made. Thus a phenomenological description of bell acoustics is here employed for understanding handbell acoustics and improving handbell design.

I. MODES OF VIBRATION

The modes of vibration of bells are described by use of the nomenclature of the normal modes in a circular plate, which they resemble. The $[m,n]$ mode has m meridian nodes extending across the crown (top), and n nodal circles around its periphery. The principal modes are the [20] mode and the [30] mode, which are shown in Fig. 1, along with several untuned modes of higher frequency. Alternate sections of the bell vibrate inward and outward; the nodes are shown as dotted lines. The strike tone of a handbell is determined by the tuning of the fundamental [20] mode, unlike tuned carillon bells and especially chimes, where the strike tone is a combination tone.[3]

The first several modes of vibration have no nodal circles or one nodal circle. In the fundamental [20] mode, the maximum amplitude of vibration occurs near the rim, whereas in the [30] mode it has moved about $\frac{1}{3}$ of the way up the side of the bell, as shown in the time-average holograms of Fig. 2 (a) and (c) (for a des-

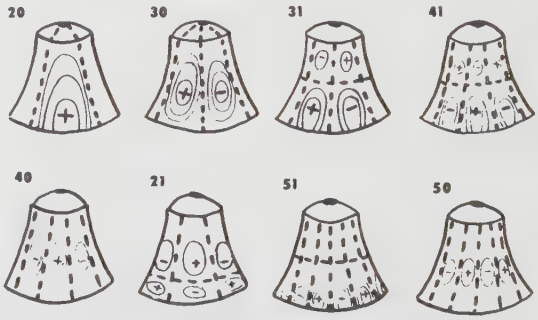

FIG. 1. Modes of vibration of a handbell. Nodes are shown as dashed lines; the mode number gives the number of meridian nodes and circular nodes. Principal modes are the [20] and [30] modes.

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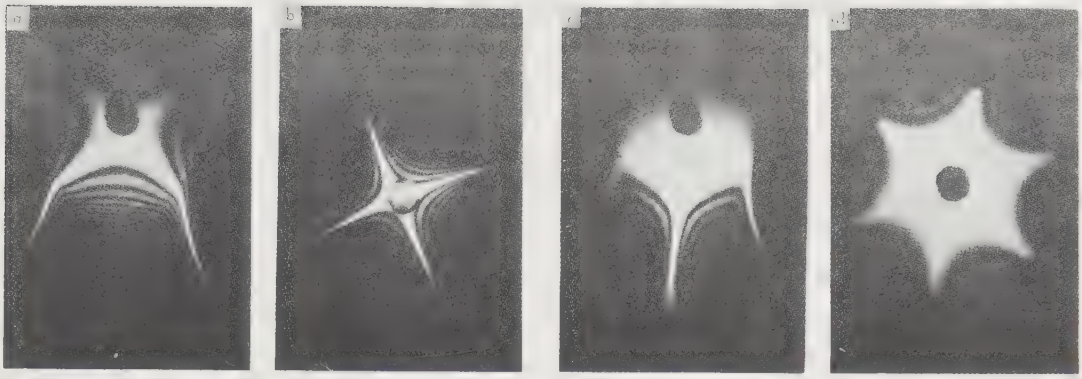

FIG. 2. Time-average hologram interferograms showing the motion of a vibrating handbell. (a) [20] mode; (b) [20] mode, top view; (c) [30] mode; (d) [30] mode, top view. (For a description of time-average holography, see Powell and Stetson.[4])

cription of the technique see Powell and Stetson[4]). Also, the crown of the bell vibrates more in the [20] mode than in the [30] mode, as might be expected.

If the bell had exact radial symmetry, the same sound would result from striking it at any point around its circumference. In practice, this is rarely the case; minor variations in thickness, internal stress, and microstructure cause the mechanical properties of the bell to vary around its circumference. Thus the frequency of vibration may vary with the strike point, which leads to "warble" in the following way:

Consider the bell as having two different fundamental [20] modes, one with frequency f_2 and one with a slightly lower frequency f_1. If the antinodes for those two modes are 45° apart, the two modes can be excited more or less independently, since an antinode of one mode coincides with a node of the other. If the bell is struck at a point between the two antinodes, however, both modes will be excited, and beats will be heard at a frequency $f_2 - f_1$. The resulting amplitude variation is called warble. Warble is one characteristic of bell sounds, and a certain amount of it is considered desirable by most bellringers. The amplitude of the warble may be regulated by selecting the strike point, which is easily done when the clapper is being mounted. Changing the warble frequency, however, requires physical changes, such as thinning the bell at selected locations.

Warble can occur in the [30] mode or higher untuned modes of vibration as well as fundamental. Selecting a strike point which minimizes warble in the [20] mode will not necessarily minimize warble in the [30] mode. Thus a compromise is called for; a strike point is usually selected by listening to the bell as it is struck at various points around its circumference. Control of warble is also an important consideration in the design of church bells.[5]

II. DETERMINATION OF MODAL FREQUENCIES

The frequencies of the modes of vibration can be determined by analyzing the spectrum of the sound emitted when the bell is struck. However, to identify the shapes of the modes of vibration it is necessary to excite each mode individually, so that the nearfield sound can be probed with a microphone. We do this either by attaching a small magnet (moving-magnet drive) or a small coil (moving-coil drive) near the point where the clapper normally strikes the bell. It is important that the mass of the attached coil or magnet be small compared to the mass of the bell (not more than one or two grams), to minimize the perturbation to the mode.

The actual scanning is done with a $\frac{1}{4}$-in.-diam electret microphone kept within 2 or 3 mm of the bell. The output of the microphone preamplifier is displayed both on the vertical axis of an oscilloscope (with the drive current on the horizontal) and on a sound level meter. A nodal line can be located with high precision by observing the phase change on the oscilloscope, as well as the minimum in sound pressure level.

The modal frequencies of some of the modes without circular nodes $[m, 0]$ and with one circular node $[m, 1]$ measured in bells tuned to C_4 and C_6 are shown in Fig. 3. The frequencies of the $[m, 1]$ group appear to follow the empirical relationship

$$f_m = km^{1.9},$$

for large m. For the modes of low m, however, the frequencies actually increase with decreasing m.

The dependence of frequency on mode shape may be compared to that of a cylindrical shell. For a shell with free ends and no circular nodes, the frequencies were found[6] to follow a relationship derived by Rayleigh[7]:

$$f_m = \frac{h}{2\pi a^2}\left(\frac{E}{12\rho(1-\nu^2)}\right)^{1/2}\frac{m(m^2-1)}{(m^2+1)^{1/2}} \qquad (1)$$

$$\approx Km^2, \quad \text{for large } m,$$

where E is Young's modulus, ρ is the density, ν is Poisson's ratio, a is the radius of the shell, and h is its thickness. For a shell with fixed ends (clamped or freely-supported), the frequency initially decreases with m until a turning point is reached, and then increases, eventually approaching the shell with free ends (see Fig. 5 in Ref. 6). The turning point occurs at

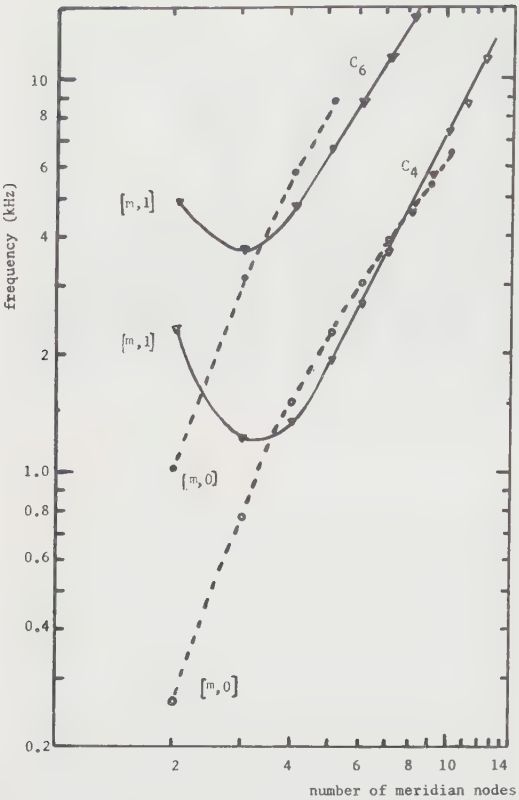

FIG. 3. Frequencies of two groups of vibrational modes in two bells: The $[m,0]$ group without circular nodes and the $[m,1]$ group with one circular node.

large mode number m for a long, thin cylinder but moves to smaller m as the ratio of radius to length decreases.

This behavior was explained by Rayleigh[7] by considering the contributions to strain energy due to stretching and bending. The modes of a free cylinder which have no nodal circles occur without stretching the middle surface of the cylinder; thus they involve only bending energy. A cylinder with fixed ends stretches as it bends, however, the stretching being greatest when the longitudinal mode number m is small. Thus a minimum energy or a turning point occurs at some value of m which depends upon the shape of the cylinder. The stretching energy in a cylindrical shell depends on the thickness h, and the bending energy on h^3. Thus the turning point occurs at a lower number m in a thick shell than in a thin one.

A cylinder with free ends vibrating in a mode with only longitudinal nodes (no nodal circles) involves essentially only bending energy as it changes its shape. However, a free cylinder with one or more nodal circles or a cylinder with fixed ends adds a considerable amount of stretching energy. The stretching decreases proportionately as m increases, resulting in a decrease in frequency with increasing m. Thus the total strain energy due to stretching and bending in a cylindrical shell with fixed ends decreases with m until it reaches a minimum, then increases with m (see Fig. 7 in Ref. 6).

Comparison of Fig. 3 with Figs. 5 and 7 in Ref. 6 indicates that the bending energy dominates in a handbell vibrating in an $[m,0]$ mode with no circular nodes. In the $[m,1]$ modes, which have a nodal circle, stretching energy constitues a substantial part of the total strain energy, as it does in a cylinder with an end plate attached.

III. SOUND RADIATION

As a bell vibrates in one of the modes shown in Fig. 1, it acts as a multipole sound source as illustrated in Fig. 4(a). For the [20] mode there are four regions of positive pressure (two on the inside, two on the outside) alternating with four regions of negative pressure; thus the bell resembles an eight-pole source. For the [30] mode there are six areas of positive pressure and six regions of negative pressure. Furthermore, each antinodal area extends some distance up the bell, either inside or outside, so a calculation of the sound field is not an easy task. Two things can be assumed about the radiated field, however: (1) The sound pressure should be proportional to the amplitude of vibration; and (2) the sound pressure should be a minimum on the axis of the bell. Both of these assumptions are consistent with our experimental observations.

In addition to the direct radiation of sound by the moving surface of the bell, there is another mechanism by which sound is radiated, in this case at twice the frequency of the vibrational mode. The mechanism is illustrated in Fig. 4(b). In the [20] mode, for example, as the mouth of the bell changes from a circle to an ellipse, some air is expelled from the mouth due to a net change in volume. The radiated sound thus includes a

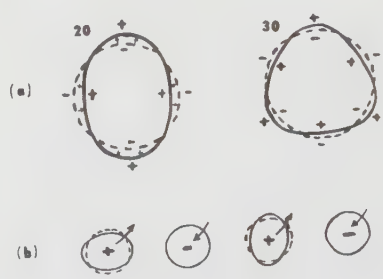

FIG. 4. Sound radiation from a vibrating bell. (a) Multipole radiation at the frequencies of the [20] and [30] modes; regions of positive and negative pressure change are shown. (b) Radiation at twice the frequency of the [20] mode due to net volume change; a similar radiation occurs at twice the frequency of the [30] mode.

second harmonic due to the [20] mode and a sixth harmonic due to the [30] mode (assuming normal tuning). To our knowledge, this mechanism for radiating bell sound has not previously been described in the published literature. It would be difficult to identify such radiation in a tuned carillon bell, because there are modes of vibration with frequencies at or near the second harmonic of each of the lowest modes.

A bell radiating in the manner just described has some resemblence to an acoustic dipole. Since the wavelength is larger than the dimensions of the bell, the sound pressure tends to decrease as $1/R$, where R is the distance from the bell mouth. If the radius of the undisturbed mouth of the bell is r, and if the amplitude of vibration in the [20] mode is δ, the momentary major and minor axes of the ellipse are $2(r+\delta)$ and $2(r-\delta)$, respectively. The change in volume is approximately $\Delta V = h\Delta A/2 = \pi h\delta^2/2$, where the bell height is h. Here we have assumed that the area of the mouth changes by an amount $\Delta A = \pi\delta^2$, but the cross section is unable to change near the crown; thus the volume change can be approximated by multiplying the change in mouth area by half the bell height.

The sound pressure at a distance $R \gg \lambda$ from a dipole source with separation h ($h \ll \lambda$) can be written, $p = (\pi h f \rho U/sR)\cos\phi$, where h is the height of the bell, f is frequency, U is volume velocity, and ρ is the density of air (it is customary to use rms values for p and U). We have estimated the volume velocity U as $\omega\pi h\delta^2/2^{1/2}$, so the sound pressure becomes

$$p = \frac{\rho\omega^3 h^2\delta^2}{8\sqrt{2}R}\cos\phi = \frac{\rho h^2 a^2}{8\sqrt{2}R\omega c}\cos\phi\,, \qquad (2)$$

where c is the speed of sound, and a is the rms acceleration of the vibrating bell. Note that the sound pressure in the farfield given by this expression increases as the square of the amplitude (or acceleration).

Sound pressure levels at one meter from the mouth of a C_5 handbell were measured in an anechoic room, while the handbell was driven at 519 or 1557 Hz. These are the respective modal frequencies of the [20] and [30] modes. The sound pressure levels were measured both at the modal frequencies and at twice (1038 and 3117 Hz, respectively) the modal frequencies. The microphone was put in the direction of maximum radiation, along the axis for 1038 and 3117 Hz, at a right angle to the axis for the modal frequencies 519 and 1557 Hz. The vibration acceleration at the driving point was measured by means of a BBN model 501 accelerometer.

The results are plotted in Fig. 5. The abscissa is the logarithm of rms acceleration divided by 1 $g_n = 9.8$ m/s^2. Lines having a slope of 20 have been fitted to the points for the modal frequencies, 519 and 1557 Hz, respectively; this slope means that sound pressure is proportional to acceleration. Lines having a slope of 40 have been fitted to points for 1038 and 3117 Hz which are twice the modal frequencies; this slope means that the twice-modal-frequency sound pressure is proportional to the square of vibration acceleration, as predicted by Eq. (2).

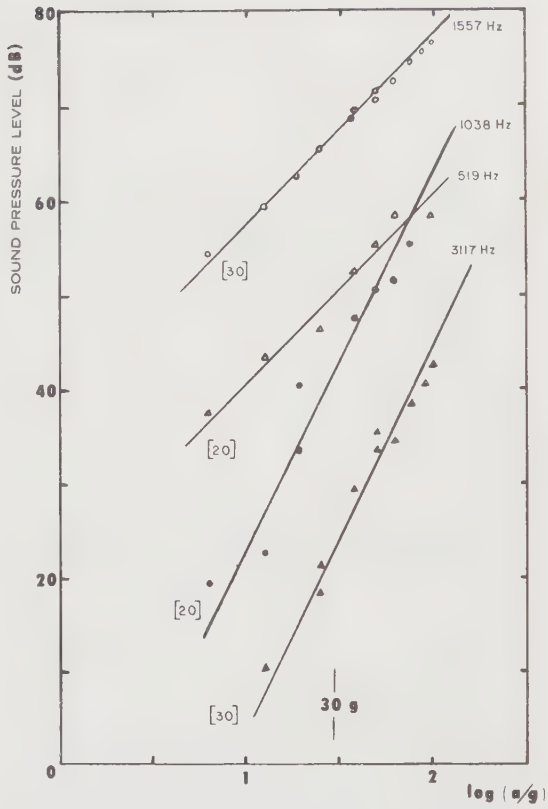

FIG. 5. Dependence of sound pressure level on vibration acceleration for the [20] and [30] modes of a handbell tuned to C_5. The horizontal axis is the logarithm of rms acceleration divided by 1 $g_n = 9.8$ m/s^2.

The pattern of sound radiation from these same two modes was determined by rotating the bell while it was being driven electromagnetically at a single frequency in the anechoic chamber. A $\frac{1}{4}$-in. electret microphone was mounted one meter from the mouth of the handbell; the signal was filtered with either a 1/3-octave or a 1/70-octave bandpass filter. Sound pressure level was recorded as a function of angle on an $X-Y$ recorder. The rms acceleration at the normal strike point was kept constant for each run at 294 m/s^2 = 30 g_n. The combined mass of the drive coil and accelerometer was about 4 g, which lowered a modal frequency by less than 1 %.

The radiation pattern for each harmonic partial is shown in Fig. 6. The radiation at the first and third harmonics, from the [20] and [30] modes, has minima along the axis of the bell (0 and 180°) and maxima at right angles to this, as expected. The radiation at the second and sixth harmonics shows small peaks at 0 and 180°, in the manner of a dipole source. [The small

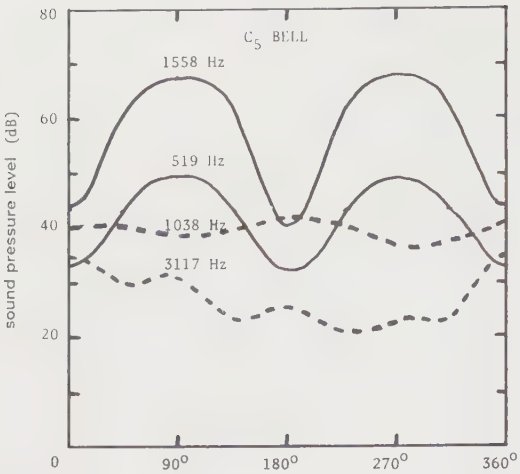

FIG. 6. Angular dependence of sound pressure level at 1 m from a C_5 handbell. Radiation at 519 and 1557 Hz shows a minimum along the axis of the bell ($\theta = 0, 180°$); radiation at 1038 and 3117 Hz shows a maximum along the axis.

peaks in the sixth harmonic (3117-Hz) radiation at 90° and 270° are due to leakage of the much larger 1558-Hz signal through the filter.]

IV. MODE TUNING AND SOUND SPECTRA

The frequencies of the [20] and [30] modes in a large number of bells were measured by means of a Conn Strobotuner. Included were sets of bells tuned by bell crafters in three different countries: Malmark and Schulmerich in the USA, Whitehapel and Shaw & Son in England, and Petit and Fritsen in the Netherlands. (The Shaw bells, crafted in 1890, have been recently retuned at the Whitechapel foundry in England.) In the Petit and Fritsen bells, the [30] mode was found to vibrate at a frequency from 2.2 to 2.8 times the [20] mode (fundamental); apparently the [30] modes are not individually tuned. In the Shaw bells, the [30] mode frequency is close to three times the [20] mode frequency up to G_5 (784 Hz), gradually decreasing to twice the frequency in the G_7 bell, the highest in the set. In the Malmark, Schulmerich, and Whitechapel bells, the [30] mode is tuned to three times the frequency of the [20] mode throughout the entire set. Ratios of the modal frequencies for some of the bells are given in Table I.

For comparison, the [30] mode of a tuned carillon bell has 2.4 times the frequency of the [20] mode (the hum tone). This is apparently the design followed in the Petit and Fritsen bells.

Sound spectra for many of the bells were recorded by means of a Honeywell SA51B constant bandwidth, real-time spectrum analyzer. The spectra of several Malmark bells are shown in Fig. 7. In these bells, three choices of clapper hardness are offered by turning the clapper on its stem. The soft clapper can be seen to emphasize the fundamental, while the hard clapper enhances the twelfth (the [30] mode), as well as several other modes.

TABLE I. Frequency ratio of two lowest modes in handbells of different manufacturers.[a]

Note	M	W	S	PF	SS	Note	M	W	S	PF	SS
G_2	3.0					C_5	3.0	3.0	3.0	2.4	3.1
•						D_5	3.0	3.0	3.0	2.8	3.0
•						E_5	3.0	3.0	3.0	2.7	3.1
•						F_5	3.0	3.0	3.0	2.3	3.1
C_3	3.0	3.0				G_5	3.0	3.0	3.0	2.4	3.1
•						A_5	3.0	3.0	3.0	2.3	2.8
•						B_5	3.0	3.0	3.0	2.4	2.9
•						C_6	3.0	3.0	3.0	2.4	2.9
G_3	3.0	3.0			3.0	•					
A_3	3.0	3.0			3.0	•					
B_3	3.0	3.0			3.0	•					
C_4	3.0	3.0	3.0	2.7	3.0	G_6	3.0	3.0	3.0	2.3	2.6
D_4	3.0	3.0	3.0	2.4	3.0	•					
E_4	3.0	3.0	3.0	2.6	3.0	•					
F_4	3.0	3.0	3.0	2.7	3.0	•					
G_4	3.0	3.0	3.0	2.6	3.0	C_7	3.0	3.0	3.0	2.2	2.5
A_4	3.0	3.0	3.0	2.8	3.1	•					
B_4	3.0	3.0	3.0	2.5	3.0	•					
						•					
						G_7	3.0	3.0			2.0
						•					
						•					
						•					
						C_8	3.0	3.0			
						•					
						•					
						•					
						C_9		3.0			

[a] M - Malmark (USA). W - Whitechapel (UK). S - Schulmerich (USA). PF - Petit and Fritsen (Netherlands). SS - Shaw and Son (UK).

Further evidence of the difference in timbre with hard and soft clappers is shown in Fig. 8 by the solid and dashed lines, respectively. The relative strengths of the first three harmonic partials are shown along the axis and at right angles to it. Each sound pressure level is the average of three strikes measured one meter from the handbell in an anechoic room.

V. SOUND DECAY

Decay of sound pressure level at one meter from the mouth of the handbell was recorded for each mode, by use of a microphone, a 1/70-octave bandpass filter, and a graphic level recorder. Since the 60-dB decay times of the handbells are long compared with the reverberation time of the laboratory, it was not necessary to make these measurements in the anechoic room. Care was taken to keep the bell at a constant distance from the microphone during decay and also to keep its orientation fixed.

Decay times for the first three partials in the sounds of G_4 and C_6 handbells are shown in Fig. 9(a) and Fig. 9(b), respectively. (T_{60} is the time that would be required

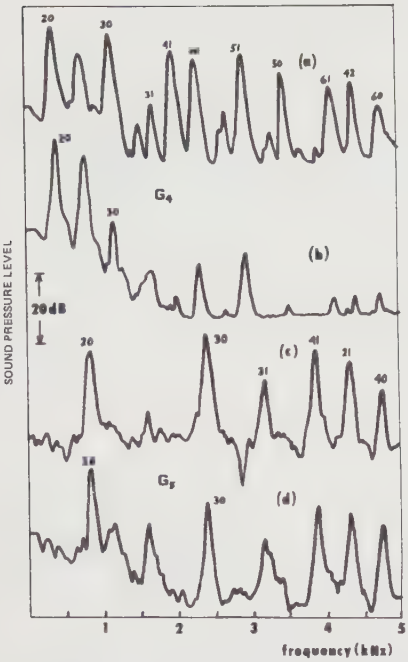

FIG. 7. Spectra of three handbells struck with "hard" and "soft" clappers in a moderately reverberant room. (a) G_4 bell, hard clapper; (b) G_4 bell, soft clapper; (c) G_5 bell, hard clapper; (d) G_5 bell, soft clapper. The harder clapper excites the [30] and higher modes of vibration; the softer clapper enhances the [20] mode (the fundamental).

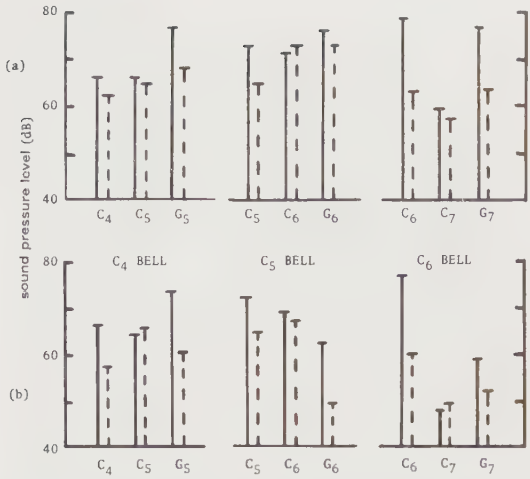

FIG. 8. Comparison of the principal partials of handbell sound radiated along the bell axis (a) and at right angles to the axis (b) with "hard" (—) and "soft" (---) clappers, respectively. Sound pressure levels were recorded 1 m from each handbell, in an anechoic room.

for the sound pressure level to fall 60 dB.) Two features are apparent. First of all, the second harmonic partial decays at twice the rate of the first harmonic. This is to be expected, of course, since they both come from the [20] mode, and the sound pressures are quadratic and linear functions, respectively, of the amplitude of vibration. The third harmonic, which is radiated by

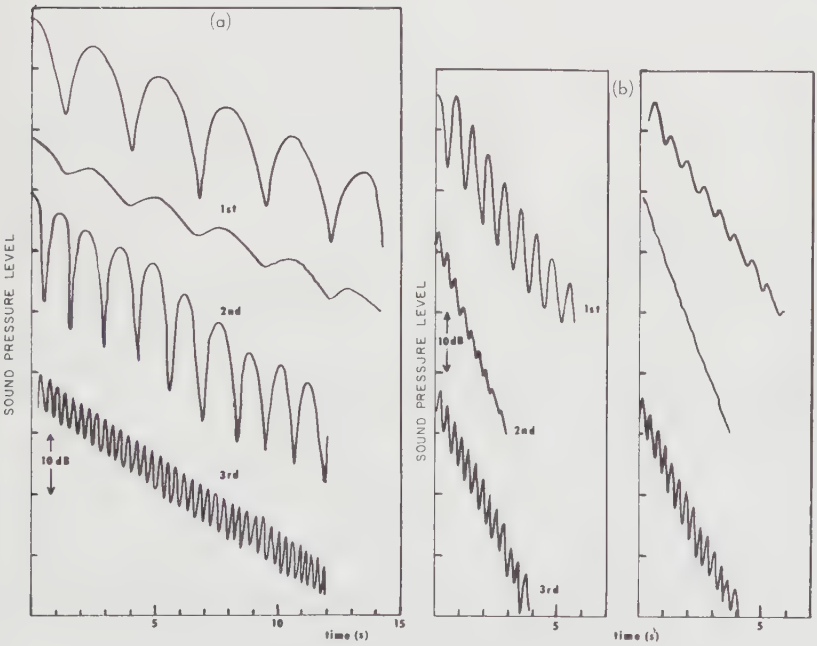

FIG. 9. Sound decay from two handbells. First and second harmonics are radiated by the [20] mode, third harmonic by the [30] mode. (a) G_4 bell in positions which give maximum and minimum warble in the fundamental; decay times are 30, 15, and 22 s for the three harmonics. (b) C_6 in two different positions; decay times are 9.0, 4.4, and 7.4 s.

the [30] mode, decays faster than the fundamental (from the [20] mode) in both of these bells, but that is not the case in all bells. Decay times of the [20] and [30] modes for a two-octave set of bells are given in Table II.

The second feature of interest in Fig. 9 is bell warble. The amplitude of the warble is strongly dependent upon the orientation of the bell with respect to the microphone. The warble frequency, on the other hand, depends only upon the maximum and minimum frequencies for a given mode around the periphery of the bell, as discussed earlier. Any mode can have warble, of course, but it is most noticeable when it occurs in the harmonically-tuned [20] and [30] modes. If there is no warble in the fundamental, there will also be no warble in the second harmonic, since both are generated by the bell vibrating in the [20] mode.

VI. MANUFACTURE AND TUNING OF HANDBELLS

Handbells are nearly always cast of bronze composed of 80% copper and 20% tin. The outside of the casting is first turned to the desired profile, and the inside is then machined on a lathe; the bell craftsman carefully checks the tuning of the [20] and [30] modes as metal is removed. The bell leaves the lathe a fraction of a semitone sharp to allow for frequency change during polishing.

Fortunately for the bell craftsman, the [20] and [30] modes have their maximum amplitudes of vibration at different heights along the bell. The [20] mode can best be tuned by removing metal near the lip of the bell, the [30] mode by removing metal just below the waist.[8] (Removing metal anywhere affects both modes, but these are the places where one mode is affected much more than the other.) Removing metal lowers the frequencies of both modes, which may seem surprising at first glance, but is easily understood by recalling that the bending energy of a plate is proportional to the cube of the thickness, whereas the mass is linear in thickness. Similarly, the frequency of a cylindrical shell vibrating in a pure bending mode is proportional to the thickness, as given in Eq. 1.

Handbell clappers have nonmetallic striking surfaces, unlike church bells, which usually have metal clappers. Harder clappers tend to excite more of the upper partials of the bell, whereas softer clappers produce a softer, darker tone dominated by the fundamental. The height of the strike point is designed to emphasize the fundamental; the location of the strike point around the circumference is selected by the tuner to control the warbel. Clappers are provided with a restraining spring so that the clapper will rebound quickly from the bell after a strike and will not restrike the bell accidentally.

Some difference of opinion apparently exists as to whether a tang, or stem, at the top of the handbell affects its acoustics. Our studies show virtually identical vibration with and without the tang, which is not surprising, since the tang is located at a node for all the vibrational modes of tonal significance. Removing the tang lowers the frequency by a small amount (20 or 30 cents, typically), as does removal of any metal from the crown, but produces no significant difference in the radiated sound pressure level, spectrum, or decay time.

VII. CONCLUDING REMARKS

The principal modes of vibration of a handbell fit into two groups: those with no nodal circles, and those with one nodal circle. The two lowest modes have no nodal circles; they are usually tuned by bellcrafters to have frequencies in the ratio 3:1, although in some sets of bells the ratio may vary from 2:1 to 3:1, depending on the shape of each bell.

Handbells have been shown to radiate sound in a rather complex way. A handbell radiates sound at the frequency of each of its modes of vibration, but, in addition, radiates at twice the frequency of each mode. The sound pressure at each modal frequency is proportional to the amplitude of vibration and has the spatial distribution of a multipole source, but the sound pressure radiated at twice the modal frequency increases with the square of the amplitude and has a spatial distribution resembling that of a dipole source.

The phenomenological description of bell radiation in this paper is given as a first step in understanding the acoustical behavior of handbells. The next step should be an exact calculation of the modes of vibration of bells of various shapes and a theoretical study of their radiation fields.

ACKNOWLEDGMENTS

The authors are grateful to Malmark, Inc. for the loan of a set of handbells, and also the interest in

TABLE II. Decay times (60 dB) of principal modes of vibration.

Note	T_{60} [20] mode	T_{60} [30] mode	Note	T_{60} [20] mode	T_{60} [30] mode
G_4	30 s	22 s	A_5	12.0 s	7.8 s
A_4	34	20	B_5	12.0	7.4
B_4	23	23	C_5	9.0	7.4
C_5	20	20	D_5	6.0	5.5
D_5	21	17	E_5	10.0	4.1
E_5	20	17	F_5	6.6	2.4
F_5	13	12	G_6	6.4	8.2
G_5	20	14			

these studies shown by J.H. Malta, the president. We also thank individuals and churches who allowed us to record their handbells, and especially Richard Peterson, who provided valuable assistance in obtaining the time-average holograms. Our thanks go to Richard Ross and Robert Shepherd who assisted in making some of the measurements in the laboratory. Some of the experiments were carried out while one of the authors (HJS) was on sabbatical leave from Bradley University.

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TUNED HANDBELLS, CHURCH BELLS AND CARILLON BELLS

- Part I -

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INTRODUCTION

Although it might be argued that all bells are musical instruments, I prefer to reserve that term for bells with two or more tuned partials which convey a strong sense of pitch. Tuned handbells, church bells, and carillon bells fulfill these conditions, and certainly deserve to be called musical instruments. The purpose of this paper is to review the acoustics of bells, and to compare and contrast the acoustical properties of handbells with those of church bells and carillon bells found throughout the world.

I was introduced to handbell ringing about 1975, and shortly thereafter I began some studies on the acoustics of handbells, some of which have been reported in this journal and elsewhere.[1-4] A short time later I started to learn change ringing on the historic bells of Old North Church in Boston. Since that time my interest in bells and bell ringing has continued to grow.

HISTORY

Although bells date from antiquity, they developed as musical instruments in the fifteenth and sixteenth centuries when bell founders learned how to tune the partials harmonically. The founders in the Low Countries took the lead in tuning bells, and their many fine bells and carillons became a source of national pride, which they still remain. Bells founded during the seventeenth century by the Hemony brothers and other Dutch founders are highly treasured today.[5,6]

Bell founding in England developed along a slightly different line. Bells were hung so that they could be rung full circle, and rung one by one in sequence. During the eighteenth century change ringing became popular in the bell towers of England. Careful tuning of partials was less important since the bells were rung one at a time. What are sometimes called "old English bells" varied in partial tuning, often having several prominent inharmonic partials.[8] At least one authority of that time criticized the English founders, and challenged them to make their bells the equal of those cast in the Low Countries, which they came to do.[9]

Tuned handbells, as we know them, developed in England during the eighteenth century, primarily to give tower bell ringers a convenient instrument on which to practice. It is during the present century that handbells have come into their own as musical instruments. There are reportedly over 2000 handbell choirs in the U.S.A. alone.[10]

FORM OF BELLS

Cross sections of a typical church bell and handbell are shown in Fig. 1, along with an ancient bell from which they are both descended. The church bell has a heavy sound bow where the metal clapper strikes, whereas a handbell has a more or less uniform thickness throughout. The thinner walls of a handbell make it much lighter in weight, and also smaller than a church bell with the same pitch. The 12th century bell in Fig. 1c has only a slight thickening at the lower lip.

Fig. 1

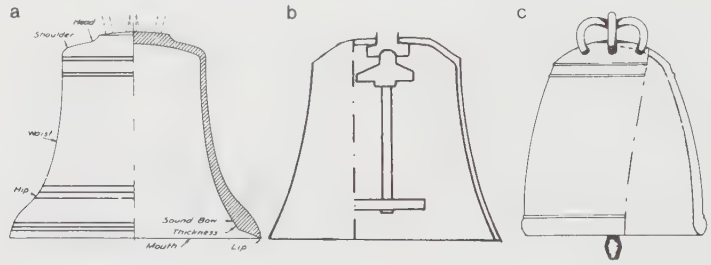

Fig. 1. Cross sections of bells: a) Church bell; b) Handbell; c) Primitive bell (12th century).

Both handbells and church bells are cast of bronze with a composition of approximately 80% copper and 20% tin. The inside of a church bell is turned down slightly on a lathe as the individual partials are tuned, but the outside is left pretty much as cast. Handbells, on the other hand, are usually cast considerably thicker than the finished bell and turned down both inside and out. Rarely are more than two partials tuned.

MODES OF VIBRATION

When struck by its clapper, a bell vibrates in many different modes These modes of vibration are conveniently described by specifying the numbers of nodal meridians and nodal circles. The (2,1) mode, for example, has one nodal circle around its waist and two nodal meridians extending across the crown. Each mode has its own natural frequency and can be excited by vibrating the bell at that frequency.

Six modes of vibration of a handbell are shown in Fig. 2. The dashed lines locate the nodes, or lines of least motion. Regions on opposite sides of the nodal lines move in opposite directions as the bell vibrates; they are denoted by + and − in the figure.[1]

Fig. 2

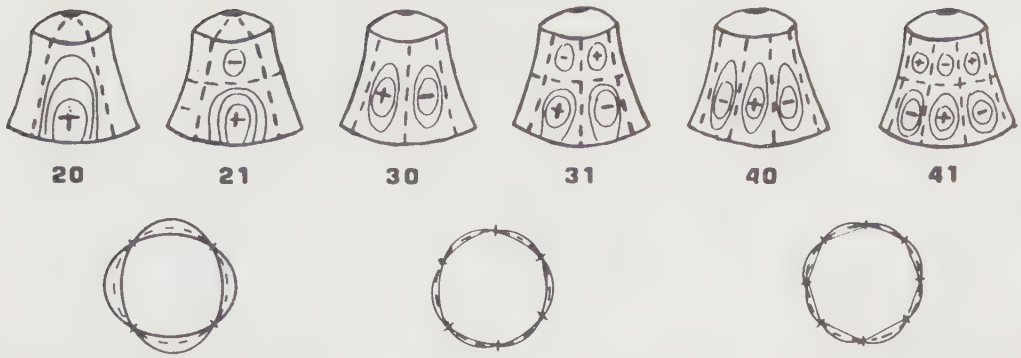

Fig. 2. Modes of vibration of a handbell. Nodes are shown as dashed lines; the mode number gives the number of meridian nodes and circular nodes. The lower diagrams show the motion (exaggerated) as seen looking at the mouth of the bell (from Ref. 1).

In most handbells, the (2,0) and (3,0) modes have a frequency ratio of 3:1. The (2,0) mode radiates a strong fundamental note, and the (3,0) mode radiates a third harmonic, which is a musical twelfth higher in pitch. In addition, the (2,0) mode radiates a second harmonic from the mouth of the bell, and the (3,0) mode radiates a weak sixth harmonic in a similar way.[2] Thus the sound spectrum of a typical tuned handbell features three strong harmonic partials which create a pleasant musical tone with a definite pitch.

The other modes of vibration of a handbell do not usually have natural frequencies that are harmonics of the fundamental, as shown in Table 1, reprinted from an earlier paper in this journal.[1] However, most of these modes die out more rapidly than the harmonic modes.

TABLE 1
Vibration Frequencies of Handbells

	C_4 bell			C_5 bell			C_6 bell		
Mode	Frequency (Hz)	Ratio to [20]	Note name	Frequency (Hz)	Ratio to [20]	Note name	Frequency (Hz)	Ratio to [20]	Note name
20	261	1	C_4	523	1	C_5	1044	1	C_6
30	781	3.0	G_5	1573	3.0	G_6	3170	3.0	G_7
31	1246	4.8	$D_5^{\#}$	2542	4.9	$D_6^{\#}$ +	3679	3.5	$A_7^{\#}$-
41	1353	5.2	F_5-	2817	5.4	F_6	4772	4.6	D_8 +
40	1542	5.9	G_5-	3094	5.9	G_6-	5899	5.7	$F_8^{\#}$
21	2369	9.1	D_7 +	3871	7.4	B_6-	4972	4.8	$D_8^{\#}$
51	1952	7.5	B_5-	3942	7.5	B_6	6647	6.4	$G_8^{\#}$
50	2314	8.9	D_7-	4735	9.1	D_8 +	8903	8.5	$C_9^{\#}$

Some of the same modes are recognizable in Fig. 3, which shows the first eight vibrational modes of a church bell or carillon bell. The modal structure is a little more complicated in a church bell because of its greater thickness and especially because of the sound bow. Note that there are pairs of modes having the same numbers of nodal meridians and circles. The third mode, for example, has a nodal circle about midway between the mouth and the shoulder; we designate that the (3,1) mode as in Fig. 2. But there is another mode with a nodal circle just above the sound bow, which we have labeled the $(3,1^{\#})$ mode, following the notation of Tyzzer.[11]

Note that the second, third, and fourth harmonics of the fundamental are represented in the modes of Fig. 2, as well as modes having frequencies 0.5, 1.2, 1.5, and 2.5 times the fundamental. (Actually they are tuned with respect to the strike tone, but in most well-tuned bells the strike tone and the prime or fundamental coincide.) We will discuss the origin of the strike tone later.

Charnley et al. have described the modes in Fig. 3. as being "ring driven" or "shell driven".[13] In the ring driven modes (2,0), (3,1), (4,1), (5,1) and (6,1), the heavy rim or sound bow vibrates in one of its inextensional ring modes and drives the remainder of the bell. In the shell driven modes $(2,1^{\#})$, $(3,1^{\#})$ and $(4,1^{\#})$, on the other hand, the sound bow is almost at rest, and a node occurs near the rim.

It is interesting to note the behavior of the crown. In our earlier paper [1] we showed hologram interferograms of the crown of a handbell vibrating in the (2,0) and (3,0) modes (the reader should note that the photos in Fig. 2a and 2c in that paper were interchanged by the printer). The crown can be seen to vibrate with greater amplitude in the (2.0) mode. On the other hand, the crown of a heavy church bell is found to vibrate very little in the low-order modes but participates actively when n reaches a certain number.[13] This is a phenomenon that merits further study.

TONAL CHARACTERISTICS

The sound of an English handbell is dominated by the fundamental plus the second and third harmonics, as we have already pointed out. Some handbells do not have the (3,0) mode tuned to the third harmonic of the fundamental; in such bells the (3,0) mode usually

Fig. 3

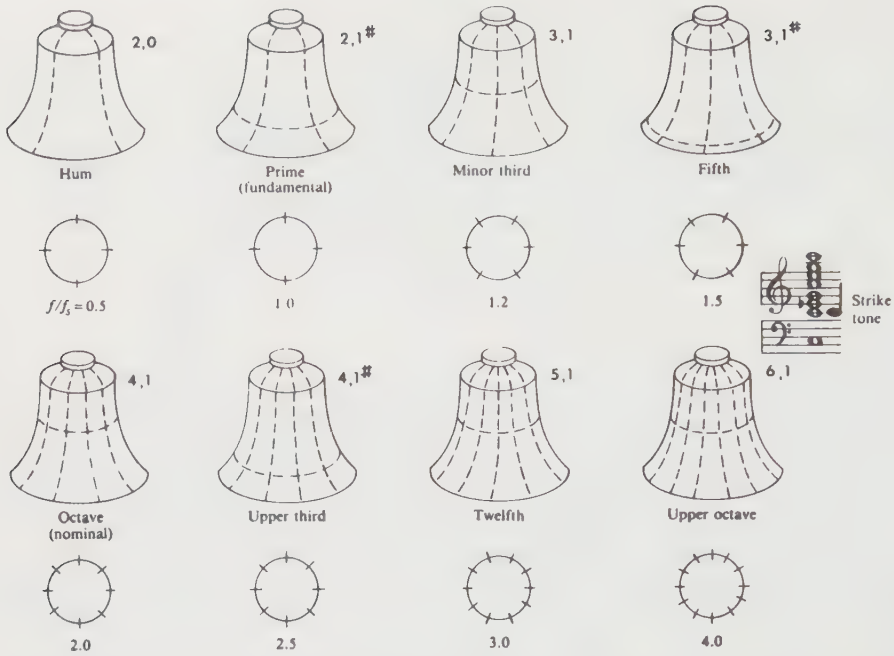

Fig. 3 **The first eight vibrational modes of a tuned bell. Dotted lines indicate the approximate locations of nodes. Frequencies relative to the strike tone are given. The notes that correspond to these in a C_4 bell are shown on a musical staff. In some bells only five modes are tuned harmonically. (From Ref. 12)**

sounds approximately a minor third above the second harmonic, somewhat in the manner of a church bell.[4]

Most church bells or carillon bells have at least five tuned partials, and often all eight modes shown in Fig. 3 are tuned. In some "old English" church bells, the (2,0) mode (hum tone) is tuned a major sixth or minor seventh below the strike tone.[8] Also the prime or fundamental may differ slightly from the strike tone. Obviously, such bells are not well suited for carillons, but they do have a distinctive sound when heard in church towers.

One characteristic component in bell sound is warble. Bell warble results from minor variations in thickness, internal stress, and microstructure around the circumference. Thus the frequency of vibration varies with strike point, and a single mode becomes a *doublet* or pair of modes with slightly different frequencies. The interaction of the sound from those two modes results in beats.[1]

While a certain amount of warble may be desirable, excessive warble is not. Warble is fairly easy to control in handbells by thinning the bell in certain places or by judiciously orienting the clapper.

In church bells, warble control is more complicated, because it may be present in several different partials.[14] Since the nodes of the various modes are in different locations, thinning the bell to suppress warble in one partial may enhance it in another. Making the casting slightly thicker at two opposite points on the bell has been proposed as a satisfactory solution.[15]

Tonal balance is an important consideration in a set of bells covering a wide range of pitch, such as a carillon or a set of handbells.

Tonal balance depends mainly upon the **scaling** of the bells and upon the size and hardness of the clapper. We will discuss bell scaling laws in a later section. At this time it is appropriate to say a few words about clappers.

Carillon bells employ metal clappers with mass ranging from 5% to 20% of the mass of the bell. The bell is always struck at the sound bow, and the timbre or tonal quality depends upon the mass of the clapper and its speed when it strikes the bell (which in turn depends upon its distance of travel). A heavy clapper tends to bring out the lower partials, while a longer stroke tends to bring out higher partials, especially the minor third. A complete discussion of clapper design is given in Chapter 3 of Bigelow's book on carillon design.[5]

Handbells nearly always have soft clappers whose mass is small compared to the bell. The striking surface may be leather, felt or plastic. Some handbells have adjustable clappers, so that the surface hardness can be changed to achieve different timbres. Soft clappers tend to emphasize the fundamental tone, whereas harder clappers bring out the overtones, including the third harmonic.[4]

VIBRATING PLATES AND CHLADNI'S LAW

Nearly 200 years have passed since E.F.F Chladni described his work on vibrating plates, including a description of his well-known method of sprinkling sand on plates to show the nodal lines.[16] Chladni observed that the addition of one nodal circle raises the vibrational frequency of a circular plate by about the same amount as adding two nodal diameters, a relationship that is sometimes referred to as Chladni's law.[17] Thus the frequency of a circular plate can be written as: $f = c(m + 2n)^2$.

We have found that vibrational frequencies in both flat and non-flat circular plates can be fairly well described by a modified version of Chladni's law: $f = c(m + 2n)^k$. In flat plates, k is close to the value of 2, but in cymbals, bells, and gongs, it may vary from about 1.4 to 2.4.

This is not surprising in view of the complicated shape of these musical instruments. In fact we were surprised to find that they showed this much resemblance to flat plates.

Fig. 4

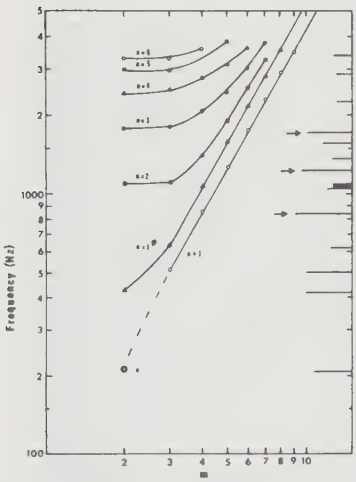

Fig. 4 Vibrational frequencies of a church bell having a strike tone of G4#. The modes of vibration are arranged into families according to the number n of nodal circles. Shown at the right side of the graph are lines denoting the strengths of the prominent partials; arrows denote the three strong partials which determine the strike tone (based on data in Ref. 18).

Fig. 5

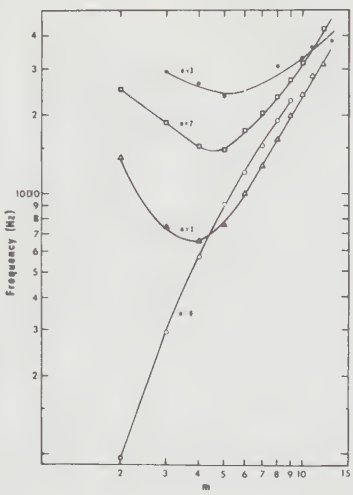

Fig. 5 Vibrational frequencies of a G2 handbell. The number n denotes the number of nodal circles, and m the number of nodal meridians.

Fig. 6a

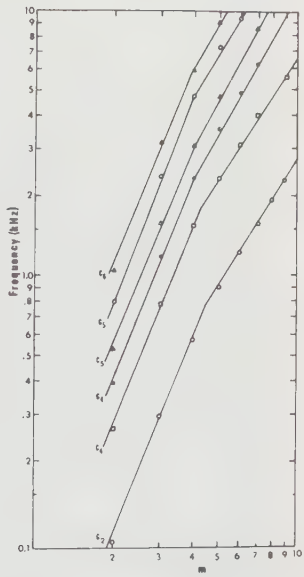

Fig. 6b

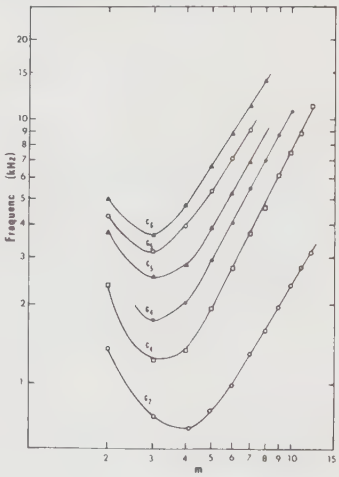

Fig. 6 Modal frequencies in Malmark handbells of various sizes: a) modes with n=0; b) modes with n=1. The n=1 families of modes with n=1 have a minimum frequency near m=3 except in the large G2 bell where the minimum occurs near m=4.

MODAL FREQUENCIES IN BELLS

Fig. 4 shows the vibrational frequencies of a church bell having a mass of 428 kg and a strike tone of $G\#_4$ (415 Hz).[18] Also shown is the sound spectrum of the bell, with arrows denoting the three strong partials which are responsible for creating the subjective strike tone. Note that all the strongest partials, except for the hum and prime, are radiated by the ring-driven modes with n = 1. (The ring-driven (2,0) mode or hum could also be included in that family.)

The modes shown in Fig. 4 appear to follow Chladni's law whenever m exceeds n by two or more. Values of k are in the neighborhood of 1.6 to 1.8 for the various modal families. At about m = 2, the modal frequencies reach a minimum value, and in some bells the modal frequencies increase for smaller m. We will discuss this phenomenon in a future paper.[19]

Fig. 5 shows the vibrational modes of a large handbell tuned to G_2. The fundamental and the principal overtones are radiated by modes with n = 0, and these lie along a line whose slope varies from k = 2.45 at low frequency to k = 1.58 at high frequency. The n = 1, n = 2, and n = 3 families of modes lie on curves that reach minima when m is approximately n + 3. The curve for n = 1 crosses the n = 0 curve and then runs parallel to it. At this time we are unable to offer a physical explanation for this rather curious behavior.

Fig. 6 compares the n = 0 and n = 1 modes in several Malmark handbells. Note that the n = 1 modes all increase in frequency at low m-number, a phenomenon that is due to the energy required to stretch the metal in modes with m < n+2. Since this energy increases with thickness h, whereas the bending energy increases with h^3, the turning point occurs at a lower m-value in thick church bells than in thin handbells.[2]

Six modes of a large G_2 handbell are illustrated by the holographic interferograms in Fig. 7, which were made for us by Professor Richard Peterson at Bethel College. Maximum motion occurs at the "bull's eye" in each segment of the bell defined by the nodes. As a general rule, in handbells this antinode or point of maximum motion moves upward as the number of nodal meridians increases (see Fig. 2 in Ref. 2, for example). However, in this large bell, an interesting thing occurs that we have not seen in smaller handbells: a node forms near the mouth of the bell as m increases. Such nodes are apparent in the (5,0) and (6,0) modes in Figs. 7b and 7c. In fact, it would be quite reasonable to call these the (5,1#) and (6,1#) modes, as in the church bell shown in Fig. 2.

Fig. 7

Fig. 7 Time-average holographic interferograms of six modes in a G2 handbell: a) (4,0) mode at 563 Hz; b) (5,0) mode at 906 Hz; c) (6,0) mode at 1200 Hz; d) (4,1) mode at 650 Hz; e) (6,0) mode at 991 Hz; f) (6,2) mode at 1765 Hz. These holograms were made by R. W. Peterson.

We note another interesting feature of the node near the mouth. It first appears near the m-value at which the (m,0) and (m,1) curves cross over in Fig. 5. That is, the (4,1) mode lies above the (4,0) mode in frequency, but the (5,0) or (5,1#) mode lies above the (5,1) mode. Likewise the (6,0) or (6,1#) mode in Fig. 7c occurs at a higher frequency than the (6,1) mode in Fig. 7e.) This phenomenon is under further investigation.

SCALING OF HANDBELLS

The fundamental frequency of a bell has roughly the same dependence on thickness h and diameter d as a circular plate: foc h/d^2. Thus it is possible to scale a set of bells by making all dimensions proportional to l/f. This scaling approximates that found in many carillons dating from the 15th and 16th centuries.[5,6] However a l/f scaling causes the smaller treble bells to have a rather weak sound, and later bell founders increased the sizes of their treble bells.[5] The average product of frequency times diameter in several fine 17th century Hemony carillons has been found to increase from 100 m/s (in bells of 30 kg or larger) to more than 150 m/s in small treble bells.[6]

The measurements of the bells in three Hemony carillons are shown in Fig. 8. The straight solid lines indicate a l/f scale. Deviations from this scaling in the treble bells are apparent.

Fig. 8

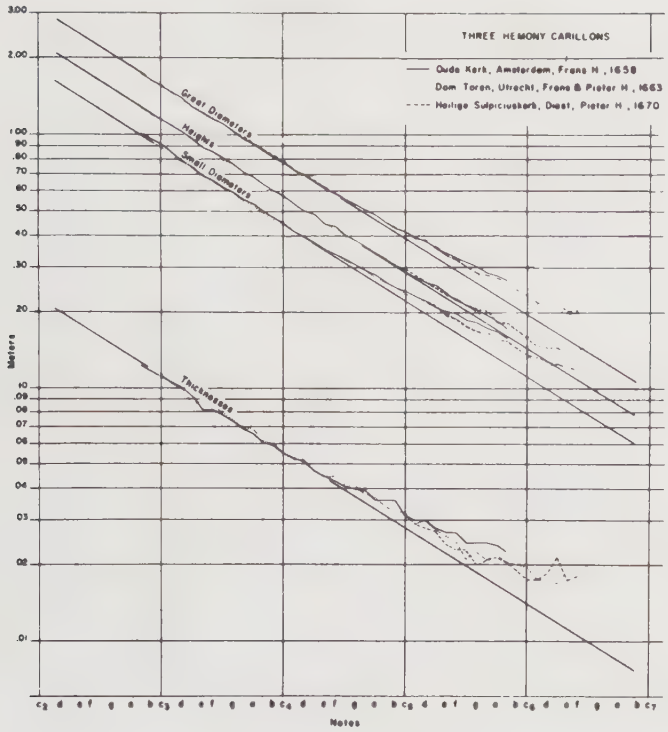

Fig. 8. Measurements of the bells of three Hemony carillons. The solid straight lines represent a 1/f scaling law (from Ref. 5).

Scaling in handbells is less regular than in church bells. We measured the diameter and thickness of handbells by four different bell crafters: Malmark, Schulmerich, Whitechapel, and Petit and Fritsen.[4]. The results are shown in Fig. 9. In the Schulmerich bells, the diameter was found to be inversely proportional to the square root of the frequency. The same was true of the Malmark bells up to about A_6, after which the diameter decreases at a slower rate. The Whitechapel bells follow the same $f^{-1/2}$ scale from about G_3 to D_6, and the Petit and Fritsen bells follow it up to about D_5.[20]

Each bell has a thickness that is appropriate to its diameter and frequency. Thus the thicknesses of individual bells vary considerably around their scaling line. Furthermore, the bells have been tuned individually by thinning them slightly. Quite frequently the thinning is done near the mouth in the region where we measure the thickness.

Fig. 9a

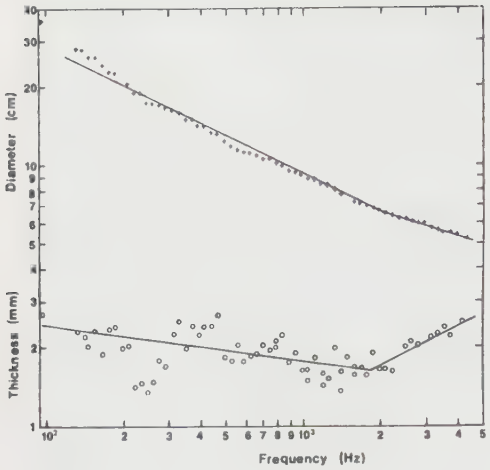

Fig. 9b

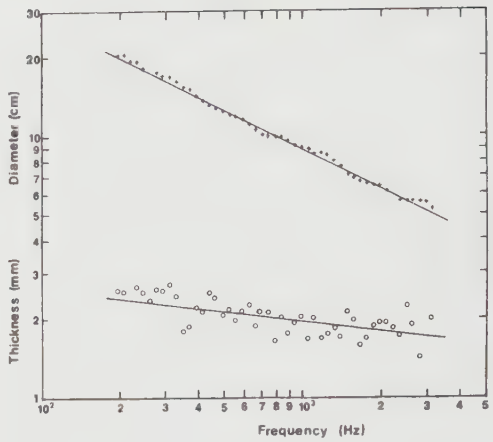

Fig. 9c

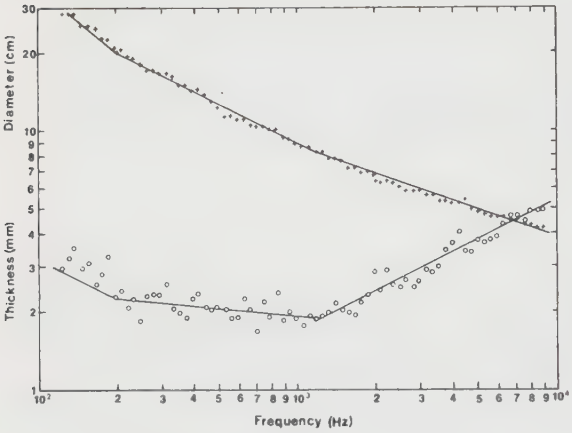

Fig. 9d

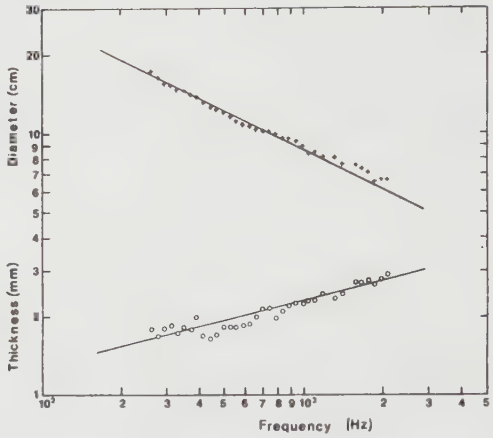

Fig. 9 Scaling in handbells by four different bell crafters: a) Malmark; b) Schulmerich; c) Whitechapel; d) Petit and Fritsen.

It is clear from Fig. 9 that sets of handbells are generally scaled such that the diameter is inversely proportional to $\sqrt{f}$ rather than to f, as are carillon bells. Also, the bells of higher pitch are designed to be a little larger than the general scale. It is not known whether this is done for tonal or mechanical reasons.

There are probably a number of reasons why handbells follow an $f^{-1/2}$ scale rather than the f^{-1} scale found in carillons and church bells. For one thing, this reduces the mass of the larger bells, making them easier to swing by hand. (The mass of a bell will be proportional to hd^2.) Since only two partials are tuned in handbells, the reduced scaling is practical, whereas tuning five or more partials (as required in carillon bells) might be difficult with an $f^{-1/4}$ scaling.

Fig. 10 shows a plot of h/d^2 vs. frequency for bells by the four manufacturers. In spite of local variations, it is quite clear that handbells show the same dependence on h/d^2 as flat circular plates.

MATERIAL

Both handbells and church bells are cast of a bronze alloy of approximately 80% copper and 20% tin, which is often called "bell metal". This metal is easily cast, readily machineable and has low internal loss when it vibrates. Most important of all, however, it has a fairly low speed of sound due to its large density and small elastic modulus compared to steel, for example.

The decay time of a bell, which is a measure of the rate at which the bell loses its energy of vibration, is determined by the internal friction of the bell material and the physical size of the bell. In most bells, the rate of internal energy loss is less than the rate at which

energy is radiated as sound, and so is the surface area and the mass of the bell which mainly determine the decay rate of the various partials and thus the timbre. If the speed of sound is great, the physical size of a bell needs to be large to attain a given pitch, and thus the bell radiates its energy rapidly.

Various attempts have been made to design and fabricate steel bells because of their great strength and low cost. Because the speed of sound is greater in steel than in bronze, steel bells are larger than bronze bells of the same pitch, and thus the sound decays more rapidly. An idea attributed to C. W. Kosten is to drill holes in the bell to decrease its rate of energy loss due to sound radiation and thus sustain the sound. [6] It has also been suggested that using steel for the bells of the highest pitch might improve tonal balance in a carillon.

Careful studies of the influence of alloy composition on timbre have shown that variations in the dimensions and mass of bells influence their sound more strongly than considerable variations in alloy content or impurity content.[21]

CONCLUSION

We have attempted to compare the acoustical properties of tuned handbells with large church bells and carillon bells. There are many similarities but also many differences. Several phenomena, described here for the first time, are under investigation, and we hope to explain them in a future scientific paper.

We thank John Sathoff, Robert Perrin, Richard Peterson, and Richard Ross for their important contributions to this paper. Thanks are also due to Jacob Malta of Malmark for his interest and support, to the owners of handbells who allowed us to measure their bells, and to Richard Litterst for some interesting discussions.

Fig. 10

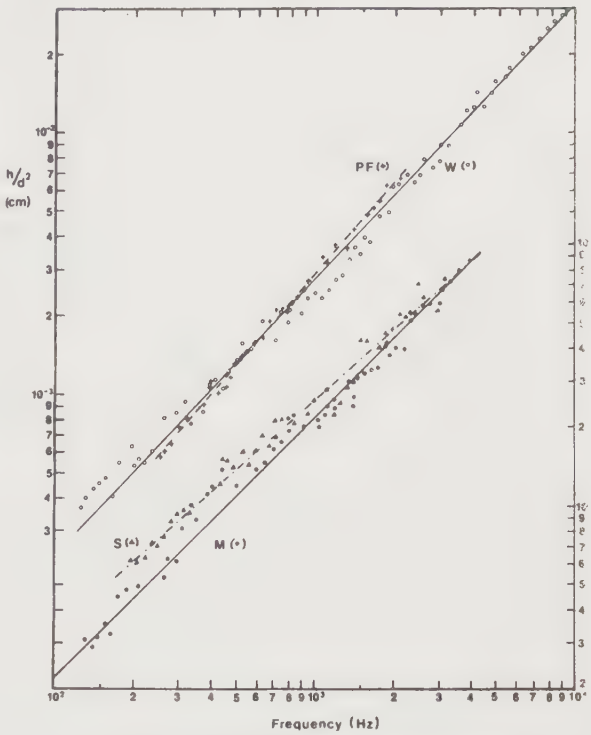

Fig. 10. Ratio of thickness to (diameter)² for handbells by the same bell crafters as in Fig. 9 (identified by initials.)

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His musical activities include playing clarinet and recorder, directing a church choir, singing solos, and ringing handbells. He has sung with the Chicago Symphony Chorus, the Schola Cantorum of Oxford, the Chorus Pro Musica of Boston, and other distinguished choruses. While a visiting researcher at MIT, he was introduced to change ringing on the historic bells of Old North Church in Boston, and thus began a love affair with bells and bell acoustics.

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